Solve The Equation By Elimination 2x - 5y = -7=-2x + 3y = 1

Solve The Equation By Elimination 2x - 5y = -7 = -2x + 3y = 1

When faced with systems of equations involving multiple variables, the elimination method is one of the most effective techniques to find solutions efficiently. In particular, solving the system:

$$2x - 5y = -7$$
$$-2x + 3y = 1$$

by elimination allows us to eliminate one variable to solve for the other. This article provides a comprehensive step-by-step guide on how to approach such equations systematically, ensuring clarity and ease of understanding for learners at any level.

Understanding the System of Equations

Before diving into the elimination process, it's essential to understand what the system of equations represents and how the elimination method works.

What Is a System of Equations?

A system of equations consists of two or more equations with the same set of variables. The goal is to find the values of these variables that satisfy all equations simultaneously.

In the given system:
- The first equation is $2x - 5y = -7$
- The second equation is $-2x + 3y = 1$

Why Use the Elimination Method?

The elimination method is particularly useful when adding or subtracting equations can eliminate one variable, making it straightforward to solve for the other. It is often more efficient compared to substitution, especially when coefficients of a variable are opposites or can be easily made to cancel out.

Step-by-Step Solution Using Elimination Method

To solve the system:

$$2x - 5y = -7$$
$$-2x + 3y = 1$$

we follow these structured steps:

Step 1: Prepare the Equations for Elimination

Notice that the coefficients of x are 2 and -2. These are already opposites, making elimination of x straightforward.

Key Point: When coefficients are opposites, simply adding the two equations will eliminate x.

Step 2: Add the Equations to Eliminate x

Adding the equations:

$$
(2x - 5y) + (-2x + 3y) = -7 + 1
$$

Simplify each side:

$$
2x - 5y - 2x + 3y = -6
$$

$$
(2x - 2x) + (-5y + 3y) = -6
$$

$$
0x - 2y = -6
$$

which simplifies to:

$$
-2y = -6
$$

Step 3: Solve for y

Divide both sides by -2:

$$
y = \frac{-6}{-2} = 3
$$

Solution for y: $y = 3$

Step 4: Substitute y Back Into One of the Original Equations

Now that we know $y = 3$, substitute into either of the original equations to solve for x.

Using the first equation:

$$
2x - 5(3) = -7
$$

Simplify:

$$
2x - 15 = -7
$$

Add 15 to both sides:

$$
2x = -7 + 15
$$

$$
2x = 8
$$

Divide both sides by 2:

$$
x = \frac{8}{2} = 4
$$

Solution for x: $x = 4$

Final Solution and Verification

The solution to the system is:

$$
x = 4, \quad y = 3
$$

To verify, substitute these values into the second original equation:

$$
-2x + 3y = 1
$$

Substitute $x = 4$ and $y = 3$:

$$
-2(4) + 3(3) = -8 + 9 = 1
$$

Since the left side equals the right side, the solution is verified.

Summary:
- The solution to the system is $(x, y) = (4, 3)$.
- The elimination method efficiently led to the solution by eliminating x through addition.

Additional Tips for Solving Systems by Elimination

Align Coefficients for Easier Cancellation

Sometimes, coefficients are not opposites. To make elimination feasible, multiply one or both equations by suitable numbers to align coefficients.

Example:
If the equations are:

$$ 3x + 4y = 7 $$
$$ 5x - 2y = 3 $$

Multiply the first equation by 5 and the second by 3:

$$
(3x + 4y) \times 5 \Rightarrow 15x + 20y = 35
$$
$$
(5x - 2y) \times 3 \Rightarrow 15x - 6y = 9
$$

Now, subtract the second from the first to eliminate x.

Choose the Variable to Eliminate

Select the variable with the easiest coefficients to eliminate. Often, coefficients that are already opposites are ideal.

Check Your Work

Always verify your solution by substituting it back into the original equations. This step confirms the correctness of your solution.

Common Mistakes to Avoid

- Not aligning equations properly before addition or subtraction.
- Forgetting to multiply equations to match coefficients.
- Neglecting to simplify after combining equations.
- Forgetting to verify the solution in both original equations.

Conclusion

Solving the system of equations:

$$ 2x - 5y = -7 $$
$$ -2x + 3y = 1 $$

by elimination is straightforward once you recognize the coefficients of x are opposites. By adding the equations, you eliminate x and can solve directly for y. Substituting back yields the value of x. This method is efficient, especially for larger systems or when coefficients are conducive to elimination.

Mastering the elimination method enhances your problem-solving skills in algebra, enabling you to approach complex systems confidently and accurately. With practice, you'll find that solving systems like these becomes quicker and more intuitive, making your algebraic toolkit even more powerful.

Frequently Asked Questions

How do I solve the system of equations 2x - 5y = -7 and -2x + 3y = 1 using elimination?

To solve by elimination, first add the two equations to eliminate x: $(2x - 5y) + (-2x + 3y) = -7 + 1$. This simplifies to $(0x - 2y) = -6$, so $-2y = -6$, giving $y = 3$. Substitute $y = 3$ into one of the original equations to find x, for example $2x - 5(3) = -7$, which simplifies to $2x - 15 = -7$, so $2x = 8$, resulting in $x = 4$.

What is the solution to the system of equations 2x - 5y = -7 and -2x + 3y = 1 using elimination method?

The solution is $x = 4$ and $y = 3$. By adding the equations, we eliminated x and found y, then substituted back to find x.

Can elimination be used directly on the equations 2x - 5y = -7 and -2x + 3y = 1? If so, how?

Yes, elimination can be used. First, add the two equations directly to eliminate x: $(2x - 5y) + (-2x + 3y) = -7 + 1$, which simplifies to $-2y = -6$, leading to $y = 3$. Then substitute $y = 3$ into either original equation to find x.

Why does adding the equations 2x - 5y = -7 and -2x + 3y = 1 help in solving for the variables?

Because adding the equations cancels out the x terms ($2x + -2x = 0$), allowing us to solve directly for y. This simplifies the process and reduces the system to one variable, making it easier to find the solution.

What are common mistakes to avoid when solving these equations by elimination?

Common mistakes include forgetting to check whether the coefficients align properly, mixing signs during addition or subtraction, and not substituting back to verify the solution. Also, ensure the equations are correctly written and simplified before proceeding.

If the equations were inconsistent or dependent, how would that affect solving by elimination?

If the equations are inconsistent, elimination would lead to a contradiction (e.g., 0 = a non-zero number), indicating no solution. If dependent, the equations are multiples of each other, resulting in infinitely many solutions. Recognizing these cases prevents incorrect conclusions.

Additional Resources

Solve The Equation By Elimination $2x - 5y = -7 = -2x + 3y = 1$

Understanding how to solve systems of equations is a fundamental skill in algebra, especially when dealing with multiple variables. One effective method is the elimination method, which involves adding or subtracting equations to eliminate one of the variables, making it easier to solve for the remaining variable. In this article, we will explore how to solve the system of equations:

$2x - 5y = -7$

$-2x + 3y = 1$

using the elimination method. We will walk through each step in detail, clarifying key concepts along the way to ensure a comprehensive understanding.

Understanding the System of Equations

Before diving into the solution process, it's crucial to understand what a system of equations represents. In essence, it's a set of two or more equations with multiple variables. The goal is to find the values of these variables that satisfy all equations simultaneously.

In our case, the system consists of two linear equations with two variables, x and y:

1. $2x - 5y = -7$
2. $-2x + 3y = 1$

Graphically, these equations represent two lines on the xy-plane. The solution to the system is the point(s) where these lines intersect.

Note: Since the equations are linear and there are two variables, the system can have:

- A unique solution (intersecting lines)
- Infinitely many solutions (coincident lines)
- No solution (parallel lines)

Our task is to determine which case applies and find the specific solution if it exists.

Preparing for the Elimination Method

The elimination method hinges on manipulating the equations to eliminate one variable, allowing us to solve for the other. The key is to align the coefficients of one variable in both equations so that adding or subtracting the equations cancels out that variable.

Step 1: Examine the coefficients

Our equations are:

- $2x - 5y = -7$
- $-2x + 3y = 1$

Notice that the coefficients of x are 2 and -2, which are opposites. This is advantageous because adding the two equations directly will eliminate x:

$(2x - 5y) + (-2x + 3y) = -7 + 1$

This provides a straightforward path to elimination without needing to multiply equations by constants.

Step 2: Add the equations

Adding the two equations term by term:

$(2x + -2x) + (-5y + 3y) = -7 + 1$

Simplifies to:

$0x + (-2y) = -6$

or simply:

$-2y = -6$

Now, this is an equation with only y, which can be solved easily.

Solving for the Remaining Variable

Having eliminated x, the next step is to find y:

$-2y = -6$

Divide both sides by -2:

$y = (-6) / (-2)$

$y = 3$

This value of y is the solution for the variable y.

Summary so far:

- $y = 3$

With y found, the next step is to substitute y back into one of the original equations to solve for x.

Back-Substitution to Find x

Choosing one of the original equations makes sense; typically, selecting the simpler one is preferable. Let's use the first:

$2x - 5y = -7$

Substitute $y = 3$:

$2x - 5(3) = -7$

Simplify:

$2x - 15 = -7$

Add 15 to both sides:

$2x = -7 + 15$

$2x = 8$

Divide both sides by 2:

$x = 8 / 2$

$x = 4$

Final solution:

x = 4
y = 3

This point, (4, 3), is the unique solution where both lines intersect, satisfying both equations simultaneously.

Verifying the Solution

Verification is an essential step to ensure the solution is correct. Plug the obtained values back into the original equations:

Equation 1:

2(4) - 5(3) = -7

8 - 15 = -7

-7 = -7 ⍰

Equation 2:

-2(4) + 3(3) = 1

-8 + 9 = 1

1 = 1 ⍰

Since both equations are satisfied, the solution (4, 3) is valid.

Summary of the Elimination Method Process

The systematic approach to solving the system through elimination can be summarized as:

1. Identify coefficients: Look for coefficients that are opposites or can be made opposites by multiplication.
2. Adjust equations if necessary: Multiply one or both equations to align coefficients.
3. Add or subtract equations: Eliminate one variable.
4. Solve for the remaining variable: Simplify and find the value.
5. Back-substitute: Insert the found value into an original equation to solve for the other variable.
6. Verify: Check the solution in both original equations.

Additional Insights and Tips

- Choose the easiest equation for substitution: After finding one variable, plugging it back into the simpler equation minimizes errors.
- Be attentive to signs: When adding or subtracting equations, keep track of positive and negative signs.
- If coefficients aren't opposites: Multiply one or both equations by suitable numbers to create opposite coefficients.
- Visualize the system: Graphing can provide a visual confirmation of the solution point.

Conclusion

Mastering the elimination method for solving systems of equations, such as $2x - 5y = -7$ and $-2x + 3y = 1$, equips students and professionals with a powerful algebraic tool. By carefully aligning coefficients, adding or subtracting equations to eliminate variables, and then solving step-by-step, complex systems become manageable. The process not only fosters problem-solving skills but also deepens understanding of linear relationships. Remember, accuracy in each step and verification at the end are critical to ensure correct solutions. With practice, this method becomes an intuitive and efficient way to approach similar systems across mathematics and applied sciences.

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