Find The Value Of X So That The Function Has The Given Value.

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Understanding how to find the value of x that makes a function equal to a specific value is a fundamental skill in algebra and mathematics as a whole. Whether you're solving equations in school, tackling real-world problems, or preparing for exams, mastering this process is essential. This article will guide you through the steps involved in determining the value of x so that a given function attains a specified output, covering different types of functions and providing practical examples to solidify your understanding.

Understanding the Basics of Functions and Values

Before diving into solving for x, it's important to understand what a function is and how it relates to the values involved.

What Is a Function?

A function is a relationship between a set of inputs and a set of possible outputs where each input is related to exactly one output. It is often written in the form $f(x)$, where x is the input variable, and $f(x)$ is the output.

Given Function and Its Value

When we say "find the value of x so that the function has the given value," it means:
- You are given a function, for example, $f(x) = 3x + 5$.
- You are also given a specific value that the function should output, for example, 20.
- Your goal is to determine the x value(s) that satisfy the condition $f(x) = 20$.

Steps to Find the Value of X for a Given Function and Value

The process of finding x depends on the type of function involved, but the general approach involves algebraic manipulation to isolate x.

Step 1: Write Down the Given Function and the Target Value

Identify the function expression and the value you want the function to output.

Example:
- Function: $f(x) = 2x + 7$
- Target value: 19

Step 2: Set the Function Equal to the Given Value

Create an equation by setting the function equal to the target value.

Example:
- $2x + 7 = 19$

Step 3: Solve for x

Use algebraic methods appropriate for the function type:
- For linear functions, isolate x by performing inverse operations.
- For quadratic functions, rearrange into standard form and solve using factoring, completing the square, or the quadratic formula.
- For other functions, apply appropriate inverse functions.

Example:
- $2x + 7 = 19$
- Subtract 7 from both sides: $2x = 12$
- Divide both sides by 2: $x = 6$

Step 4: Verify the Solution

Plug the value of x back into the original function to confirm it yields the target value.

Example:
- $f(6) = 2(6) + 7 = 12 + 7 = 19$ (which matches the target)

Examples of Finding X in Different Types of Functions

To gain a clearer understanding, let's examine various types of functions and how to find x when given a specific output.

1. Linear Functions

A linear function has the form $f(x) = mx + b$.

Example:
Find x such that $f(x) = 10$, where $f(x) = 3x - 4$.

Solution:
- Set equal to 10: $3x - 4 = 10$
- Add 4 to both sides: $3x = 14$
- Divide both sides by 3: $x = 14/3 \approx 4.67$

Verification:
$f(14/3) = 3(14/3) - 4 = 14 - 4 = 10$

2. Quadratic Functions

Quadratic functions are of the form $f(x) = ax^2 + bx + c$.

Example:
Find x such that $f(x) = 0$, where $f(x) = x^2 - 5x + 6$.

Solution:
- Set equal to 0: $x^2 - 5x + 6 = 0$
- Factor: $(x - 2)(x - 3) = 0$
- Solutions: $x = 2$ or $x = 3$

Verification:
$f(2) = 4 - 10 + 6 = 0$
$f(3) = 9 - 15 + 6 = 0$

3. Rational and Radical Functions

When dealing with rational functions or radicals, extra steps like clearing denominators or squaring may be necessary.

Example:
Find x such that $f(x) = 4$, where $f(x) =$ ⍰ $(x + 1)$.

Solution:
- Set equal to 4: ⍰ $(x + 1) = 4$
- Square both sides: $x + 1 = 16$
- Solve: $x = 15$

Verification:
$f(15) =$ ⍰ $(15 + 1) =$ ⍰ $16 = 4$

Handling Special Cases and Common Challenges

Sometimes, equations may be more complex or involve multiple steps, requiring careful attention.

1. No Solution Cases

Some equations have no real solutions, such as when the algebra leads to a contradiction.

Example:
Find x such that $x^2 + 4x + 5 = 0$.

Discriminant: $\Delta = (4)^2 - 415 = 16 - 20 = -4$

Since the discriminant is negative, there are no real solutions.

2. Multiple Solutions

Quadratic and higher-degree functions often have more than one solution.

Example:
Find x such that $f(x) = 0$, where $f(x) = x^3 - 8$.

Solution:
- Set: $x^3 - 8 = 0$
- $x^3 = 8$
- $x =$ ⍰ $8 = 2$

Only one real solution here, but for quadratic functions, you may have two solutions.

3. Domain Restrictions

Always consider the domain of the function to ensure solutions are valid.

Example:
$f(x) = \sqrt{(x - 3)}$. Find x such that $f(x) = 2$.

Solution:
- Set: $\sqrt{(x - 3)} = 2$
- Square both sides: $x - 3 = 4$
- $x = 7$

Domain restriction:
- The expression under the square root must be ≥ 0: $x - 3 \geq 0 \Rightarrow x \geq 3$
- Since $x = 7$ satisfies $x \geq 3$, it is valid.

Practical Tips for Finding X in Various Contexts

- Always isolate the term containing x first.
- When dealing with quadratic equations, consider factoring, completing the square, or quadratic formula.
- For equations involving radicals, remember to check for extraneous solutions introduced by squaring.
- For rational functions, clear denominators to simplify the equation.
- Verify solutions thoroughly to avoid errors from domain restrictions or extraneous roots.

Conclusion: Mastering the Skill of Finding X for a Given Function Value

The ability to find the value of x so that a function attains a specific value is a core skill in algebra, serving as a foundation for more advanced mathematical concepts. By understanding the structure of different types of functions, applying appropriate algebraic techniques, and verifying your solutions, you can confidently solve a wide array of problems. Practice with diverse functions and problem types will sharpen your skills, making you proficient in this essential area of mathematics.

Remember, the key steps involve setting the function equal to the given value, algebraic manipulation to solve for x, and verification of solutions. With consistent practice, you'll find this process becomes intuitive and an invaluable tool in your mathematical toolkit.

Frequently Asked Questions

Find the value of x so that f(x) = 10, where f(x) = 2x + 4.

To find x, set $2x + 4 = 10$. Subtract 4 from both sides: $2x = 6$. Divide both sides by 2: $x = 3$.

If f(x) = x^2 - 5 and f(x) = 4, what is the value of x?

Set $x^2 - 5 = 4$. Add 5 to both sides: $x^2 = 9$. Take square roots: $x = \pm 3$.

Determine x so that the quadratic function f(x) = 3x^2 + 2x - 1 equals 0.

Set $3x^2 + 2x - 1 = 0$. Use quadratic formula: $x = [-2 \pm \sqrt{(2^2 - 43(-1))}]/(23)$. Calculate discriminant: 4 +

12 = 16. So, x = [-2 ± 4]/6. Therefore, x = (2/6) = 1/3 or x = (-6/6) = -1.

Find x if f(x) = |x - 3| and the function value is 5.

Set |x - 3| = 5. This gives two equations: x - 3 = 5 or x - 3 = -5. Solving: x = 8 or x = -2.

Given the linear function f(x) = -4x + 7, find x when f(x) = 3.

Set -4x + 7 = 3. Subtract 7 from both sides: -4x = -4. Divide both sides by -4: x = 1.

If the function f(x) = (x + 2)/(x - 1) equals 3, what is the value of x?

Set (x + 2)/(x - 1) = 3. Cross-multiplied: x + 2 = 3(x - 1). Expand: x + 2 = 3x - 3. Bring all to one side: x + 2 - 3x + 3 = 0 ⇒ -2x + 5 = 0. Solve: -2x = -5 ⇒ x = 5/2.

Additional Resources

Find The Value Of X So That The Function Has The Given Value

Understanding how to determine the value of x such that a function attains a specific value is a fundamental concept in algebra and calculus. This process involves solving equations where the variable x is unknown, and the goal is to find all possible solutions that satisfy the given condition. Whether you're dealing with linear functions, quadratic functions, polynomial functions, rational functions, or more complex expressions, the core principles remain consistent. This comprehensive guide aims to explore this topic in depth, covering various types of functions, methods of solving, common pitfalls, and practical applications.

Introduction to the Concept

At its core, the problem "Find the value of x so that the function has the given value" asks us to solve an equation of the form:

$$f(x) = c$$

where:
- $f(x)$ is a function (linear, quadratic, polynomial, etc.)
- c is a constant, representing the given value the function should reach.

For example, suppose you are asked:

"Find x such that $f(x) = 5$."

This means you need to solve the equation:

$$f(x) = 5$$

to find all values of x that make the function equal to 5.

Why is this important?
This type of problem appears in numerous real-world contexts:
- Determining input values for a profit function to achieve a target revenue
- Finding time instances when a physical quantity reaches a certain level
- Solving for parameters in engineering to meet specified performance criteria

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Fundamental Approach to the Problem

The general approach involves these steps:

1. Identify the function $f(x)$ and the target value c.
2. Set up the equation $f(x) = c$.
3. Solve the resulting equation for x.
4. Verify the solutions within the context (domain considerations).

Let's break down each step:

Step 1: Understanding the Function and the Given Value

Before solving, it's crucial to understand the nature of $f(x)$:
- Is it linear, quadratic, polynomial, rational, exponential, logarithmic, or composite?
- What is the domain of $f(x)$?
- Are there restrictions that might exclude certain solutions?

Step 2: Formulate the Equation

Replace $f(x)$ with its explicit expression and set it equal to the given constant c.

Example:
Suppose $f(x) = 2x + 3$, and the target value is 7:
$$2x + 3 = 7$$

Step 3: Solve the Equation

Use algebraic methods suitable for the type of equation:
- Linear equations: isolate x
- Quadratic equations: factorization, quadratic formula, completing the square
- Polynomial equations: synthetic division, factoring, or numerical methods
- Rational equations: find common denominators, check for extraneous solutions
- Exponential/logarithmic equations: logarithm properties, exponential rules

Step 4: Verify Solutions

Solutions must be checked against the domain:
- For example, if the original function involves $\sqrt{x}$, then $x \geq 0$.
- For rational functions, ensure the denominator is not zero.
- For solutions introduced during algebraic manipulation (such as squaring both sides), verify that they do not create extraneous solutions.

Solving Different Types of Functions

The method of finding x varies depending on the function type. Here, we delve into the most common cases.

Linear Functions

Form:
$$f(x) = ax + b$$

Equation to solve:
$$ax + b = c$$

Solution:
$$x = \frac{c - b}{a}$$

Example:
Find x so that $3x - 4 = 11$:

$$3x - 4 = 11 \Rightarrow 3x = 15 \Rightarrow x = 5$$

Note: Linear functions are straightforward, with only one solution unless the equation is inconsistent.

Quadratic Functions

Form:
$$f(x) = ax^2 + bx + c$$

Equation to solve:
$$ax^2 + bx + c = d$$

which rearranges to:

$$ax^2 + bx + (c - d) = 0$$

Solution methods:
- Quadratic formula:
$$
x = \frac{-b \pm \sqrt{b^2 - 4a(c - d)}}{2a}
$$
- Factoring (if possible)
- Completing the square

Discriminant analysis:
- $b^2 - 4a(c - d) > 0$: Two real solutions
- $b^2 - 4a(c - d) = 0$: One real solution (repeated root)
- $b^2 - 4a(c - d) < 0$: No real solutions

Example:
Find x such that $x^2 - 4x + 3 = 0$:

$$
x^2 - 4x + 3 = 0
$$
$$
\Rightarrow (x - 1)(x - 3) = 0
$$
$$
\Rightarrow x = 1 \text{ or } x = 3
$$

Polynomial Functions of Higher Degree

For cubic, quartic, and higher-degree polynomials:

- Use factoring, synthetic division, or Rational Root Theorem to find roots.
- Numerical methods like Newton-Raphson may be employed when algebraic solutions are complex or not feasible.
- Graphing can help identify approximate solutions before refining.

Example:
Solve $x^3 - 6x^2 + 11x - 6 = 0$:

- Factorization:

$$
(x - 1)(x - 2)(x - 3) = 0
$$

- Solutions: $x=1, 2, 3$

Rational Functions

Form:

$$f(x) = \frac{P(x)}{Q(x)}$$

Equation to solve:

$$\frac{P(x)}{Q(x)} = c$$

Solution steps:

1. Cross-multiplied form:

$$
P(x) = c \cdot Q(x)
$$

2. Solve the polynomial equation $P(x) - c \cdot Q(x) = 0$.
3. Check for extraneous solutions where $Q(x) = 0$.

Example:
Find x such that $\frac{2x + 1}{x - 3} = 4$:

$$
2x + 1 = 4(x - 3)
$$

$$
2x + 1 = 4x - 12
$$

$$
-2x = -13
$$

$$
x = \frac{13}{2}
$$

Verify that $x \neq 3$ (denominator zero), which it isn't, so solution is valid.

Exponential and Logarithmic Functions

Exponential form:
$$f(x) = a^x$$

Logarithmic form:
$$f(x) = \log_a x$$

To solve for x:

- When $a^x = c$:
$$
x = \log_a c
$$
- When $\log_a x = c$:
$$
x = a^c
$$

Example:
Find x such that $2^x = 16$:

$$
x = \log_2 16 = 4
$$

or

$$
x = \log_2 16 = 4
$$

Note:
- The base a must be positive and not equal to 1.
- Domain restrictions: $x > 0$ for $\log_a x$.

Special Techniques and Considerations

When solving equations involving the function value, be alert to various complexities:

Handling Absolute Values

Equation:
$$|f(x)| = c$$

Solution involves splitting into two cases:

- $f(x) = c$
- $f(x) = -c$

Dealing with Domain Restrictions

Always verify the solutions within the domain of the original function:

- For square roots $\sqrt{x}$, require $x \geq 0$.
- For logarithms $\log x$, require $x > 0$.
- For rational functions, ensure denominator isn't zero.

Extraneous Solutions

Algebraic manipulations like squaring both sides can introduce extraneous solutions. Always substitute solutions back into the original equation to verify.

Use of Graphing Tools

Graphing the function $f(x)$ and the horizontal line $y = c$ provides visual insight:

- Intersection points correspond to solutions.
- Helps identify approximate solutions and the number of solutions.

Practical Examples and Applications

Let's explore some real-world scenarios where finding the value

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