How Do You Solve A Quadratic Inequality Step By Step Example?

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Quadratic inequalities are a fundamental part of algebra that often challenge students due to their multiple solution steps and the need for careful analysis. Whether you're working on a problem for homework or preparing for an exam, understanding how to solve quadratic inequalities systematically is crucial. This article will guide you through the process step by step, using clear explanations and a practical example to help you master this skill.

Understanding Quadratic Inequalities

Before diving into the solving process, it’s important to understand what quadratic inequalities are and how they differ from quadratic equations.

What Is a Quadratic Inequality?

A quadratic inequality is an inequality that involves a quadratic expression—an algebraic expression where the highest degree of the variable (usually x) is 2—being related to zero through inequality symbols like <, >, ≤, or ≥.

For example:
- $x^2 - 5x + 6 > 0$
- $2x^2 + 3x - 4 \leq 0$

These inequalities describe ranges of x values that satisfy the inequality condition, rather than a single solution.

Difference Between Quadratic Equations and Inequalities

While quadratic equations set the expression equal to zero, quadratic inequalities involve inequalities that require us to find all x-values where the quadratic expression is either greater than, less than, or equal to zero.

Step-by-Step Process for Solving Quadratic Inequalities

The general approach to solving quadratic inequalities involves several key steps. We will outline each step and then demonstrate the process with an example.

Step 1: Bring the Inequality to Standard Form

Ensure the quadratic inequality is written in the form:
$$ax^2 + bx + c \; \text{(inequality symbol)} \; 0$$
This standard form makes it easier to analyze and solve.

Step 2: Find the Critical Points (Roots of the Quadratic)

Solve the quadratic equation $ax^2 + bx + c = 0$ to find the critical points where the quadratic expression equals zero. These roots divide the number line into intervals that will be tested later.

- Use the quadratic formula:
$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$
- Or factor if possible.

Step 3: Determine the Sign of the Quadratic in Each Interval

The roots partition the real number line into intervals. To find where the quadratic expression satisfies the inequality, select a test point from each interval and substitute it into the quadratic expression to check its sign.

Step 4: Write the Solution Set Based on the Inequality

Using the sign analysis, determine which intervals satisfy the inequality. Remember:

- For $ax^2 + bx + c > 0$, the solution is where the quadratic is positive.
- For $ax^2 + bx + c < 0$, the solution is where it is negative.
- For ≥ 0 or ≤ 0, include the roots where the quadratic equals zero.

Step 5: Express the Solution

Express the solution set in interval notation, set builder notation, or graphically, depending on the context.

Practical Example: Solving a Quadratic Inequality Step By Step

Let’s walk through an example to illustrate the entire process.

Example:
Solve the quadratic inequality:
$$x^2 - 4x - 5 \leq 0$$

Step 1: Bring into standard form
The inequality is already in standard form:
$$x^2 - 4x - 5 \leq 0$$

Step 2: Find the roots of the quadratic
Solve $x^2 - 4x - 5 = 0$:

- Use quadratic formula:
$$
x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 1 \times (-5)}}{2 \times 1} = \frac{4 \pm \sqrt{16 + 20}}{2} = \frac{4 \pm \sqrt{36}}{2}
$$

- Simplify:
$$
x = \frac{4 \pm 6}{2}
$$

- Roots:
$$
x = \frac{4 + 6}{2} = \frac{10}{2} = 5
$$
$$
x = \frac{4 - 6}{2} = \frac{-2}{2} = -1
$$

Step 3: Analyze the sign of the quadratic in each interval
The roots split the number line into three intervals:

- $(-\infty, -1)$
- $(-1, 5)$
- $(5, \infty)$

Choose test points from each interval:

- For $(-\infty, -1)$, pick $x = -2$:
$$
(-2)^2 - 4(-2) - 5 = 4 + 8 - 5 = 7 > 0
$$

- For $(-1, 5)$, pick $x = 0$:
$$
0 - 0 - 5 = -5 < 0
$$

- For $(5, \infty)$, pick $x=6$:
$$

36 - 24 - 5= 7 > 0
\]

Step 4: Determine solution based on the inequality
Since the original inequality is \(\leq 0\), we are interested in where the quadratic is less than or equal to zero. The quadratic is negative between the roots and equals zero at the roots.

From the sign analysis:

- The quadratic is positive outside the roots.
- The quadratic is negative between the roots.

Therefore, the solution set is:

\[
[-1, 5]
\]

Step 5: Final solution in interval notation
\[
\boxed{[-1, 5]}
\]

This means all x-values between -1 and 5, inclusive, satisfy the inequality \(x^2 - 4x - 5 \leq 0\).

Additional Tips for Solving Quadratic Inequalities

- Always verify whether your quadratic can be factored easily; if yes, factoring can be faster than using the quadratic formula.
- Remember to include the roots in your solution if the inequality involves \(\leq\) or \(\geq\).
- Be cautious with the direction of the inequality when multiplying or dividing by negative numbers during the process.
- Use graphing as a visual aid to confirm your solution, especially for complex inequalities.

Conclusion

Solving quadratic inequalities might seem daunting at first, but by following the systematic steps—standardizing the inequality, finding roots, testing signs, and carefully analyzing the solution intervals—you can approach any problem with confidence. Practice with different examples to strengthen your understanding and recognize patterns for quicker solutions. Mastery of this skill not only helps in algebra but also builds a strong foundation for more advanced mathematics topics.

Frequently Asked Questions

What is the first step to solve a quadratic inequality like $x^2 - 5x + 6 > 0$?

The first step is to factor the quadratic expression, if possible. In this case, factor $x^2 - 5x + 6$ into $(x - 2)(x - 3)$.

After factoring the quadratic expression, what do you do next when solving a quadratic inequality?

Next, determine the critical points by setting each factor equal to zero: $x - 2 = 0$ and $x - 3 = 0$, giving $x = 2$ and $x = 3$. These points divide the number line into intervals.

How do you test the intervals to find where the inequality holds true?

Choose a test point from each interval created by the critical points and substitute it back into the factored inequality to check if the expression is positive or negative.

What is the solution set for the inequality $(x - 2)(x - 3) > 0$?

The solution set is where the product is positive, which occurs when $x < 2$ or $x > 3$. So, the solution is $(-\infty, 2) \cup (3, \infty)$.

How do you interpret the solution when solving a quadratic inequality step by step?

Interpreting the solution involves understanding the intervals where the inequality holds true, based on the sign of the factored expression, and expressing the solution in interval notation or inequalities accordingly.

Additional Resources

How Do You Solve A Quadratic Inequality Step By Step Example?

Quadratic inequalities are fundamental concepts in algebra that often challenge students due to their layered approach involving both algebraic manipulations and understanding of solution regions. Mastering how to solve a quadratic inequality step by step is essential for progressing in algebra, and it provides a foundation for more complex mathematical topics. This comprehensive guide will walk you through the process, illustrating with detailed examples, and offering tips to ensure clarity and confidence.

Understanding Quadratic Inequalities

Before diving into solving methods, it's crucial to grasp what quadratic inequalities are and how they differ from quadratic equations.

Definition

A quadratic inequality is an inequality involving a quadratic expression, typically written in the form:

$$ ax^2 + bx + c \ \text{(relation)} \ 0 $$

where:
- a, b, c are constants with $a \neq 0$,
- The relation can be $>, <, \geq, \leq$.

Examples:
- $x^2 - 4x + 3 > 0$
- $2x^2 + 3x \leq 5$

Difference from Quadratic Equations

Quadratic equations are specific cases where the inequality is replaced with an equality:

$$ ax^2 + bx + c = 0 $$

Solutions to quadratic inequalities involve finding all x values that make the inequality true, which typically results in an interval or union of intervals.

Step-by-Step Process to Solve a Quadratic Inequality

The general approach involves several key steps:

1. Rewrite the inequality in standard form.
2. Find the roots of the corresponding quadratic equation.
3. Determine the sign of the quadratic expression in the intervals defined by these roots.
4. Choose the solution set based on the inequality symbol.

Let's explore each step in detail, using a concrete example:

Example: Solve the inequality $x^2 - 5x + 6 > 0$.

Step 1: Rewrite the inequality in standard form

Ensure the quadratic expression is on one side and zero on the other:

$$ x^2 - 5x + 6 > 0 $$

This is already in standard form. The goal here is to analyze when this quadratic is greater than zero.

Step 2: Find the roots of the quadratic equation

Set the quadratic equal to zero:

$$ x^2 - 5x + 6 = 0 $$

Solve for x:

- Factorization method:

Identify two numbers that multiply to 6 and add to -5:

$$ -2 \text{ and } -3 $$

Expressed as factors:

$$ (x - 2)(x - 3) = 0 $$

Thus, roots are:

$$ x = 2, 3 $$

- Alternative methods: Quadratic formula or completing the square, but factoring is easiest here.

Note: These roots partition the real number line into intervals:

- $(-\infty, 2)$
- $(2, 3)$
- $(3, \infty)$

Step 3: Determine the sign of the quadratic expression in each interval

Since the quadratic is factored as $(x - 2)(x - 3)$, its sign depends on the signs of these factors in each interval.

Method:

- Pick a test point in each interval.
- Plug the test point into the factored form.
- Record whether the quadratic is positive or negative there.

Intervals and test points:

- For $(-\infty, 2)$, choose $x = 0$:
$$(0 - 2)(0 - 3) = (-2)(-3) = 6 > 0$$
- For $(2, 3)$, choose $x = 2.5$:
$$(2.5 - 2)(2.5 - 3) = (0.5)(-0.5) = -0.25 < 0$$
- For $(3, \infty)$, choose $x = 4$:
$$(4 - 2)(4 - 3) = (2)(1) = 2 > 0$$

Summary:

Interval	Test Point	Sign of $(x - 2)(x - 3)$	Quadratic Sign
$(-\infty, 2)$	0	$(+)(+)$	Positive
$(2, 3)$	2.5	$(+)(-)$	Negative
$(3, \infty)$	4	$(+)(+)$	Positive

Step 4: Interpret the signs for the inequality

Recall the original inequality:

$$x^2 - 5x + 6 > 0$$

which translates to:

$$(x - 2)(x - 3) > 0$$

From the sign analysis:

- The quadratic is positive on $(-\infty, 2)$ and $(3, \infty)$.
- It is negative between 2 and 3.

Since the inequality is strict (> 0), the solution set includes all x where the quadratic is positive, excluding the roots because at the roots the expression equals zero.

Solution:

$$ (-\infty, 2) \cup (3, \infty) $$

Special Cases and Additional Tips

1. When the inequality includes equality ($\geq$ or $\leq$)

- Include the roots if the inequality is ≥ 0 or ≤ 0.
- For example, if solving $x^2 - 5x + 6 \geq 0$, the solution set becomes:

$$ (-\infty, 2] \cup [3, \infty) $$

2. When the quadratic coefficient a is negative

- The parabola opens downward.
- The sign analysis remains the same, but the intervals where the quadratic is positive or negative will be inverted accordingly.

3. Dealing with inequalities involving $<$ or $>$

- Use strict inequalities to exclude roots where the quadratic equals zero.
- Use $\geq$ or $\leq$ to include roots in the solution.

4. When the quadratic cannot be factored easily

- Use the quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

- Then analyze the sign of the quadratic using the roots obtained.

Additional Example: Solving a More Complex

Quadratic Inequality

Example: Solve $3x^2 - 4x - 1 \leq 0$.

Step 1: Write in standard form:

$$3x^2 - 4x - 1 \leq 0$$

Step 2: Find the roots using the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 3 \times (-1)}}{2 \times 3}$$

Calculate discriminant:

$$\Delta = 16 - (-12) = 16 + 12 = 28$$

Compute roots:

$$x = \frac{4 \pm \sqrt{28}}{6} = \frac{4 \pm 2 \sqrt{7}}{6} = \frac{2 \pm \sqrt{7}}{3}$$

Approximate roots:

- $x \approx \frac{2 - 2.6458}{3} \approx -0.215$
- $x \approx \frac{2 + 2.6458}{3} \approx 1.5486$

Step 3: Sign analysis:

- Since $a = 3 > 0$, the parabola opens upward.
- The quadratic is ≤ 0 between the roots.

Step 4: Solution set:

$$\left[\frac{2 - \sqrt{7}}{3}, \frac{2 + \sqrt{7}}{3} \right]$$

because the quadratic is less than or equal to zero between roots, including the roots (since inequality is $\leq$).

Common Mistakes and How to Avoid Them

- Ignoring the direction of the parabola: Remember that the sign of a determines whether the parabola opens upward or downward, which affects the interpretation.
- Forgetting to include or exclude roots appropriately: For inequalities with $\geq$ or $\leq$, roots are included; for strict inequalities, they are excluded.
- Mis

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