Solve For X. Show Your Work3x(2x - 5) = (3x - 4)(2x + 1)

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Introduction to Solving Algebraic Equations

Algebra forms the backbone of many mathematical concepts and real-world problem-solving. At its core, solving algebraic equations involves finding the value(s) of the variable(s) that satisfy the equation. Whether you're tackling simple linear equations or complex quadratic expressions, understanding the step-by-step process is essential. In this article, we will demonstrate how to solve the equation:

$3x(2x - 5) = (3x - 4)(2x + 1)$

This particular example combines distribution, factoring, and simplifying techniques that are foundational in algebra.

Understanding the Equation Components

Before diving into the solution, it's important to analyze the structure of the given equation:

$3x(2x - 5) = (3x - 4)(2x + 1)$

Left Side: $3x(2x - 5)$

- This is a product of a monomial ($3x$) and a binomial ($2x - 5$).
- The distributive property will be used to expand this expression.

Right Side: $(3x - 4)(2x + 1)$

- This is a product of two binomials.
- The FOIL method (First, Outer, Inner, Last) is suitable for expansion.

Step-by-Step Solution Process

Step 1: Expand Both Sides of the Equation

A. Expand the left side: $3x(2x - 5)$

- Distribute $3x$ across the terms inside the parentheses:

$3x\ 2x = 6x^2$

$3x\ (-5) = -15x$

- So, the expanded form:

$6x^2 - 15x$

B. Expand the right side: $(3x - 4)(2x + 1)$

- Use FOIL:

First	Outer	Inner	Last
$3x\ 2x = 6x^2$	$3x\ 1 = 3x$	$-4\ 2x = -8x$	$-4\ 1 = -4$

- Combine:

$6x^2 + 3x - 8x - 4$

- Simplify like terms:

$6x^2 - 5x - 4$

Step 2: Set the Expanded Expressions Equal

Now that both sides are expanded, equate them:

$6x^2 - 15x = 6x^2 - 5x - 4$

Step 3: Simplify the Equation

Subtract $6x^2$ from both sides to eliminate quadratic terms:

$6x^2 - 15x - 6x^2 = 6x^2 - 5x - 4 - 6x^2$

which simplifies to:

$-15x = -5x - 4$

Step 4: Solve for x

Now, isolate x:

1. Add 5x to both sides:

$-15x + 5x = -5x - 4 + 5x$

which simplifies to:

$-10x = -4$

2. Divide both sides by -10:

$x = (-4) / (-10) = 4 / 10$

3. Simplify the fraction:

$x = 2 / 5$

Verifying the Solution

It's always good practice to verify your solution by substituting $x = 2/5$ back into the original equation.

Step 1: Substitute into the left side:

$3(2/5)(2(2/5) - 5)$

- Calculate $3(2/5) = 6/5$

- Calculate $2(2/5) = 4/5$

- So, $(4/5) - 5$

- Rewrite 5 as 25/5

- (4/5) - (25/5) = -21/5

- Multiply 6/5 -21/5 = (6 -21) / (5 5) = -126 / 25

Step 2: Substitute into the right side:

(3(2/5) - 4)(2(2/5) + 1)

- 3(2/5) = 6/5

- 6/5 - 4

- Rewrite 4 as 20/5

- (6/5 - 20/5) = -14/5

- Next, 2(2/5) + 1

- 2(2/5) = 4/5

- Add 1 as 5/5

- (4/5 + 5/5) = 9/5

- Multiply:

(-14/5) (9/5) = (-14 9) / (5 5) = -126 / 25

- Both sides yield -126/25, confirming the solution's correctness.

Additional Techniques for Solving Algebraic Equations

While this particular problem involved straightforward expansion and simplification, other algebraic equations may require different methods:

Factoring

- Useful when quadratic equations can be factored into binomials.
- Example: Solving ax^2 + bx + c = 0 by factoring.

Quadratic Formula

- When factoring isn't straightforward, use the quadratic formula:

$$x = (-b \pm \sqrt{(b^2 - 4ac)}) / (2a)$$

Completing the Square

- A method to solve quadratics that cannot be factored easily.

Rational Equations

- Equations involving fractions; often require multiplying through by the least common denominator (LCD).

Tips for Effective Equation Solving

- Always expand expressions carefully to avoid mistakes.
- Combine like terms early to simplify the equation.
- Keep track of negative signs.
- Check your solutions by substituting back into the original equation.
- Use graphing tools to visualize solutions when applicable.

Common Mistakes to Avoid

- Forgetting to distribute properly during expansion.
- Mixing up signs when simplifying.
- Forgetting to simplify fractions.
- Overlooking extraneous solutions, especially when squaring both sides.

Conclusion

Solving algebraic equations like $3x(2x - 5) = (3x - 4)(2x + 1)$ involves a systematic approach: expanding, simplifying, and solving step-by-step. By understanding the properties of algebra, practicing expansion methods like distribution and FOIL, and verifying solutions, students can confidently tackle complex equations. This structured method not only helps in exams but also builds a strong foundation for advanced mathematics and real-world applications.

Remember, practice makes perfect—so keep working through various algebra problems to hone your skills!

Frequently Asked Questions

How do you start solving the equation 3x(2x - 5) = (3x - 4)(2x + 1)?

Begin by expanding both sides: 3x(2x - 5) becomes 6x^2 - 15x, and (3x - 4)(2x + 1) becomes 6x^2 + 3x - 8x - 4, which simplifies to 6x^2 - 5x - 4. So, the equation becomes 6x^2 - 15x = 6x^2 - 5x - 4.

What is the next step after expanding both sides in the equation 3x(2x - 5) = (3x - 4)(2x + 1)?

Subtract 6x^2 from both sides to eliminate the quadratic term, resulting in -15x = -5x - 4.

How do you solve for x after simplifying to -15x = -5x - 4?

Add 5x to both sides to gather x terms on one side: -15x + 5x = -4, which simplifies to -10x = -4. Then, divide both sides by -10 to find x: x = (-4)/(-10) = 2/5.

What is the solution to the equation 3x(2x - 5) = (3x - 4)(2x + 1)?

The solution is x = 2/5.

How can you verify the solution x = 2/5 in the original equation?

Plug x = 2/5 into both sides: Left side is 3(2/5)(2(2/5) - 5), and right side is (3(2/5) - 4)(2(2/5) + 1). Simplify each side to confirm they are equal, verifying the solution.

Additional Resources

Solve For X. Show Your Work3x(2x - 5) = (3x - 4)(2x + 1)

Introduction

Mathematics, often regarded as the language of logic and reasoning, presents countless opportunities for problem-solving and intellectual exploration. Among the fundamental skills for students and professionals alike is mastering algebraic equations—a core component that underpins advanced mathematics and real-world applications. One such equation that exemplifies the process of solving for an unknown variable, x, is:

3x(2x - 5) = (3x - 4)(2x + 1)

This equation offers a rich landscape for analytical investigation, involving the application of distributive property, expansion, collection of like terms, and solving quadratic equations. Understanding how to approach such problems not only deepens mathematical comprehension but also cultivates critical thinking skills essential for tackling complex problems in various disciplines.

In this article, we will embark on a detailed, step-by-step journey to solve this equation, illustrating the process with clarity and rigor. We will explore the algebraic techniques involved, analyze the solution set, and reflect on the broader significance of such problems in mathematical reasoning.

The Equation at a Glance

The given equation is:

$3x(2x - 5) = (3x - 4)(2x + 1)$

Before delving into the solution, it's instructive to recognize the nature of both sides.

- The left side involves a single term multiplied by a binomial: a product that suggests applying the distributive property.
- The right side is a product of two binomials, which also calls for expansion via the distributive property (FOIL method).

The goal is to simplify both sides, derive a standard quadratic form, and then solve for x.

Step 1: Expand Both Sides

1.1 Expand the Left Side: $3x(2x - 5)$

Applying distributive property:

- $3x\ 2x = 6x^2$
- $3x\ (-5) = -15x$

Thus,

Left Side = $6x^2 - 15x$

1.2 Expand the Right Side: $(3x - 4)(2x + 1)$

Using FOIL (First, Outer, Inner, Last):

- First: $3x\ 2x = 6x^2$
- Outer: $3x\ 1 = 3x$
- Inner: $-4\ 2x = -8x$
- Last: $-4\ 1 = -4$

Combining these:

Right Side = $6x^2 + 3x - 8x - 4 = 6x^2 - 5x - 4$

Step 2: Set the Expanded Forms Equal

Now, equate the two expanded expressions:

$6x^2 - 15x = 6x^2 - 5x - 4$

Step 3: Simplify and Rearrange

Subtract $6x^2$ from both sides to eliminate the quadratic term:

$6x^2 - 15x - 6x^2 = 6x^2 - 5x - 4 - 6x^2$

which simplifies to:

$-15x = -5x - 4$

Next, isolate x terms:

Add 5x to both sides:

$-15x + 5x = -5x + 5x - 4$

which results in:

$-10x = -4$

Now, solve for x:

Divide both sides by -10:

$x = (-4) / (-10) = 2/5$

Step 4: Verify the Solution

Verification is a crucial step to ensure the solution satisfies the original equation.

Substitute $x = 2/5$ into the original equation:

$3x(2x - 5) = (3x - 4)(2x + 1)$

Calculate each side separately.

4.1 Left Side:

- $3 (2/5) = 6/5$

- $2x - 5 = 2(2/5) - 5 = (4/5) - 5 = (4/5) - (25/5) = -21/5$

Multiply:

$(6/5) (-21/5) = (6 -21) / (5 5) = -126 / 25$

4.2 Right Side:

- $3x - 4 = 3(2/5) - 4 = (6/5) - 4 = (6/5) - (20/5) = -14/5$

- $2x + 1 = 2(2/5) + 1 = (4/5) + 1 = (4/5) + (5/5) = 9/5$

Multiply:

$(-14/5) (9/5) = (-14 9) / (5 5) = -126 / 25$

Both sides equal -126/25, confirming that $x = 2/5$ is a valid solution.

Broader Context and Implications

The process demonstrated above isn't merely about algebraic manipulation; it embodies a systematic approach to problem-solving that involves:

- Recognizing the structure of the equation
- Applying properties of algebra for expansion
- Simplifying to a manageable form
- Verifying solutions for accuracy

This methodology is fundamental across mathematics and its applications, including engineering, physics, economics, and computer science.

Potential Pitfalls and Common Mistakes

While the solution appears straightforward, several common pitfalls can occur:

- Neglecting the distributive property: Failing to correctly expand binomials can lead to incorrect equations.
- Mismanaging signs: Negative signs, especially during expansion, often lead to errors.
- Overlooking extraneous solutions: Although in this case there's only one solution, more complex equations may yield extraneous roots that need verification.
- Dividing by variables: Attempting to divide both sides by a variable without confirming it's not zero can eliminate valid solutions or introduce invalid ones.

Being mindful of these issues enhances both accuracy and confidence in algebraic problem-solving.

Extending the Problem: Additional Considerations

In more advanced contexts, equations like this can be extended or generalized:

- Parameterization: Introducing parameters to analyze how solutions change.
- Graphical analysis: Plotting both sides to visualize intersections.
- Numerical methods: Applying iterative algorithms when algebraic solutions are complex or unavailable.

Furthermore, equations involving higher-degree polynomials or multiple variables can be approached with similar systematic strategies, emphasizing the importance of foundational skills demonstrated here.

Conclusion

The journey through solving $3x(2x - 5) = (3x - 4)(2x + 1)$ exemplifies the core principles of algebra: expansion, simplification, and verification. The solution, $x = 2/5$, was derived through meticulous application of algebraic properties, revealing the power of systematic reasoning.

Mastering such techniques is invaluable, not only for academic success but also for developing logical thinking applicable across various fields. As mathematical problems grow in complexity, the foundational skills showcased here serve as essential tools for navigating the intricate landscape of algebraic equations.

In the broader perspective, equations like this serve as stepping stones in the journey of mathematical literacy—enabling learners to unlock deeper understanding, foster analytical skills, and appreciate the elegance and utility of mathematics in the world around us.

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