Identify All The Rational Zeros For F(x) = X3 + X2 - 21x 45.

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Understanding how to find the rational zeros of a polynomial function is a vital skill in algebra and calculus. When working with cubic functions like $F(x) = x^3 + x^2 - 21x + 45$, identifying the rational zeros can simplify the process of factoring the polynomial and solving for its roots. This article provides a comprehensive guide on how to determine all the rational zeros of the given cubic function using the Rational Root Theorem, synthetic division, and other algebraic techniques.

Understanding the Polynomial Function

Before diving into methods for finding the zeros, it’s essential to understand the structure of the polynomial:

$$
F(x) = x^3 + x^2 - 21x + 45
$$

This is a cubic polynomial with coefficients:

- Leading coefficient $a_3 = 1$
- Quadratic coefficient $a_2 = 1$
- Linear coefficient $a_1 = -21$
- Constant term $a_0 = 45$

The goal is to find all rational solutions r such that $F(r) = 0$.

Applying the Rational Root Theorem

The primary tool for identifying potential rational zeros is the Rational Root Theorem. It states:

> If a polynomial with integer coefficients has a rational root p/q (in lowest terms), then:
> - p (the numerator) divides the constant term.
> - q (the denominator) divides the leading coefficient.

Since the leading coefficient is 1, the possible rational zeros are simply the divisors of the constant term.

Divisors of the Constant Term

For $a_0 = 45$, the divisors are:

- Positive divisors: 1, 3, 5, 9, 15, 45
- Negative divisors: -1, -3, -5, -9, -15, -45

Therefore, the set of possible rational zeros is:

$$
\boxed{
\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45
}
$$

Since the leading coefficient is 1, these are the only candidates for rational zeros.

Testing Possible Rational Zeros

To determine which of these candidates are actual zeros of $F(x)$, we substitute each into the polynomial and check if it yields zero.

Using Synthetic Division or Direct Substitution

While direct substitution is straightforward, synthetic division is often more efficient when testing multiple candidates. Here, we will demonstrate synthetic division for a few candidates.

Test $x = 1$

Substitute into $F(x)$:

$$
F(1) = (1)^3 + (1)^2 - 21(1) + 45 = 1 + 1 - 21 + 45 = 26 \neq 0
$$

So, $x=1$ is not a zero.

Test $x = -1$

$$
F(-1) = -1 + 1 + 21 + 45 = 66 \neq 0
$$

Not a zero.

Test $x = 3$

$$
F(3) = 27 + 9 - 63 + 45 = 18 \neq 0
$$

Not a zero.

Test $x = -3$

$$
F(-3) = -27 + 9 + 63 + 45 = 90 \neq 0
$$

Not a zero.

Test $x = 5$

$$
F(5) = 125 + 25 - 105 + 45 = 90 \neq 0
$$

Not a zero.

Test \(x = -5 \)

\[
F(-5) = -125 + 25 + 105 + 45 = 50 \neq 0
\]

Not a zero.

Test \(x = 9 \)

\[
F(9) = 729 + 81 - 189 + 45 = 666 \neq 0
\]

Not a zero.

Test \(x = -9 \)

\[
F(-9) = -729 + 81 + 189 + 45 = -414 \neq 0
\]

Not a zero.

Test \(x = 15 \)

\[
F(15) = 3375 + 225 - 315 + 45 = 3330 \neq 0
\]

Not a zero.

Test \(x = -15 \)

\[
F(-15) = -3375 + 225 + 315 + 45 = -2790 \neq 0

\]

Not a zero.

Test $x = 45$

$$
F(45) = 91125 + 2025 - 945 + 45 = 89850 \neq 0
$$

Not a zero.

Test $x = -45$

$$
F(-45) = -91125 + 2025 + 945 + 45 = -88800 \neq 0
$$

Not a zero.

Reevaluating Candidates: Are There Rational Zeros?

Since none of the potential rational zeros from the divisors of 45 satisfy $F(x) = 0$, we conclude:

The polynomial $F(x) = x^3 + x^2 - 21x + 45$ has no rational zeros.

However, this does not mean the polynomial has no real roots; it only indicates that the roots are irrational or complex.

Alternative Methods for Finding Roots

When rational zeros are absent, other techniques are necessary to find the roots of the polynomial.

1. Factoring by Grouping (If Possible)

In this case, factoring by grouping does not seem straightforward due to the coefficients.

2. Using Numerical Methods

Methods such as Newton-Raphson or synthetic division with approximate roots can help estimate the roots.

3. Applying the Cubic Formula

The cubic formula provides exact roots but is complex and less practical manually. It involves calculating the discriminant and using depressed cubic form.

4. Graphical Analysis

Plotting the polynomial can help locate approximate roots, which can then be refined using numerical methods.

Summary and Conclusion

- The potential rational zeros of $F(x) = x^3 + x^2 - 21x + 45$ are $\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45$.
- Substituting these candidates into the polynomial reveals that none satisfy $F(x) = 0$. Therefore, there are no rational zeros for this polynomial.
- To find the actual roots, one must resort to numerical methods, graphing, or the cubic formula.

Key Takeaways

- The Rational Root Theorem is a powerful tool for identifying possible rational zeros but does not guarantee their existence.
- Systematic substitution or synthetic division helps verify these candidates efficiently.
- When rational zeros are absent, alternative approaches like numerical approximation or factoring techniques are necessary.
- Understanding the nature of roots—whether rational, irrational, or complex—is crucial in solving

polynomial equations effectively.

Final Remarks

While the process may seem exhaustive, mastering the Rational Root Theorem and associated techniques enables students and mathematicians to analyze polynomial functions methodically. For the polynomial $F(x) = x^3 + x^2 - 21x + 45$, the absence of rational zeros directs attention toward more advanced methods to uncover its roots, ensuring a comprehensive understanding of polynomial behavior.

Remember: Practice with various polynomials enhances your skill in identifying zeros and understanding their properties, a fundamental aspect of algebra and calculus.

Frequently Asked Questions

What is the first step to find all rational zeros of the polynomial $F(x) = x^3 + x^2 - 21x + 45$?

The first step is to use the Rational Root Theorem to list all possible rational zeros, which are factors of the constant term (45) divided by factors of the leading coefficient (1).

What are the possible rational zeros of the polynomial $F(x) = x^3 + x^2 - 21x + 45$?

The possible rational zeros are all factors of 45 over factors of 1, which are ±1, ±3, ±5, ±9, ±15, and ±45.

How do I test whether a candidate rational zero is actually a zero of the polynomial?

Substitute the candidate into the polynomial and see if it evaluates to zero. If it does, then it is a rational zero.

Can synthetic division help in finding rational zeros of the polynomial?

Yes, synthetic division can be used to test each possible rational zero efficiently and to factor the polynomial further once a zero is found.

What is the process to find all rational zeros after testing possible candidates?

Once a rational zero is identified, factor the polynomial using synthetic division or polynomial division, then repeat the process on the reduced polynomial to find all zeros.

Are all zeros of the polynomial necessarily rational?

No, only some zeros are rational; others may be irrational or complex. Rational zeros are specifically those that can be expressed as a ratio of integers.

What is the complete set of rational zeros for F(x) = x^3 + x^2 - 21x + 45?

By testing the possible zeros, the rational zeros are x = 3 and x = 5. Further factoring reveals the polynomial's roots are 3, 5, and -3.

How can I verify that all rational zeros have been found for this polynomial?

After identifying the zeros, factor the polynomial completely and confirm that the remaining factors are quadratic or linear with no additional rational zeros. The factorization should match the original polynomial when expanded.

Additional Resources

Identify All The Rational Zeros For $F(x) = x^3 + x^2 - 21x + 45$

When tackling polynomial functions, especially those of higher degree, one of the fundamental steps is to identify all rational zeros. This process not only simplifies the factorization but also provides insights into the roots of the polynomial. In this comprehensive review, we will explore the systematic approach to finding all rational zeros for the polynomial function $F(x) = x^3 + x^2 - 21x + 45$, including the theoretical background, step-by-step methods, and practical considerations.

Understanding Rational Zeros of Polynomial Functions

Before diving into the specific polynomial, it’s essential to understand what rational zeros are and why they matter.

Definition of Rational Zeros

Rational zeros of a polynomial function are roots that can be expressed as a fraction p/q, where p and q are integers with no common factors other than 1, $q \neq 0$, and p is a divisor of the constant term of the polynomial, while q is a divisor of the leading coefficient.

For the polynomial $F(x) = x^3 + x^2 - 21x + 45$:
- The constant term is 45.
- The leading coefficient is 1.

Significance of Rational Zeros

Identifying rational zeros allows us to:
- Factor the polynomial completely over the rational numbers.
- Find all real roots efficiently.
- Simplify the process of solving polynomial equations.

The Rational Root Theorem

The primary tool for identifying rational zeros is the Rational Root Theorem (also called the Rational Zero Theorem).

Statement of the Theorem

If a polynomial

$$a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$$

has a rational zero p/q (in lowest terms), then:
- p divides the constant term a_0.
- q divides the leading coefficient a_n.

Given that the leading coefficient $a_n = 1$, the possible rational zeros are simply the divisors of the constant term.

Application to Our Polynomial

For $F(x) = x^3 + x^2 - 21x + 45$:
- Constant term: 45
- Leading coefficient: 1

Possible values of p (divisors of 45): ±1, ±3, ±5, ±9, ±15, ±45
Possible values of q (divisors of 1): ±1

Therefore, the list of possible rational zeros is:
\[\pm 1, \pm 3, \pm 5, \pm 9, \pm 15, \pm 45 \]

Since q = 1, the potential rational zeros are simply these divisors.

Testing Candidate Rational Zeros

Having compiled the list of potential rational zeros, the next step is to test each candidate by substitution into the polynomial or by synthetic division to see if it yields zero.

Using Synthetic Division

Synthetic division provides a quick way to evaluate whether a candidate is a root:
- Set up synthetic division with the candidate zero.
- If the remainder is zero, the candidate is a root.

Step-by-Step Testing

Let's examine the candidates systematically.

Step-by-Step Evaluation of Rational Zeros

Candidate x = 1

- Synthetic division:

| 1 | 1 | 1 | -21 | 45 |
		1	2	-19
	1	2	-19	26

Remainder: 26 ≠ 0 → 1 is not a zero.

Candidate x = -1

| -1 | 1 | 1 | -21 | 45 |
		-1	0	21
	1	0	-21	66

Remainder: 66 ≠ 0 → -1 is not a zero.

Candidate x = 3

| 3 | 1 | 1 | -21 | 45 |
		3	12	-27
	1	4	-9	18

Remainder: 18 ≠ 0 → 3 is not a zero.

Candidate x = -3

| -3 | 1 | 1 | -21 | 45 |
		-3	6	45
	1	-2	-15	0

Remainder: 0 → -3 is a rational zero.

This is a significant finding; now, we know (x + 3) is a factor of the polynomial.

Candidate x = 5

5	1	1	-21	45
		5	30	45
	1	6	9	90

Remainder: 90 ≠ 0 → 5 is not a zero.

Candidate x = -5

-5	1	1	-21	45
		-5	0	105
	1	-4	-21	150

Remainder: 150 ≠ 0 → -5 is not a zero.

Candidate x = 9

9	1	1	-21	45
		9	90	693
	1	10	69	738

Remainder: 738 ≠ 0 → 9 is not a zero.

Candidate x = -9

-9	1	1	-21	45
		-9	0	189
	1	-8	-21	234

Remainder: 234 ≠ 0 → -9 is not a zero.

Candidate x = 15

| 15 | 1 | 1 | -21 | 45 |
		15	240	3510
	1	16	219	3555

Remainder: 3555 ≠ 0 → 15 is not a zero.

Candidate x = -15

| -15 | 1 | 1 | -21 | 45 |
		-15	0	315
	1	-14	-21	360

Remainder: 360 ≠ 0 → -15 is not a zero.

Candidate x = 45

| 45 | 1 | 1 | -21 | 45 |
		45	1980	3510
	1	46	1960	3555

Remainder: 3555 ≠ 0 → 45 is not a zero.

Candidate x = -45

| -45 | 1 | 1 | -21 | 45 |
		-45	0	945
	1	-44	-21	990

Remainder: 990 ≠ 0 → -45 is not a zero.

Summary of Tests
The only candidate that yielded a zero remainder is $x = -3$.

Factoring the Polynomial

Since $x = -3$ is a root, we can factor out $(x + 3)$ from the polynomial using synthetic division.

Synthetic division with root $x = -3$:

| -3 | 1 | 1 | -21 | 45 |
		-3	6	45
	1	-2	-15	0

The quotient polynomial is:

$$x^2 - 2x - 15$$

which factors further.

Factoring the Quadratic Polynomial

Factor $x^2 - 2x - 15$:

Find two numbers that multiply to -15 and add to -2.

- The numbers are -5 and 3.

Thus,

$$x^2 - 2x - 15 = (x - 5)(x + 3)$$

Complete Factorization and Roots

Putting it all together:

\[F(x) = (

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