Solve For X In The Equation X Squared + 11 X + 121/4 = 125/4.

Solve For X In The Equation X Squared + 11 X + 121/4 = 125/4. This algebraic problem requires us to find the value(s) of the variable X that satisfy the given quadratic equation. Solving such equations is fundamental in algebra, and understanding the methods involved can help in tackling various mathematical and real-world problems. In this article, we will explore step-by-step solutions, analyze the structure of the equation, and discuss different approaches to find the value(s) of X effectively.

Understanding the Equation

Before diving into solving the equation, let's analyze its structure:

> X2 + 11X + 121/4 = 125/4

This is a quadratic equation, which generally takes the form:

> ax2 + bx + c = 0

In our case, to compare with this standard form, we need to bring all terms to one side:

> X2 + 11X + 121/4 - 125/4 = 0

Simplifying the constants:

> 121/4 - 125/4 = (121 - 125)/4 = -4/4 = -1

Therefore, the equation becomes:

> X2 + 11X - 1 = 0

This simplified form makes it easier to identify the coefficients for applying solving methods such as factoring, completing the square, or using the quadratic formula.

Method 1: Using the Quadratic Formula

The quadratic formula is a universal method suitable for solving any quadratic equation of the form ax2 + bx + c = 0:

> X = [-b ± √(b2 - 4ac)] / 2a

Let's identify the coefficients from our simplified equation:

- a = 1
- b = 11
- c = -1

Now, we substitute these into the quadratic formula:

> X = [-11 ± √(112 - 41(-1))] / (21)

Calculating the discriminant:

> D = 112 - 41(-1) = 121 + 4 = 125

Since the discriminant is positive, we will have two real solutions. Computing the solutions:

> X = [-11 ± √125] / 2

Note that √125 can be simplified:

> √125 = √(255) = 5√5

Therefore,

> X = [-11 ± 5√5] / 2

This gives us the two solutions:

1. X = [-11 + 5√5] / 2
2. X = [-11 - 5√5] / 2

These are exact solutions expressed in radical form. For approximate decimal values, compute √5 ≈ 2.23607:

- First solution:

> X ≈ [-11 + 52.23607] / 2 = [-11 + 11.18034] / 2 ≈ 0.18034 / 2 ≈ 0.09017

- Second solution:

> X ≈ [-11 - 11.18034] / 2 = [-22.18034] / 2 ≈ -11.09017

Thus, the approximate solutions are:

- X ≈ 0.09017
- X ≈ -11.09017

Method 2: Completing the Square

Completing the square is another powerful method, especially for equations where the quadratic coefficient is 1, as in our case. Let's revisit the simplified form:

> X2 + 11X - 1 = 0

To complete the square:

1. Move the constant to the other side:

> X2 + 11X = 1

2. Take half of the coefficient of X, square it, and add to both sides:

Half of 11 is 11/2; square it:

> (11/2)2 = 121/4

3. Add this to both sides:

> X2 + 11X + 121/4 = 1 + 121/4

4. The left side becomes a perfect square:

> (X + 11/2)2 = 1 + 121/4

Now, convert 1 to quarters:

> 1 = 4/4

Adding:

> 4/4 + 121/4 = 125/4

So,

> $(X + 11/2)^2 = 125/4$

Taking square roots:

> $X + 11/2 = \pm \sqrt{125/4} = \pm \sqrt{125} / 2 = \pm 5\sqrt{5} / 2$

Finally, solving for X:

> $X = -11/2 \pm 5\sqrt{5} / 2$

Expressed as two solutions:

1. $X = (-11/2) + (5\sqrt{5} / 2) = (-11 + 5\sqrt{5}) / 2$
2. $X = (-11/2) - (5\sqrt{5} / 2) = (-11 - 5\sqrt{5}) / 2$

This matches the solutions obtained via the quadratic formula, confirming their correctness.

Additional Insights and Applications

Understanding how to solve quadratic equations such as this one has practical applications across various fields.

Applications in Real-World Scenarios

- Physics: Calculating projectile motion parameters.
- Engineering: Analyzing forces and stresses.
- Economics: Modeling quadratic cost or revenue functions.
- Computer Science: Algorithm analysis involving quadratic time complexity.

Key Takeaways

- Always simplify the equation first to identify the quadratic form.
- Use the quadratic formula when the quadratic does not factor easily.
- Completing the square provides an alternative, especially for understanding the structure of the quadratic.
- The discriminant indicates the nature of solutions: positive for two real solutions, zero for one real solution, negative for complex solutions.

Conclusion

Solving the quadratic equation X2 + 11X + 121/4 = 125/4 involves recognizing its simplified form, applying the quadratic formula or completing the square, and interpreting the solutions. The exact solutions are:

> X = [-11 ± 5√5] / 2

Or approximately:

> X ≈ 0.09017 and -11.09017

Mastering these methods enhances problem-solving skills and deepens understanding of quadratic equations, which are foundational in algebra and numerous scientific disciplines. Whether you're preparing for exams, tackling engineering problems, or analyzing data, knowing how to solve for X in such equations is an invaluable skill.

Frequently Asked Questions

How can I solve the quadratic equation X^2 + 11X + 121/4 = 125/4?

First, rewrite the equation in standard form: X^2 + 11X + 121/4 - 125/4 = 0, which simplifies to X^2 + 11X - 1/4 = 0. Then, use the quadratic formula to find X.

What is the quadratic formula used to solve equations like X^2 + 11X - 1/4 = 0?

The quadratic formula is X = [-b ± √(b^2 - 4ac)] / 2a. For the equation X^2 + 11X - 1/4 = 0, identify a=1, b=11, c=-1/4 and substitute into the formula.

What are the solutions to the equation X^2 + 11X + 121/4 = 125/4?

The solutions are X = [-11 ± √(11^2 - 4×1×(-1/4))]/2. Simplifying inside the square root gives √(121 + 1) = √122. Therefore, X = [-11 ± √122]/2.

Can I complete the square to solve X^2 + 11X + 121/4 = 125/4?

Yes. Rewrite the equation as (X + 11/2)^2 = 125/4 + (11/2)^2. Since (11/2)^2 = 121/4, the right side becomes 125/4 + 121/4 = 246/4 = 123/2. Then, solve for X.

What is the value of X when solving X^2 + 11X + 121/4 = 125/4?

Using the completed square method, $X + 11/2 = \pm\sqrt{(123/2)}$, so $X = -11/2 \pm \sqrt{(123/2)}$.

Is the quadratic equation X^2 + 11X + 121/4 = 125/4 solvable by factoring?

No, because the discriminant ($b^2 - 4ac$) is 122, which is not a perfect square, so the solutions involve irrational numbers and are best found using the quadratic formula.

What are the approximate decimal solutions for the equation?

Calculating $\sqrt{122} \approx 11.045$, so the solutions are approximately $X = (-11 + 11.045)/2 \approx 0.0225$ and $X = (-11 - 11.045)/2 \approx -11.0225$.

How do I verify the solutions for the equation X^2 + 11X + 121/4 = 125/4?

Substitute each solution back into the original equation to check if both sides are equal. If they hold true, the solutions are correct.

What is the significance of the constant term 121/4 in the quadratic equation?

The constant term 121/4 relates to the square of $(X + 11/2)$ when completing the square, helping to find solutions more straightforwardly.

Can this quadratic equation be solved using graphing methods?

Yes. Graphing the quadratic function $y = X^2 + 11X + 121/4$ and finding where it intersects $y=125/4$ can help visualize the solutions.

Additional Resources

Mathematics

Unlocking the Solution: A Comprehensive Guide to Solving \(x^2 + 11x + \frac{121}{4} = \frac{125}{4} \)

Mathematics, often regarded as the universal language, provides us with powerful tools to analyze and interpret the world around us. Among these, algebra stands as a fundamental pillar, allowing us to solve for unknown variables within equations. Today, we delve into a specific quadratic equation:

$$
x^2 + 11x + \frac{121}{4} = \frac{125}{4}
$$

Our goal? To solve for x in this equation with clarity, precision, and an in-depth understanding of each step involved. Whether you're a student, educator, or enthusiast, this comprehensive review aims to demystify the process, explore multiple solution methods, and deepen your appreciation for algebraic problem-solving.

Understanding the Equation: An Overview

The Structure of the Equation

At first glance, the equation appears complex due to the fractional coefficients:

$$
x^2 + 11x + \frac{121}{4} = \frac{125}{4}
$$

However, with a strategic approach, we can simplify and solve it efficiently.

Recognizing the Type of Equation

This is a quadratic equation—an algebraic expression where the highest power of the variable x is 2. Quadratic equations typically take the form:

$$
ax^2 + bx + c = 0
$$

or, as in our case, can be rearranged into this standard form. Solving such equations involves methods like:

- Factoring
- Completing the square
- The quadratic formula

In this review, we will explore each of these methods, emphasizing their advantages and applicability.

Step 1: Simplifying the Equation

Before applying any solution method, it's advantageous to express the equation in a more manageable form.

Move All Terms to One Side

Subtract $\frac{125}{4}$ from both sides:

$$
x^2 + 11x + \frac{121}{4} - \frac{125}{4} = 0
$$

Combine the fractions:

$$
x^2 + 11x + \left(\frac{121}{4} - \frac{125}{4} \right) = 0
$$

$$
x^2 + 11x - \frac{4}{4} = 0
$$

$$
x^2 + 11x - 1 = 0
$$

Now, the quadratic is simplified to:

$$
x^2 + 11x - 1 = 0
$$

This form is ideal for applying standard solution methods.

Step 2: Solving the Quadratic Equation

Method 1: Factoring (Not Applicable Here)

Factoring involves expressing the quadratic as a product of binomials:

$$
(ax + m)(bx + n) = 0
$$

Given the coefficients, especially the constant term -1, the factors are limited. Since -1 has only factors ±1, and the middle term is $11x$, it's unlikely to factor neatly into rational numbers. Therefore, factoring is not straightforward here.

Method 2: Completing the Square (Preferred for Educational Clarity)

Completing the square transforms the quadratic into a perfect square trinomial plus or minus some value, enabling direct extraction of x.

Method 3: Quadratic Formula (Most Universal)

The quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

provides a quick and reliable way to find solutions for any quadratic.

Step 3: Applying the Quadratic Formula

Given the quadratic:

$$
x^2 + 11x - 1 = 0
$$

Identify coefficients:

- $a = 1$
- $b = 11$
- $c = -1$

Plug into the quadratic formula:

$$
x = \frac{-11 \pm \sqrt{11^2 - 4 \times 1 \times (-1)}}{2 \times 1}
$$

Calculate discriminant:

$$
D = 11^2 - 4 \times 1 \times (-1) = 121 + 4 = 125
$$

Since the discriminant $D = 125$ is positive and not a perfect square, the solutions will involve irrational numbers.

Compute the square root:

$$
\sqrt{125} = \sqrt{25 \times 5} = 5 \sqrt{5}
$$

Now, write the solutions explicitly:

$$
x = \frac{-11 \pm 5 \sqrt{5}}{2}
$$

Thus, the two solutions are:

$$
\boxed{
x = \frac{-11 + 5 \sqrt{5}}{2} \quad \text{and} \quad x = \frac{-11 - 5 \sqrt{5}}{2}
}
$$

Deep Dive: Interpreting the Solutions

Exact vs. Approximate Values

While the exact solutions involve radicals, it can be insightful to approximate their decimal equivalents:

Calculate $\sqrt{5} \approx 2.23607$:

- First solution:

$$
x \approx \frac{-11 + 5 \times 2.23607}{2} = \frac{-11 + 11.18035}{2} = \frac{0.18035}{2} \approx 0.09017
$$

- Second solution:

$$
x \approx \frac{-11 - 11.18035}{2} = \frac{-22.18035}{2} \approx -11.09017
$$

Summary of Solutions

Exact Form	Approximate Value
$\frac{-11 + 5 \sqrt{5}}{2}$	≈ 0.09017
$\frac{-11 - 5 \sqrt{5}}{2}$	≈ -11.09017

These solutions can be interpreted as the points where the quadratic function intersects the x-axis.

Additional Insight: Completing the Square Method

While the quadratic formula provides a straightforward solution, completing the square offers valuable conceptual understanding.

Step-by-step process:

1. Write the quadratic:

$$
x^2 + 11x - 1 = 0
$$

2. Move the constant to the other side:

$$
x^2 + 11x = 1
$$

3. Complete the square:

- Take half of the coefficient of x, which is $\frac{11}{2}$.
- Square it: $\left(\frac{11}{2} \right)^2 = \frac{121}{4}$.

4. Add and subtract this value:

$$
x^2 + 11x + \frac{121}{4} = 1 + \frac{121}{4}
$$

5. Express the left side as a perfect square:

$$
\left(x + \frac{11}{2} \right)^2 = \frac{4}{4} + \frac{121}{4} = \frac{125}{4}
$$

6. Take the square root of both sides:

$$
x + \frac{11}{2} = \pm \sqrt{\frac{125}{4}} = \pm \frac{\sqrt{125}}{2} = \pm \frac{5 \sqrt{5}}{2}
$$

7. Solve for x:

$$
x = -\frac{11}{2} \pm \frac{5 \sqrt{5}}{2}
$$

which simplifies to the same solutions obtained via the quadratic formula.

Final Summary and Takeaways

- The original equation simplifies to a standard quadratic form after algebraic manipulation.
- The quadratic formula is a reliable method, especially when the discriminant isn't a perfect square.
- Exact solutions involve radicals and can be expressed as:

$$
x = \frac{-11 \pm 5 \sqrt{5}}{2}
$$

- Approximate solutions are approximately 0.09017 and -11.09017.

Practical Applications and Broader Context

Understanding how to solve quadratic equations like this is foundational in various fields, including physics, engineering, finance, and computer science. For example:

- Physics: Calculating projectile trajectories.
- Engineering: Analyzing stress and strain models.
- Economics: Optimizing profit functions.
- Computer Graphics: Curve plotting and intersection calculations.

Mastering these methods enhances problem-solving skills, enabling professionals and students to tackle complex real-world problems with confidence.

Closing Thoughts

Solving for x in the equation $x^2 + 11x + \frac{121}{4} = \frac{125}{4}$ exemplifies the elegance and power of algebraic techniques. Whether through the quadratic formula, completing the square, or factoring (when applicable), each approach deepens your understanding of quadratic relationships.

By breaking down complex fractional coefficients and systematically applying these methods, we've demonstrated that even seemingly complicated equations can be unraveled with clarity and precision. Embrace these techniques as part of your mathematical toolkit, and continue exploring the rich landscape

Solve For X In The Equation X Squared 11 X 121 4 125 4

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