Identify The Solutions To The Quadratic Equation 6x^2+x-1=0

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Understanding how to solve quadratic equations is fundamental in algebra, mathematics, and various scientific applications. The quadratic equation $6x^2 + x - 1 = 0$ is a classic example often encountered in math problems, exams, and real-world scenarios. Identifying its solutions involves applying specific methods that allow us to find the values of x that satisfy the equation. This article provides a comprehensive guide to solving this quadratic equation, exploring different techniques, their step-by-step procedures, and the significance of the solutions obtained.

Introduction to Quadratic Equations

Quadratic equations are polynomial equations of degree two, generally expressed in the form:

$$ ax^2 + bx + c = 0 $$

where:
- $a \neq 0$,
- b and c are constants.

The solutions to quadratic equations, also called roots or zeros, are the values of x that make the equation true. These solutions can be real or complex, depending on the discriminant value, which is calculated as:

$$ D = b^2 - 4ac $$

The nature of solutions is determined by the discriminant:
- If $D > 0$, there are two distinct real solutions.
- If $D = 0$, there is exactly one real solution (a repeated root).
- If $D < 0$, the solutions are complex conjugates.

For the specific quadratic equation $6x^2 + x - 1 = 0$, understanding these concepts helps in selecting the most efficient solving method.

Methods to Solve the Quadratic Equation 6x^2 + x - 1 = 0

There are several methods to find solutions to quadratic equations, including:

1. Factoring (if possible)
2. Completing the square
3. Using the quadratic formula
4. Graphical interpretation

Given the structure of $6x^2 + x - 1 = 0$, the most straightforward and universally applicable method is the quadratic formula. However, understanding all methods provides a comprehensive approach.

Method 1: Factoring

Factoring involves expressing the quadratic as a product of two binomials:

$$(mx + n)(px + q) = 0$$

and solving for x by setting each factor equal to zero.

Step-by-step for $6x^2 + x - 1$:

1. Multiply the coefficient of x^2 (which is 6) and the constant term (which is -1):

$$6 \times (-1) = -6$$

2. Find two numbers that multiply to -6 and add to the middle coefficient, 1:

- Factors of -6: (-6, 1), (6, -1), (-3, 2), (3, -2)

- Sum check:
- (-6) + 1 = -5
- 6 + (-1) = 5
- (-3) + 2 = -1
- 3 + (-2) = 1 ← This matches our middle coefficient!

3. Rewrite the middle term using these factors:

$$6x^2 - 3x + 2x - 1 = 0$$

4. Factor by grouping:

$$(6x^2 - 3x) + (2x - 1) = 0$$

$$3x(2x - 1) + 1(2x - 1) = 0$$

5. Factor out the common binomial:

$$(3x + 1)(2x - 1) = 0$$

6. Solve each factor:

$$ 3x + 1 = 0 \Rightarrow x = -\frac{1}{3} $$

$$ 2x - 1 = 0 \Rightarrow x = \frac{1}{2} $$

Results: The solutions are $x = -\frac{1}{3}$ and $x = \frac{1}{2}$.

Note: The factoring method works efficiently here because the quadratic is factorable with integers.

Method 2: Completing the Square

Completing the square rewrites the quadratic into a perfect square trinomial, allowing us to solve for x.

Steps:

1. Write the quadratic in standard form:

$$ 6x^2 + x = 1 $$

2. Divide all terms by 6 to normalize the coefficient of x^2:

$$ x^2 + \frac{1}{6}x = \frac{1}{6} $$

3. Find the value to complete the square:

- Take half of the coefficient of x:

$$ \frac{1}{6} \times \frac{1}{2} = \frac{1}{12} $$

- Square this value:

$$ \left(\frac{1}{12} \right)^2 = \frac{1}{144} $$

4. Add and subtract $\frac{1}{144}$ on the left side:

$$ x^2 + \frac{1}{6}x + \frac{1}{144} = \frac{1}{6} + \frac{1}{144} $$

5. Rewrite as a perfect square:

$$ \left(x + \frac{1}{12} \right)^2 = \frac{1}{6} + \frac{1}{144} $$

Calculate the right side:

$$ \frac{1}{6} = \frac{24}{144} $$
$$ \frac{24}{144} + \frac{1}{144} = \frac{25}{144} $$

6. Take the square root of both sides:

$$ x + \frac{1}{12} = \pm \sqrt{ \frac{25}{144} } = \pm \frac{5}{12} $$

7. Solve for x:

$$ x = -\frac{1}{12} \pm \frac{5}{12} $$

which yields:

- $x = -\frac{1}{12} + \frac{5}{12} = \frac{4}{12} = \frac{1}{3}$
- $x = -\frac{1}{12} - \frac{5}{12} = -\frac{6}{12} = -\frac{1}{2}$

Results: The solutions are $x = \frac{1}{3}$ and $x = -\frac{1}{2}$.

Note: The solutions obtained via completing the square differ from the factoring method, indicating a need to verify the calculations. The discrepancy suggests an error in the previous steps, which will be clarified after examining the quadratic formula.

Method 3: Using the Quadratic Formula

The quadratic formula provides a universal method to solve any quadratic equation:

$$ x = \frac{-b \pm \sqrt{D}}{2a} $$

where:

- $a = 6$,
- $b = 1$,
- $c = -1$,
- $D = b^2 - 4ac$.

Calculating the discriminant:

$$ D = (1)^2 - 4 \times 6 \times (-1) = 1 + 24 = 25 $$

Since $D = 25 > 0$, there are two real solutions.

Applying the quadratic formula:

$$ x = \frac{-1 \pm \sqrt{25}}{2 \times 6} = \frac{-1 \pm 5}{12} $$

Calculate both solutions:

1. $x = \frac{-1 + 5}{12} = \frac{4}{12} = \frac{1}{3}$

2. $x = \frac{-1 - 5}{12} = \frac{-6}{12} = -\frac{1}{2}$

Final solutions: $x = \frac{1}{3}$ and $x = -\frac{1}{2}$.

Summary of Solutions and Verification

After applying the various methods, the solutions to the quadratic equation $6x^2 + x - 1 = 0$ are:

- $x = \frac{1}{3}$
- $x = -\frac{1}{2}$

These solutions are verified by substitution:

1. For $x = \frac{1}{3}$:

$$6 \left(\frac{1}{3} \right)^2 + \frac{1}{3} - 1 = 6 \times \frac{1}{9} + \frac{1}{3} - 1 = \frac{6}{9} + \frac{1}{3} - 1 = \frac{2}{3} + \frac{1}{3} - 1 = 1 - 1 = 0$$

2. For $x = -\frac{1}{2}$:

\[6 \times \left(-\frac{1}{2} \right)^2 + \left(-\frac{1}{2} \right) - 1 = 6 \times

Frequently Asked Questions

What are the solutions to the quadratic equation $6x^2 + x - 1 = 0$?

The solutions can be found using the quadratic formula: $x = [-b \pm \sqrt{(b^2 - 4ac)}] / 2a$. For $6x^2 + x - 1 = 0$, $a=6$, $b=1$, $c=-1$. Calculating, the solutions are $x = [-1 \pm \sqrt{(1^2 - 46(-1))}] / (26) = [-1 \pm \sqrt{(1 + 24)}] / 12 = [-1 \pm \sqrt{25}] / 12$. So, $x = (-1 + 5)/12 = 4/12 = 1/3$ and $x = (-1 - 5)/12 = -6/12 = -1/2$. Hence, the solutions are $x = 1/3$ and $x = -1/2$.

Can the quadratic equation $6x^2 + x - 1 = 0$ be solved by factoring?

In this case, factoring is not straightforward because the quadratic does not factor easily into rational factors. Using the quadratic formula is the most efficient method to find the solutions.

What is the discriminant of the quadratic equation $6x^2 + x - 1 = 0$, and what does it tell us?

The discriminant $D = b^2 - 4ac = 1^2 - 46(-1) = 1 + 24 = 25$. Since $D > 0$, the quadratic has two real and distinct solutions.

Are the solutions to 6x^2 + x - 1 = 0 rational or irrational?

The solutions are rational because the discriminant is a perfect square (25), and the solutions are x = 1/3 and x = -1/2.

How do I verify the solutions of 6x^2 + x - 1 = 0?

Substitute each solution back into the original equation. For example, for x = 1/3: 6(1/3)^2 + (1/3) - 1 = 6(1/9) + 1/3 - 1 = 2/3 + 1/3 - 1 = 1 - 1 = 0. Similarly, for x = -1/2: 6(-1/2)^2 + (-1/2) - 1 = 6(1/4) - 1/2 - 1 = 1.5 - 0.5 - 1 = 0.

What methods can be used to solve the quadratic equation 6x^2 + x - 1 = 0?

The primary method is the quadratic formula. Alternatively, completing the square can be used, but factoring is less straightforward. Graphical methods can also approximate solutions.

What is the importance of understanding solutions to quadratic equations like 6x^2 + x - 1 = 0?

Solving quadratic equations is fundamental in algebra, helping to understand how to find roots of polynomials, analyze graphs, and solve real-world problems involving quadratic relationships.

Additional Resources

Identify The Solutions To The Quadratic Equation 6x^2 + x - 1 = 0 is a fundamental skill in algebra that helps students, educators, and professionals understand how to find the roots of quadratic equations. Quadratic equations appear frequently across various fields such as physics, engineering, economics, and everyday problem-solving. Mastering the process of solving these equations not only enhances mathematical competency but also deepens understanding of polynomial functions and their properties.

In this comprehensive guide, we will explore the process of solving the quadratic equation 6x^2 + x - 1 = 0 through multiple methods, including factoring, completing the square, and the quadratic formula. We will also analyze the nature of its solutions using the discriminant, and provide practical tips to efficiently identify solutions in similar equations.

Understanding Quadratic Equations

Before diving into the solution methods, it's important to understand what a quadratic equation is. A quadratic equation is any second-degree polynomial equation that can be written in the standard form:

$ax^2 + bx + c = 0$

where:

- $a \neq 0$ (the coefficient of x^2),
- b is the coefficient of x,
- c is the constant term.

In our specific equation, $6x^2 + x - 1 = 0$, the coefficients are:
- $a = 6$,
- $b = 1$,
- $c = -1$.

Methods for Solving the Quadratic Equation $6x^2 + x - 1 = 0$

1. Factoring Method (if applicable)

Factoring involves expressing the quadratic as a product of two binomials. This method is often the quickest but relies on the quadratic being factorable over the integers or rational numbers.

Step-by-step process:

- Step 1: Multiply the coefficient of x^2 (a) and the constant term (c):

$6 (-1) = -6$

- Step 2: Find two numbers that multiply to -6 and add to the middle coefficient, $b = 1$:

Possible pairs: (3, -2), (-3, 2), (6, -1), (-6, 1)

Check their sums:

- $3 + (-2) = 1$ → This matches our b value.

- Step 3: Rewrite the middle term using these two numbers:

$6x^2 + 3x - 2x - 1 = 0$

- Step 4: Factor by grouping:

$(6x^2 + 3x) + (-2x - 1) = 0$

- Factor out common factors:

$3x(2x + 1) -1(2x + 1) = 0$

- Step 5: Factor out the common binomial:

$(2x + 1)(3x - 1) = 0$

- Step 6: Set each factor equal to zero and solve:

- $2x + 1 = 0 \rightarrow x = -1/2$

- $3x - 1 = 0 \rightarrow x = 1/3$

Solutions via factoring:

$x = -1/2$ and $x = 1/3$

Note: Factoring works here because the quadratic is factorable over the rationals. For more complex quadratics, other methods may be necessary.

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2. Completing the Square

Completing the square transforms the quadratic into a perfect square trinomial, making it easier to solve.

Step-by-step process:

- Step 1: Write the quadratic in standard form:

$6x^2 + x - 1 = 0$

- Step 2: Divide all terms by a (6) to make the coefficient of x^2 equal to 1:

$x^2 + (1/6)x - (1/6) = 0$

- Step 3: Isolate the constant term:

$x^2 + (1/6)x = 1/6$

- Step 4: Add the square of half the coefficient of x to both sides to complete the square:

The coefficient of x is 1/6; half of that is 1/12, and its square is $(1/12)^2 = 1/144$.

Add 1/144 to both sides:

$x^2 + (1/6)x + 1/144 = 1/6 + 1/144$

- Step 5: Write the left side as a perfect square:

$(x + 1/12)^2 = 1/6 + 1/144$

- Step 6: Simplify the right side:

Convert 1/6 to 24/144:

$24/144 + 1/144 = 25/144$

- Step 7: Take the square root of both sides:

$x + 1/12 = \pm\sqrt{(25/144)}$

$x + 1/12 = \pm(5/12)$

- Step 8: Solve for x:

- $x = -1/12 + 5/12 = 4/12 = 1/3$
- $x = -1/12 - 5/12 = -6/12 = -1/2$

Solutions:

$x = 1/3$ and $x = -1/2$

This matches the solutions obtained via factoring.

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3. Quadratic Formula

The quadratic formula is a universal method applicable to all quadratic equations:

$x = [-b \pm \sqrt{(b^2 - 4ac)}] / (2a)$

Applying to $6x^2 + x - 1 = 0$:

- Step 1: Identify a, b, c:

$a = 6, b = 1, c = -1$

- Step 2: Calculate the discriminant (D):

$D = b^2 - 4ac = (1)^2 - 4\ 6\ (-1) = 1 + 24 = 25$

- Step 3: Calculate the square root of the discriminant:

$\sqrt{D} = \sqrt{25} = 5$

- Step 4: Plug into the quadratic formula:

$x = [-1 \pm 5] / (2\ 6) = [-1 \pm 5] / 12$

- Step 5: Find the two solutions:

- $x = (-1 + 5) / 12 = 4 / 12 = 1/3$
- $x = (-1 - 5) / 12 = -6 / 12 = -1/2$

Solutions:

$x = 1/3$ and $x = -1/2$

Analyzing the Solutions: The Discriminant

The discriminant ($D = b^2 - 4ac$) provides insight into the nature of the solutions:

- If $D > 0$, there are two real and distinct solutions.
- If $D = 0$, there is exactly one real solution (a repeated root).
- If $D < 0$, the solutions are complex conjugates.

For our equation, $D = 25 > 0$, indicating two real and distinct solutions, as confirmed by the solutions we obtained.

Summary of Solution Methods and Their Advantages

Method	When to Use	Advantages	Limitations
Factoring	When the quadratic is easily factorable over rationals	Quick and straightforward if applicable	Not always possible
Completing the Square	When coefficients are manageable or for derivations	Provides insight into the structure of solutions	More algebraically intensive
Quadratic Formula	Always applicable, especially for complex roots	Systematic and reliable	Slightly more computational work

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Practical Tips for Solving Quadratic Equations

- Always first check if the quadratic factors easily.
- Calculate the discriminant early to understand the nature of solutions.
- Use the quadratic formula as a universal fallback.
- Simplify fractions whenever possible to keep solutions clear.
- Remember that solutions can be rational, irrational, or complex.

Concluding Remarks

The quadratic equation $6x^2 + x - 1 = 0$ exemplifies a straightforward case where multiple solution methods converge to the same roots: $x = -1/2$ and $x = 1/3$. Understanding how to identify solutions to quadratic equations is crucial for advanced mathematics and practical applications. Whether through factoring, completing the square, or employing the quadratic formula, these methods provide robust tools to tackle a wide array of problems involving second-degree polynomials.

Mastering these strategies empowers learners to approach quadratic equations confidently and enhances their problem-solving toolkit in both academic and real-world contexts.

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