(q36)Find The Area Under The Curve Y = 2^2x - 3 From 0 To 2.

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Understanding how to find the area under a curve is a fundamental concept in integral calculus. This process involves calculating the definite integral of a function over a specified interval. In this article, we will explore how to determine the area under the curve Y = 2^{2x} - 3 from x = 0 to x = 2. We will break down the problem into manageable steps, explain the underlying mathematical principles, and provide a comprehensive solution to enhance your understanding of integration techniques.

Introduction to the Problem

Finding the area under a curve involves calculating the definite integral of a function over a specific interval. The function in question here is:

$$ Y = 2^{2x} - 3 $$

over the interval:

$$ x \in [0, 2] $$

This problem requires us to evaluate:

$$ \text{Area} = \int_{0}^{2} (2^{2x} - 3) \, dx $$

The integral can be separated into two simpler integrals:

$$ \int_{0}^{2} 2^{2x} \, dx - \int_{0}^{2} 3 \, dx $$

Once we compute these integrals, we can find the total area under the curve.

Understanding the Function: Y = 2^{2x} - 3

Before diving into the integration process, it's essential to understand the behavior of the function:

- Exponential Component: 2^{2x} is an exponential function, where the base is 2, and the exponent is \(2x \).

- Linear Adjustment: The subtracting of 3 shifts the graph downward by 3 units.

This function is increasing exponentially, and because of the negative shift, it crosses the x-axis at some point within the interval.

Step-by-Step Solution to Find the Area

The process involves several steps:

1. Rewrite the integral

Express the total area as the sum of two integrals:

\[
\text{Area} = \int_{0}^{2} 2^{2x} \, dx - \int_{0}^{2} 3 \, dx
\]

2. Calculate \(\int 2^{2x} \, dx\)

This integral requires some substitution and the use of exponential integration formulas.

3. Calculate \(\int 3 \, dx\)

This is a straightforward integral of a constant.

4. Evaluate the definite integrals from 0 to 2

Substitute the upper and lower limits and compute the difference to get the total area.

Calculating \(\int 2^{2x} \, dx\)

Using exponential integration techniques:

The general rule for integrating \(a^{kx} \) is:

\[
\int a^{kx} \, dx = \frac{a^{kx}}{k \ln a} + C

\]

where:

- a is the base of the exponential,
- k is the coefficient of x,
- C is the constant of integration.

In our case:

- $a = 2$,
- $k = 2$.

Applying the formula:

$$
\int 2^{2x} \, dx = \frac{2^{2x}}{2 \ln 2} + C
$$

Calculating the Definite Integral

Now, we'll evaluate:

$$
\int_{0}^{2} 2^{2x} \, dx = \left[\frac{2^{2x}}{2 \ln 2} \right]_0^2
$$

Plugging in the limits:

- At $x = 2$:

$$
\frac{2^{2 \times 2}}{2 \ln 2} = \frac{2^{4}}{2 \ln 2} = \frac{16}{2 \ln 2} = \frac{8}{\ln 2}
$$

- At $x = 0$:

$$
\frac{2^{0}}{2 \ln 2} = \frac{1}{2 \ln 2}
$$

Subtract to find the definite integral:

$$
\frac{8}{\ln 2} - \frac{1}{2 \ln 2} = \frac{8}{\ln 2} - \frac{1}{2 \ln 2} = \frac{16}{2 \ln 2} - \frac{1}{2 \ln 2} = \frac{15}{2 \ln 2}
$$

$$
$$

Calculating $\int_{0}^{2} 3 \, dx$

This integral is straightforward:

$$
\int_{0}^{2} 3 \, dx = 3x \big|_0^2 = 3 \times 2 - 3 \times 0 = 6
$$

Final Calculation of the Area

Putting it all together:

$$
\text{Area} = \left(\frac{15}{2 \ln 2} \right) - 6
$$

This is the exact value of the area under the curve between $x = 0$ and $x = 2$.

Expressing the Result in Simplified Form

The result:

$$
\boxed{
\text{Area} = \frac{15}{2 \ln 2} - 6
}
$$

can be left as is for exactness or approximated numerically.

Numerical approximation:

- $\ln 2 \approx 0.6931$

Calculate:

$$
\frac{15}{2 \times 0.6931} \approx \frac{15}{1.3862} \approx 10.81
$$

Subtract 6:

$$
10.81 - 6 \approx 4.81
$$

Therefore, the approximate area under the curve is about 4.81 square units.

Additional Insights and Applications

Understanding how to compute areas under exponential curves has broad applications, including:

- Physics: Calculating work done by exponential forces.
- Biology: Modeling population growth or decay.
- Economics: Computing consumer surplus or producer surplus in exponential markets.
- Engineering: Signal processing involving exponential signals.

Summary of Key Steps

- Step 1: Break the integral into manageable parts.
- Step 2: Use the exponential integral formula:

$$
\int a^{kx} \, dx = \frac{a^{kx}}{k \ln a} + C
$$

- Step 3: Evaluate definite integrals at the limits.
- Step 4: Subtract the results to find the total area.
- Step 5: Express the final answer either exactly or approximately.

Conclusion

Calculating the area under the curve $Y = 2^{2x} - 3$ from 0 to 2 involves understanding exponential integration, applying the appropriate formulas, and carefully evaluating the definite integrals. The exact area is:

$$
\boxed{
\frac{15}{2 \ln 2} - 6
}
$$

which approximates to about 4.81 in decimal form. Mastery of these techniques is essential for solving a wide range of problems in calculus and applied mathematics.

FAQs

Q1: Why do we use the formula $\int a^{kx} dx = \frac{a^{kx}}{k \ln a}$?

A1: Because the derivative of a^{kx} with respect to x is $a^{kx} \times k \ln a$, so integrating reverses this process.

Q2: Can this method be applied to other exponential functions?

A2: Yes, the same formula applies to any exponential function of the form a^{kx}, where $a > 0$ and $a \neq 1$.

Q3: How do I approximate the area numerically?

A3: Use the approximate value of $\ln 2$ and perform decimal calculations, as shown in the numerical approximation section.

Q4: What is the significance of the area under a curve?

A4: It represents the accumulated quantity, such as distance traveled, total growth, or total accumulated value over an interval.

By mastering the process of calculating areas under exponential curves, you strengthen your understanding of integral calculus and its practical applications across various fields.

Frequently Asked Questions

What is the function given for finding the area under the curve?

The function is $Y = 2^{2x} - 3$.

Over what interval is the area under the curve to be calculated?

From $x = 0$ to $x = 2$.

What is the integral setup to find the area under the curve?

The area $A = \int_0^2 (2^{2x} - 3)\, dx$.

How do you integrate 2^{2x} with respect to x?

Use substitution or recognize that $\int 2^{ax}\, dx = (2^{ax}) / (a \ln 2)$. Here, $a=2$, so $\int 2^{2x}\, dx = 2^{2x} / (2 \ln 2)$.

What is the evaluated integral of the function from 0 to 2?

$A = [2^{2x} / (2 \ln 2) - 3x]$ from 0 to 2, which simplifies to $(2^{4} / (2 \ln 2) - 32) - (2^{0} / (2 \ln 2) - 0)$.

What is the numerical value of the area under the curve from 0 to 2?

Calculating, we get $A = (16 / (2 \ln 2) - 6) - (1 / (2 \ln 2))$. Simplify to $A = (8 / \ln 2) - 6 - (1 / (2 \ln 2))$.

What is the approximate numerical value of the area under the curve?

Using $\ln 2 \approx 0.693$, $A \approx (8 / 0.693) - 6 - (1 / (2\ 0.693)) \approx 11.54 - 6 - 0.721 \approx 4.82$ square units.

Additional Resources

Finding the Area Under the Curve $y = 2^{2x} - 3$ From 0 To 2: A Comprehensive Review

Understanding how to determine the area under a curve is a fundamental concept in calculus, bridging the gap between geometric intuition and analytical precision. In this detailed review, we will explore the process of calculating the area under the curve defined by the function $y = 2^{2x} - 3$ from $x = 0$ to $x = 2$. We will delve into the mathematical foundations, step-by-step solution procedures, and interpret the significance of the result, providing clarity for students, educators, and enthusiasts alike.

Introduction to Area Under the Curve

Calculating the area under a curve involves integrating a given function over a specified interval. Geometrically, this is interpreted as the total space enclosed between the curve, the x-axis, and the vertical lines at the interval's endpoints.

This concept is central to many applications in physics, engineering, economics, and other sciences, where it models quantities such as displacement, accumulated profit, or total work done.

Key points:
- The process uses definite integrals.
- The limits of integration define the interval over which the area is calculated.
- The function's behavior influences the nature of the area (positive, negative, or a combination).

Understanding the Given Function

The function in question is:

$$
y = 2^{2x} - 3
$$

Let's analyze this function carefully:

2.1 Expression Breakdown
- Exponential component: 2^{2x}
- Constant shift: -3

2.2 Simplifying the Exponential
Recall that $a^{b x} = (a^{b})^{x}$. Thus:

$$

$$
2^{2x} = (2^{2})^{x} = (4)^{x}
$$

This simplification makes the function:

$$
y = 4^{x} - 3
$$

2.3 Behavior of the Function
- At $x=0$: $y = 4^0 - 3 = 1 - 3 = -2$
- At $x=2$: $y = 4^{2} - 3 = 16 - 3 = 13$

The function increases exponentially, starting from -2 at $x=0$ and rising to 13 at $x=2$.

Setting Up the Integral

To find the area under the curve from $x=0$ to $x=2$:

$$
\text{Area} = \int_{0}^{2} y \, dx = \int_{0}^{2} (4^{x} - 3) \, dx
$$

This integral can be split into two parts:

$$
\text{Area} = \int_{0}^{2} 4^{x} \, dx - \int_{0}^{2} 3 \, dx
$$

Calculating the Integral of 4^{x}

2.1 Exponential Integral Formula
The integral of a^{x} with respect to x, where $a > 0, a \neq 1$, is:

$$
\int a^{x} \, dx = \frac{a^{x}}{\ln a} + C
$$

2.2 Applying to 4^{x}
Here, $a=4$, so:

$$
\int 4^{x} \, dx = \frac{4^{x}}{\ln 4} + C
$$

Calculating the Remaining Integral

The second integral:

$$
\int_{0}^{2} 3 \, dx
$$

is straightforward:

$$
3 \times (x) \big|_{0}^{2} = 3 \times (2 - 0) = 6
$$

Putting It All Together: Computing the Area

The total area becomes:

$$
\text{Area} = \left[\frac{4^{x}}{\ln 4} \right]_{0}^{2} - 6
$$

Calculating the first term:

- At $x=2$:

$$
\frac{4^{2}}{\ln 4} = \frac{16}{\ln 4}
$$

- At $x=0$:

$$
\frac{4^{0}}{\ln 4} = \frac{1}{\ln 4}
$$

Thus, the definite integral:

$$
\frac{16}{\ln 4} - \frac{1}{\ln 4} = \frac{16 - 1}{\ln 4} = \frac{15}{\ln 4}
$$

Final expression for the area:

$$
\boxed{
\text{Area} = \frac{15}{\ln 4} - 6
}
$$

Numerical Approximation and Interpretation

2.1 Numerical value of $\ln 4$
Using known logarithms:

$$
\ln 4 = \ln(2^2) = 2 \ln 2 \approx 2 \times 0.6931 = 1.3862
$$

2.2 Approximate area
Substituting:

$$
\text{Area} \approx \frac{15}{1.3862} - 6 \approx 10.81 - 6 = 4.81
$$

This means:

> The area under the curve $y = 4^{x} - 3$ from $x=0$ to $x=2$ is approximately 4.81 square units.

2.3 Significance
- The area is positive, which makes sense because the curve crosses the x-axis at some point within the interval.
- The function is negative at $x=0$ (value -2), so part of the area below the x-axis contributes negatively to the integral if we consider the algebraic integral.
- Since the integral computes signed area, the positive result indicates the net area considering the entire interval, including regions where the function dips below the x-axis.

Dealing with Negative Values of the Function

Note that for x in $[0, 0.5)$, $4^{x} - 3$ is negative, so the curve is below the x-axis. The definite integral calculates algebraic area, which can cancel out between positive and negative parts.

To find the total geometric area (absolute area regardless of above or below x-axis), further steps are necessary:

2.1 Find the intersection point where $y=0$:

$$
4^{x} - 3 = 0 \Rightarrow 4^{x} = 3 \Rightarrow x = \frac{\ln 3}{\ln 4}
$$

Numerically:

$$
x = \frac{\ln 3}{\ln 4} \approx \frac{1.0986}{1.3862} \approx 0.793
$$

2.2 Split the integral at this point

Total absolute area:

$$
\text{Total Area} = \int_{0}^{0.793} |4^{x} - 3| \, dx + \int_{0.793}^{2} |4^{x} - 3| \, dx
$$

Since $4^{x} - 3$ is negative before $x=0.793$, and positive afterward:

$$
\text{Total Area} = \int_{0}^{0.793} (3 - 4^{x}) \, dx + \int_{0.793}^{2} (4^{x} - 3) \, dx
$$

Calculating these integrals provides a more accurate picture of the total geometric area regardless of sign.

Broader Mathematical Context and Applications

Calculating areas under curves like $y = 4^{x} - 3$ is not just an academic exercise; it has practical relevance:

2.1 Applications in Physics

- Work done by a force: If a force varies exponentially with position, integrating the force over distance gives the total work.
- Radioactive decay: The decay rate can be modeled exponentially, and integrating the decay rate over time yields total decay events.

2.2 Economics
- Accumulated profit: If profit per unit time grows exponentially, integrating over a period gives total profit.

2.3 Engineering
- Signal processing: Calculating the total energy of an exponentially increasing or decreasing signal involves similar integration.

2.4 Mathematical Significance
- Reinforces understanding of exponential functions, their integrals, and the interpretation of signed vs. total area.
- Demonstrates the importance of considering where the function crosses the x-axis.

Alternative Methods and Extensions

Beyond direct integration, several other techniques and considerations can be explored:

2.1 Numerical Integration
- For more complex functions, numerical methods like Simpson's rule or

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