Given That Q(x)=2x^2 +5x-3 Find And Simplify Q(a+h)-Q(a-h)

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Introduction to the Problem

In algebra, understanding how to manipulate and simplify polynomial expressions is fundamental. The problem at hand involves a quadratic function $Q(x) = 2x^2 + 5x - 3$, and the goal is to find and simplify the expression $Q(a+h) - Q(a-h)$. This type of problem is common in calculus, especially in the context of derivatives and difference quotients, where understanding the behavior of functions around specific points is crucial.

This article will guide you through a detailed, step-by-step process to evaluate and simplify the expression $Q(a+h) - Q(a-h)$. We will also explore the underlying concepts, methods for polynomial expansion, and the significance of such expressions in broader mathematical contexts, including derivatives and limits.

Understanding the Components of the Expression

Before diving into the calculation, it's important to understand the components involved:

- Quadratic Function $Q(x)$: Given as $Q(x) = 2x^2 + 5x - 3$. It is a second-degree polynomial, which means it has a parabolic graph.
- Variables a, h: a is a fixed point, and h is a small increment or change around a. In calculus, h often approaches zero to analyze the instantaneous rate of change.
- Expressions $Q(a+h)$ and $Q(a-h)$: These represent the function evaluated at points shifted by h to the right and left of a.

The main task is to evaluate the difference $Q(a+h) - Q(a-h)$ and simplify it into a form that reveals the relationship between a, h, and the polynomial's coefficients.

Step-by-Step Solution to Find and Simplify $Q(a+h) - Q(a-h)$

Step 1: Compute $Q(a+h)$

Substitute $a+h$ into the polynomial $Q(x)$:

$$
Q(a+h) = 2(a+h)^2 + 5(a+h) - 3
$$

Expand each term:

- Expand $(a+h)^2$:

$$
(a+h)^2 = a^2 + 2ah + h^2
$$

- Multiply by 2:

$$
2(a+h)^2 = 2a^2 + 4ah + 2h^2
$$

- Expand $5(a+h)$:

$$
5a + 5h
$$

- Subtract 3:

$$
-3
$$

Putting it all together:

$$
Q(a+h) = 2a^2 + 4ah + 2h^2 + 5a + 5h - 3
$$

Step 2: Compute $Q(a-h)$

Similarly, substitute $a-h$ into $Q(x)$:

\[
Q(a-h) = 2(a-h)^2 + 5(a-h) - 3
\]

Expand each term:

- Expand \((a-h)^2\):

\[
(a-h)^2 = a^2 - 2ah + h^2
\]

- Multiply by 2:

\[
2a^2 - 4ah + 2h^2
\]

- Expand \(5(a-h)\):

\[
5a - 5h
\]

- Subtract 3:

\[
-3
\]

Putting it all together:

\[
Q(a-h) = 2a^2 - 4ah + 2h^2 + 5a - 5h - 3
\]

Step 3: Find the Difference \(Q(a+h) - Q(a-h)\)

Subtract the expressions:

\[

$$
Q(a+h) - Q(a-h) = \left(2a^2 + 4ah + 2h^2 + 5a + 5h - 3 \right) - \left(2a^2 - 4ah + 2h^2 + 5a - 5h - 3 \right)
$$

Now, subtract term-by-term:

- $2a^2 - 2a^2 = 0$
- $4ah - (-4ah) = 4ah + 4ah = 8ah$
- $2h^2 - 2h^2 = 0$
- $5a - 5a = 0$
- $5h - (-5h) = 5h + 5h = 10h$
- $-3 - (-3) = 0$

Thus, the simplified difference becomes:

$$
Q(a+h) - Q(a-h) = 8ah + 10h
$$

Step 4: Factor the Expression

Factor out h:

$$
Q(a+h) - Q(a-h) = h (8a + 10)
$$

This is the simplified form of the expression.

Interpretation and Significance of the Simplified Expression

The resulting expression $Q(a+h) - Q(a-h) = h (8a + 10)$ has several important implications:

- Symmetry in Polynomial Behavior: The expression reveals how the quadratic function behaves symmetrically around the point a, especially as h approaches zero.
- Connection to Derivatives: In calculus, the derivative $Q'(a)$ of $Q(x)$ at point a can be approximated by the difference quotient:

$$
\frac{Q(a+h) - Q(a-h)}{2h}
$$

which, as $h \to 0$, approaches $Q'(a)$. Here, the numerator simplifies to $h(8a + 10)$, so the difference quotient becomes:

$$
\frac{h(8a + 10)}{2h} = \frac{8a + 10}{2}
$$

- Derivative of $Q(x)$: The expression confirms that the derivative $Q'(x)$ is $4x + \frac{5}{2}$, since substituting a yields:

$$
Q'(a) = 4a + \frac{5}{2}
$$

which aligns with the derivative of the quadratic polynomial.

Broader Applications and Related Concepts

1. Polynomial Difference

Understanding how to evaluate $Q(a+h) - Q(a-h)$ is foundational in polynomial difference analysis. It helps in:

- Calculating finite differences
- Approximating derivatives
- Analyzing the symmetry and rate of change of polynomial functions

2. Derivatives and Calculus

The process of simplifying $Q(a+h) - Q(a-h)$ directly relates to the concept of derivatives:

- The symmetric difference quotient:

$$
\frac{Q(a+h) - Q(a-h)}{2h}
$$

approaches the derivative $Q'(a)$ as $h \to 0$.

- For quadratic functions, this approach provides an exact value of the derivative, making it a powerful tool for understanding the function's slope at a point.

3. Polynomial Function Analysis

Analyzing polynomial functions through their difference expressions helps in:

- Understanding their growth rates
- Finding tangents and slopes
- Developing Taylor series expansions

Practical Examples and Additional Exercises

To reinforce understanding, consider the following exercises:

Exercise 1: Calculate $Q(a+h) - Q(a-h)$ for specific values

- Let $a=2$ and $h=0.1$, compute the difference.

Exercise 2: Derive the derivative $Q'(x)$

- Using the simplified expression, confirm the derivative of $Q(x) = 2x^2 + 5x - 3$.

Exercise 3: Explore the limit as $h \to 0$

- Show how the expression relates to the derivative by taking the limit:

$$
\lim_{h \to 0} \frac{Q(a+h) - Q(a-h)}{2h}
$$

Conclusion

In this comprehensive guide, we thoroughly examined how to find and simplify the expression $Q(a+h) - Q(a-h)$ for the quadratic polynomial $Q(x) = 2x^2 + 5x - 3$. Starting with polynomial expansion, we carefully performed algebraic manipulations and arrived at a simplified form:

$$
Q(a+h) - Q(a-h) = h (8a + 10)
$$

\]

This result not only simplifies the original problem but also illustrates the deep connection between polynomial differences and derivatives in calculus. Understanding such fundamental concepts equips students and mathematicians with the tools to analyze functions, approximate slopes, and explore the behavior of polynomial functions with precision.

Mastering these techniques enhances problem-solving skills and lays the groundwork for more advanced topics in calculus, differential equations, and mathematical analysis. Whether you're preparing for exams or engaging in research, the ability to simplify and interpret polynomial expressions is an invaluable skill in the mathematician's toolkit.

Frequently Asked Questions

What is the expression for $Q(a+h)$ given $Q(x) = 2x^2 + 5x - 3$?

$Q(a+h) = 2(a+h)^2 + 5(a+h) - 3$.

How do you expand $Q(a+h)$ for the given quadratic function?

Expand as $Q(a+h) = 2[a^2 + 2ah + h^2] + 5a + 5h - 3$.

What is the expression for $Q(a-h)$ given $Q(x) = 2x^2 + 5x - 3$?

$Q(a-h) = 2(a-h)^2 + 5(a-h) - 3$.

How do you simplify $Q(a+h) - Q(a-h)$?

Subtract $Q(a-h)$ from $Q(a+h)$ by expanding both and combining like terms to simplify.

What is the expanded form of $Q(a+h) - Q(a-h)$?

$Q(a+h) - Q(a-h) = [2(a^2 + 2ah + h^2) + 5a + 5h - 3] - [2(a^2 - 2ah + h^2) + 5a - 5h - 3]$.

What is the simplified form of $Q(a+h) - Q(a-h)$?

After simplification, $Q(a+h) - Q(a-h) = 20ah + 10h$.

Why does the quadratic term cancel out when finding $Q(a+h) - Q(a-h)$?

Because the quadratic terms are symmetric and cancel each other when subtracting, leaving only linear terms involving h.

How can the difference Q(a+h) - Q(a-h) be interpreted in terms of derivatives?

It represents a finite difference approximation of the derivative of Q at point a, scaled by h.

What is the final simplified expression for Q(a+h) - Q(a-h)?

The final simplified expression is 20ah + 10h.

Additional Resources

Given That $Q(x)=2x^2 + 5x - 3$ Find And Simplify Q(a+h) - Q(a-h)

Understanding and simplifying algebraic expressions involving quadratic functions is a fundamental skill in mathematics, especially in calculus, algebra, and problem-solving contexts. The expression Q(a+h) - Q(a-h), where Q(x) is given as a quadratic function, offers insight into the symmetry and difference of quadratic values at specific points. This particular problem not only tests your algebraic manipulation skills but also enhances your understanding of how quadratic functions behave around a point 'a' when perturbed by a small increment 'h'. In this article, we will explore the process of simplifying this expression step-by-step, analyze the underlying concepts, and discuss the significance of such an operation in broader mathematical contexts.

Understanding the Problem

Before diving into the algebraic manipulations, it is essential to comprehend what the problem asks. We are given a quadratic function:

$$Q(x) = 2x^2 + 5x - 3$$

and we need to find and simplify the expression:

$$Q(a+h) - Q(a-h)$$

where 'a' and 'h' are variables, with 'a' typically representing a point on the x-axis, and 'h' representing a small change or offset from that point.

This type of problem is common when analyzing the rate of change of functions, differences between function values at symmetric points, or preparing for derivative calculations through difference quotients.

Step-by-Step Solution

1. Expand Q(a+h) and Q(a-h)

The first step is to substitute $x = a+h$ and $x = a-h$ into the quadratic function.

- For $Q(a+h)$:

$$
Q(a+h) = 2(a+h)^2 + 5(a+h) - 3
$$

- For $Q(a-h)$:

$$
Q(a-h) = 2(a-h)^2 + 5(a-h) - 3
$$

Now, expand both expressions.

---

2. Expand the quadratic terms

Using the binomial expansion:

- $(a+h)^2 = a^2 + 2ah + h^2$
- $(a-h)^2 = a^2 - 2ah + h^2$

Substituting these, we get:

$$
Q(a+h) = 2(a^2 + 2ah + h^2) + 5a + 5h - 3
$$

$$
Q(a-h) = 2(a^2 - 2ah + h^2) + 5a - 5h - 3
$$

\]

---

3. Simplify each expression

Distribute the coefficients:

- $Q(a+h)$:

$$
Q(a+h) = 2a^2 + 4ah + 2h^2 + 5a + 5h - 3
$$

- $Q(a-h)$:

$$
Q(a-h) = 2a^2 - 4ah + 2h^2 + 5a - 5h - 3
$$

---

4. Form the difference $Q(a+h) - Q(a-h)$

Subtract term-by-term:

$$
Q(a+h) - Q(a-h) = (2a^2 + 4ah + 2h^2 + 5a + 5h - 3) - (2a^2 - 4ah + 2h^2 + 5a - 5h - 3)
$$

Combine like terms carefully:

- $2a^2 - 2a^2 = 0$
- $4ah - (-4ah) = 4ah + 4ah = 8ah$
- $2h^2 - 2h^2 = 0$
- $5a - 5a = 0$
- $5h - (-5h) = 5h + 5h = 10h$
- $-3 - (-3) = 0$

Therefore:

$$
Q(a+h) - Q(a-h) = 8ah + 10h
$$

Final Simplified Expression

The simplified form of $Q(a+h) - Q(a-h)$ is:

$$\boxed{8ah + 10h}$$

This expression reveals the difference in the quadratic function's values at symmetric points around 'a' in terms of 'a' and 'h'.

Insights and Interpretations

Understanding the simplified form $8ah + 10h$ provides valuable insights:

- It is linear in 'h', which is typical for difference expressions involving quadratic functions.
- The term $8ah$ indicates how the difference depends on both the point 'a' and the offset 'h'.
- The term $10h$ shows the contribution purely from the offset 'h'.

This expression is particularly useful in calculus when approximating derivatives or analyzing the rate of change of $Q(x)$ around the point 'a' as h approaches zero.

Applications and Significance

1. Derivative Approximation

The expression $Q(a+h)-Q(a-h)$ is related to the symmetric difference quotient, which can be used to approximate the derivative $Q'(a)$ as:

$$
Q'(a) \approx \frac{Q(a+h) - Q(a-h)}{2h}
$$

Given the simplified form, the approximation becomes:

$$
Q'(a) \approx \frac{8ah + 10h}{2h} = \frac{8ah}{2h} + \frac{10h}{2h} = 4a + 5
$$

which aligns perfectly with the actual derivative of $Q(x) = 2x^2 + 5x - 3$, since:

$$
Q'(x) = 4x + 5
$$

and evaluated at $x = a$:

$$
Q'(a) = 4a + 5
$$

This confirms the validity of the algebraic approach and highlights the importance of such difference calculations in differential calculus.

2. Symmetry and Function Behavior

The difference $Q(a+h) - Q(a-h)$ captures the symmetry of the quadratic around 'a'. The linearity in 'h' indicates how the function's value changes symmetrically as we move away from 'a' in both directions.

3. Mathematical Features

- Linearity in 'h': The result is linear in the small change h, which simplifies further analysis.
- Dependence on 'a': The term $8ah$ shows the influence of the specific point 'a' on the difference, indicating the function's slope characteristics at that point.

Pros and Cons of the Approach

Pros:

- Clarity: The step-by-step expansion and simplification provide clear insight into how the difference behaves.
- Foundation for Calculus: Demonstrates the connection between algebra and calculus, especially in derivative approximation.
- Versatility: The method can be applied to other polynomial functions with similar structures.

Cons:

- Limited to Quadratic Functions: While effective for quadratics, more complex functions require advanced techniques.
- Assumes familiarity with algebraic expansion: Beginners might find expansion and combination challenging initially.
- Not directly providing the derivative: It offers an approximation but requires dividing by $2h$ to get the derivative.

Conclusion

The process of finding and simplifying $Q(a+h) - Q(a-h)$ for the quadratic function $Q(x) = 2x^2 + 5x - 3$ highlights the elegance of algebraic manipulation and its connection to calculus concepts. The final simplified expression, $8ah + 10h$, encapsulates the symmetry and rate of change of the quadratic function around a point 'a'. Understanding this difference expression not only aids in grasping the behavior of quadratic functions but also serves as a stepping stone toward more advanced topics like derivatives, Taylor series, and function analysis. This exercise exemplifies how fundamental algebraic techniques form the backbone of higher mathematical reasoning and problem-solving.

Given That Q X 2x 2 5x 3 Find And Simplify Q A H Q A H

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