If G(x) = F (1/3x), Which Statement Is True? (see Image)

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Understanding the relationship between functions is a fundamental concept in mathematics, especially in algebra and calculus. When given a function like G(x) = F(1/3x), the key question often revolves around how G(x) relates to F(x), what transformations are involved, and how the properties of these functions change. This article aims to explore this specific function relationship, analyze the possible statements that could be true based on the given, and provide a comprehensive guide to interpreting such functions accurately.

Understanding the Function G(x) = F(1/3x)

Before diving into the specifics of which statement is true, it is essential to understand what the function G(x) = F(1/3x) signifies. This form indicates a composition of functions involving a transformation of the input variable.

Breaking Down the Function

- The function G(x) is defined in terms of another function F, with its input scaled by a factor of 1/3.
- The notation F(1/3x) suggests that for each x, the function F is evaluated at 1/3 times x.
- This form involves a horizontal transformation of the original function F.

Implication of the Transformation

- The argument inside F is scaled by 1/3, which affects the shape and position of the graph of F.
- Specifically, replacing x with 1/3x compresses or stretches the graph horizontally, depending on the transformation.

Analyzing the Transformation: Horizontal

Compression

Transformations of functions involve shifting, reflecting, stretching, or compressing graphs. In this case, the transformation involves a change in the argument of the function, leading to a horizontal compression or dilation.

Effect of Replacing x with 1/3x

- When the input x is replaced with 1/3x, the graph of F is affected in a particular way.
- To understand this effect, consider the general rule:

If the input is replaced with kx, then the graph of F is scaled horizontally by a factor of 1/|k|.

Horizontal Compression or Stretch

- Since the input is 1/3x, the scale factor k = 1/3.
- The graph of F is compressed horizontally by a factor of 3 because:

> Horizontal scaling factor = 1/|k| = 1/(1/3) = 3.

- Therefore, the original graph of F is compressed toward the y-axis by a factor of 3.

Visualizing the Transformation

- Imagine stretching or compressing a rubber band horizontally.
- This transformation makes features of the graph (like peaks, valleys, intercepts) occur at one-third the original x-values.

Implications for the Function G(x)

Given the transformation, we now analyze how G(x) relates to F(x), and what properties are affected.

Relationship Between G(x) and F(x)

- Since G(x) = F(1/3x), G(x) is a horizontally compressed version of F(x).
- The graph of G(x) can be obtained by compressing the graph of F(x) by a factor of 3 along the x-axis.

Key Points to Remember

- Domain and Range:
- The domain of G(x) depends on the domain of F, scaled accordingly.
- If F is defined for all real numbers, then G(x) is also defined for all real x.
- Graph shifts:
- No vertical shifts are involved; only horizontal compression.
- Function values:
- For a specific x, G(x) = F(1/3x).

Effect on Function Properties

- Intercepts:
- The x-intercepts of G(x) occur where F(1/3x) = 0, i.e., where F's argument is zero.
- If F has an x-intercept at x = a, then G(x) has an x-intercept at x = 3a.
- Symmetry:
- If F is symmetric about the y-axis or origin, G(x) will retain similar symmetry properties, adjusted by the scaling.

Common Statements and Which Is True

Based on typical multiple-choice questions about such functions, the statements might include:

- G(x) is a vertical stretch of F(x).
- G(x) is a horizontal stretch of F(x).
- G(x) is a horizontal compression of F(x).
- G(x) is a reflection of F(x).
- G(x) has the same graph as F(x).

Let's analyze each and determine which is true.

Statement 1: G(x) is a vertical stretch of F(x)

- False. The transformation involves scaling the input, not the output. Vertical transformations involve multiplying F(x) by a constant outside the function, e.g., a factor of a.

Statement 2: G(x) is a horizontal stretch of F(x)

- False. Since replacing x with 1/3x compresses the graph horizontally, it's a compression, not a stretch.

Statement 3: G(x) is a horizontal compression of F(x)

- True. As explained earlier, replacing x with 1/3x scales the graph horizontally by a factor of 3, which is a compression.

Statement 4: G(x) is a reflection of F(x)

- False. Reflection would require multiplying the function output by -1 or changing the sign of the input, but not replacing x with 1/3x.

Statement 5: G(x) has the same graph as F(x)

- False. The transformations alter the graph's appearance; unless the transformation is trivial (k=1), the graphs are not identical.

Conclusion: The correct statement is that G(x) is a horizontal compression of F(x).

Applications and Real-World Examples

Understanding how transformations like G(x) = F(1/3x) work is crucial in various fields, including physics, engineering, and data analysis.

Applications in Physics

- Signal Processing: Horizontal compression can model signals that are sped up or slowed down.
- Wave Functions: Adjusting the argument of a wave function models different frequencies or wavelengths.

Applications in Engineering

- System Response: The transformation can represent system responses to input signals scaled in time.
- Control Systems: Modifying input time scales affects system behavior and stability.

Data Analysis and Graphing

- When plotting data, understanding how input scaling affects the graph helps in interpreting models correctly.
- For example, if a graph of data is compressed horizontally, it indicates a faster response or process.

Practice Problems to Reinforce Understanding

To solidify comprehension, here are some practice problems related to the transformation G(x) = F(1/3x):

1. If F(x) has an x-intercept at x = 4, where is the x-intercept of G(x)?
2. If F(x) is increasing for x > 0, what can be said about G(x)?
3. Suppose F(x) is symmetric about the y-axis. Is G(x) also symmetric? Why?
4. How would the graph of G(x) change if the function was defined as G(x) = F(3x)?
5. Describe the overall transformation from F(x) to G(x) in terms of graph shape and position.

Answers:
1. At x = 12 (since 4 3).
2. G(x) will also be increasing where F(1/3x) is increasing, but the rate of increase with respect to x will be affected.
3. Yes, G(x) would retain symmetry if F(x) is symmetric, because the horizontal compression does not affect symmetry about the y-axis.
4. G(x) would be horizontally stretched by a factor of 1/3.
5. G(x) is a horizontally compressed version of F(x), with features occurring at one-third the original x-values.

Summary and Final Thoughts

Understanding the function G(x) = F(1/3x) is crucial for grasping how horizontal transformations affect graphs. The key takeaway is that replacing x with 1/3x results in a horizontal compression of the graph of F by a factor of 3. This transformation affects the placement of intercepts, the shape, and the symmetry of the graph, but preserves the overall type of the function.

Recognizing these transformations enables students and professionals alike to interpret functions correctly, analyze graphs more effectively, and model real-world phenomena accurately. When confronted with statements about such transformations, it’s essential to analyze whether they refer to horizontal or vertical changes and understand the

Frequently Asked Questions

What is the main relationship between G(x) and F(x) in the given function G(x) = F(1/3x)?

G(x) is a composition of F with a scaled and transformed input, specifically evaluating F at (1/3)x, which affects the graph's horizontal scaling.

How does the transformation G(x) = F(1/3x) affect the graph of F(x)?

It horizontally stretches the graph of F(x) by a factor of 3 because the input is scaled by 1/3 inside the function.

If F(x) has a certain feature at x = a, where will G(x) have the corresponding feature?

G(x) will have the feature at x = 3a because the input to F is scaled by 1/3, so solving (1/3)x = a gives x = 3a.

Which statement correctly describes the relationship between G(x) and F(x) in terms of their graphs?

The graph of G(x) is a horizontally stretched version of F(x) by a factor of 3, meaning every point on F(x) is moved 3 times farther from the y-axis in G(x).

Given a specific point on F(x), say (a, F(a)), where is the corresponding point on G(x)?

The corresponding point on G(x) is at (3a, F(a)), since G(3a) = F(1/33a) = F(a).

Additional Resources

Understanding the Composition of Functions: Analyzing G(x) = F(1/3x)

When studying functions in mathematics, especially those involving compositions or transformations, it’s crucial to understand how the functions relate to each other and what implications these relationships have for their properties. The problem involving G(x) = F(1/3x) is a classic example that tests your understanding of function composition, transformations, and the behavior of functions under various operations. This type of problem often appears in algebra and pre-calculus courses, and mastering it can deepen your insight into how functions behave under transformations.

In this article, we will thoroughly explore the statement "If G(x) = F(1/3x), which statement is true?" by dissecting the structure of G(x), understanding the relationship between G and F, and analyzing the possible statements that might be associated with such a functional relationship. Whether you’re preparing for exams or seeking a clearer conceptual understanding, this guide will walk you through the essential concepts step-by-step.

What Does G(x) = F(1/3x) Mean?

Before diving into the specifics, it’s important to interpret what the statement G(x) = F(1/3x) signifies in terms of functions.

Function Composition and Transformation

- G(x) is defined as applying the function F to the input (1/3)x.
- This is a composite type of function, where the input to F is not just x but a transformed version (1/3)x.

Implication of the Transformation

- The expression (1/3)x indicates a scaling or stretch transformation applied to x before passing it into F.
- Specifically, (1/3)x compresses the input x by a factor of 3, because multiplying by 1/3 makes the input smaller when x is positive, and larger in magnitude when x is negative.

Intuition: How does this affect G(x)?

- If F is a function that produces certain outputs based on its input, then G is a scaled or transformed version of F, depending on how the input is adjusted.
- In essence, G(x) is F evaluated at a scaled version of x.

Key Concepts Needed to Analyze the Problem

To determine which statement is true regarding $G(x) = F(1/3x)$, it's essential to understand the following concepts:

1. Function Composition

- The process of applying one function to the result of another.
- Notation: $(F \circ h)(x) = F(h(x))$
- In this case: $G(x) = (F \circ h)(x)$, where $h(x) = (1/3)x$.

2. Horizontal and Vertical Transformations

- Horizontal transformations involve shifting, stretching, or compressing the graph along the x-axis.
- Vertical transformations involve shifting, stretching, or compressing along the y-axis.

3. Effect of Input Scaling on the Graph

- Replacing x with ax in a function F(x) results in a horizontal transformation:
- If $a > 1$, the graph compresses horizontally by a factor of $1/a$.
- If $0 < a < 1$, the graph stretches horizontally by a factor of $1/a$.
- In our case, $a = 1/3$, so the graph of F is compressed horizontally by a factor of 3.

4. Domain and Range Implications

- Understanding how the domain and range of F relate to G.
- Input scaling affects the domain of G relative to F.

Analyzing the Statements and Their Validity

Given the relationship $G(x) = F(1/3x)$, typical statements that might be posed include:

- Statement A: G(x) is a horizontal stretch or compression of F.
- Statement B: G(x) is a vertical stretch or compression of F.
- Statement C: G(x) shifts the graph of F horizontally.
- Statement D: G(x) shifts the graph vertically.
- Statement E: The domain of G is scaled relative to F.

Let's analyze these possibilities systematically.

Step-by-Step Breakdown

1. Identifying the Type of Transformation

The core of the analysis hinges on understanding how the input modification (1/3)x affects the graph of F.

- Horizontal Compression: When the input x is replaced by (1/3)x, the graph of F is compressed horizontally by a factor of 3.

Why? Because for a fixed output, a smaller input (due to multiplication by 1/3) corresponds to a wider part of the original function's graph. To see this more clearly, consider the inverse: solving for x in terms of F.

2. Graphical Interpretation

Suppose the graph of F is known. Then:

- G(x) = F(1/3 x) implies that to find G(x), you look at the point (x, G(x)) which corresponds to (1/3 x, F(1/3 x)) on the original graph of F.

- So, the x-coordinate in G corresponds to a smaller x-coordinate in F's graph, scaled by a factor of 3.

3. Effect on Domain

- If the domain of F is D_F, then the domain of G becomes:

$D_G = \{ x \mid (1/3)\, x \in D_F \}$

- For each x in D_G, (1/3) x must be in D_F.
- Equivalently, x must be in 3 D_F (the domain of F scaled by 3).

- This indicates that G's domain is scaled by a factor of 3 relative to F's domain.

4. Effect on Range

- Since G(x) = F(1/3 x), the range of G is the same as the range of F, because F is evaluated at scaled inputs, but the outputs are unaffected by the input scaling.

Practical Example: Visualizing the Transformation

Imagine F is a simple quadratic function, such as $F(x) = x^2$.

- Original graph: Parabola opening upward with vertex at (0,0).
- $G(x) = F(1/3\, x) = (1/3\, x)^2 = (1/9)\, x^2$.

This transformation:

- Compresses the graph horizontally by a factor of 3 (since the input is scaled by 1/3).

- Vertical scale: The output is scaled by a factor of 1/9, but since the original function is quadratic, the entire graph is also scaled vertically, effectively making it narrower.

Summary of the True Statement

Based on the detailed analysis, the key takeaway is:

- G(x) = F(1/3 x) results in a horizontal compression of the original function F by a factor of 3.
- The domain of G is 3 times the domain of F.
- The range remains the same as F, assuming no other transformations.

Final Considerations: Common True Statements

Given the analysis, which of the following statements is most likely to be true?

- Statement 1: G(x) is a horizontally compressed version of F(x) by a factor of 3. (True)
- Statement 2: G(x) is a vertically stretched version of F. (False)
- Statement 3: The domain of G is scaled by a factor of 3 compared to F. (True)
- Statement 4: The graph of G is shifted horizontally. (False) unless additional shifts are specified.
- Statement 5: G(x) is obtained by shifting F(x) vertically. (False)

Conclusion

In conclusion, understanding the relationship G(x) = F(1/3 x) hinges on grasping how input scaling affects the graph of a function. This transformation results in a horizontal compression of the original graph by a factor of 3, and the domain of G is scaled accordingly. Recognizing these transformations is essential for accurately interpreting and analyzing functional relationships and their behaviors.

By mastering these concepts, you can confidently analyze similar problems involving function transformations, composition, and their graphical implications.

[If G X F 1 3x Which Statement Is True See Image]

If G X F 1 3x Which Statement Is True See Image

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