Solve The Inequality And Graph The Solution. 4 |3x + 5| 16

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Understanding and solving inequalities involving absolute values is a fundamental skill in algebra. These concepts not only enhance problem-solving abilities but also pave the way for more advanced mathematical topics. In this comprehensive guide, we will walk through the process of solving the inequality 4 |3x + 5| ≤ 16 step-by-step, explaining each concept clearly. Moreover, we'll explore how to graph the solutions on a coordinate plane, providing visual insight into the problem. Whether you're a student seeking to improve your algebra skills or someone interested in mathematical reasoning, this article offers detailed guidance to master solving inequalities involving absolute values.

Understanding the Absolute Value Inequality

Before diving into the solution, it's essential to understand the nature of the absolute value and its properties.

What Is Absolute Value?

The absolute value of a number refers to its distance from zero on the number line, regardless of direction. It is always non-negative.

- For any real number a, |a| ≥ 0.
- |a| = a if a ≥ 0.
- |a| = -a if a < 0.

Interpreting the Inequality 4 |3x + 5| ≤ 16

The inequality involves a scaled absolute value:

$$
4 |3x + 5| \leq 16
$$

This inequality states that four times the absolute value of 3x + 5 is less than or equal to 16.

Step-by-Step Solution of the Inequality

Let's systematically solve 4 |3x + 5| ≤ 16.

Step 1: Isolate the Absolute Value

To isolate the absolute value, divide both sides of the inequality by 4:

$$
|3x + 5| \leq \frac{16}{4}
$$

Simplify the right side:

$$
|3x + 5| \leq 4
$$

This indicates that the expression 3x + 5 is within a distance of 4 units from zero.

Step 2: Convert the Absolute Value Inequality to a Compound Inequality

By definition, |A| ≤ B (where B ≥ 0) can be rewritten as:

$$
- B \leq A \leq B
$$

Applying this to our inequality:

$$
-4 \leq 3x + 5 \leq 4
$$

This compound inequality encapsulates all values of x where 3x + 5 is within 4 units of zero.

Step 3: Solve the Compound Inequality for x

Now, solve for x in the combined inequality:

1. Subtract 5 from all parts:

$$

-4 - 5 \leq 3x + 5 - 5 \leq 4 - 5
$$

Simplify:

$$
-9 \leq 3x \leq -1
$$

2. Divide all parts by 3:

$$
\frac{-9}{3} \leq x \leq \frac{-1}{3}
$$

Simplify:

$$
-3 \leq x \leq -\frac{1}{3}
$$

This interval contains all solutions to the inequality.

Solution Set

The solution to $4 |3x + 5| \leq 16$ is all x in the interval:

$$
\boxed{
x \in \left[-3,\, -\frac{1}{3} \right]
}
$$

This means that any value of x between -3 and -1/3, inclusive, satisfies the original inequality.

Graphing the Solution on the Number Line

Visualizing the solution set is an essential part of understanding inequalities. Graphing helps to see the range of solutions clearly.

Steps to Graph the Solution

1. Draw a horizontal number line with appropriate markers for -3 and -1/3.

2. Use closed circles at -3 and -1/3 to indicate that these endpoints are included (since the inequality is ≤, which is inclusive).
3. Shade the segment between -3 and -1/3 to represent all solutions.

This visual confirms that the solution set is a closed interval on the number line.

Additional Tips for Solving Absolute Value Inequalities

To enhance your understanding and problem-solving skills, consider these tips:

1. Recognize the Types of Absolute Value Inequalities

- $|A| \leq B$ (less than or equal to): leads to a double inequality.
- $|A| < B$ (less than): similar to the above but with strict inequalities.
- $|A| \geq B$ (greater than or equal to): splits into two separate inequalities.
- $|A| > B$ (greater than): also splits but with strict inequalities.

2. Handling Inequalities with Negative or Zero Values

- When the right side is negative, the inequality $|A| \leq$ negative number has no solution because absolute value cannot be negative.
- When the right side is zero, $|A| \leq 0$, the only solution is $A = 0$.

3. Remember the Domain Restrictions

Always check if the solution makes sense within the domain of the original problem, especially if additional constraints are present.

Common Mistakes to Avoid

- Forgetting to split the inequality: When dealing with $|A| \leq B$, always consider both $A \leq B$ and $A \geq -B$.
- Incorrect algebraic steps: Ensure proper division, particularly when dividing by negative numbers (which reverses inequalities).
- Ignoring endpoints: When the inequality includes ≤ or ≥, include endpoints in the solution set.

Practice Problems for Mastery

To reinforce the concepts, try solving these inequalities:

1. $|2x - 7| < 5$
2. $3 |x + 4| \geq 6$
3. $|x - 2| > 3$
4. $4 |x - 1| \leq 8$

Solutions to these will help solidify understanding of various inequality forms involving absolute values.

Conclusion

Solving inequalities involving absolute values, such as $4 |3x + 5| \leq 16$, involves understanding the fundamental properties of absolute values and translating inequalities into manageable algebraic steps. By isolating the absolute value, converting to a compound inequality, and solving for the variable, you can find the solution set efficiently. Visualizing the solutions on a number line further enhances comprehension, making the abstract concepts more tangible.

Mastering these techniques provides a strong foundation for tackling more complex problems in algebra and beyond. Remember to verify solutions and be cautious with inequality directions, especially when dividing by negative numbers. With practice, solving absolute value inequalities becomes an intuitive and straightforward process, empowering you to approach a wide range of mathematical challenges confidently.

Keywords: solve inequality, absolute value inequality, graph solution, algebra, compound inequality, number line, inequality solving steps, absolute value properties, math practice

Frequently Asked Questions

How do I start solving the inequality $4 |3x + 5| \leq 16$?

First, divide both sides of the inequality by 4 to isolate the absolute value: $|3x + 5| \leq 4$.

What is the next step after simplifying to $|3x + 5| \leq 4$?

Next, rewrite the absolute value inequality as a compound inequality: $-4 \leq 3x + 5 \leq 4$.

How do I solve for x in the inequality -4 ≤ 3x + 5 ≤ 4?

Subtract 5 from all parts: -4 - 5 ≤ 3x ≤ 4 - 5, which simplifies to -9 ≤ 3x ≤ -1. Then, divide all parts by 3: -3 ≤ x ≤ -1/3.

What is the solution set for the inequality 4 |3x + 5| ≤ 16?

The solution set is all x such that -3 ≤ x ≤ -1/3.

How do I graph the solution on a number line?

Draw a line, mark the points -3 and -1/3, and shade the region between them, including the endpoints since the inequality is less than or equal to.

What does the graph of the solution look like?

It is a closed interval between -3 and -1/3 on the number line, indicating all x-values within this range satisfy the inequality.

Additional Resources

Solve The Inequality And Graph The Solution. 4 |3x + 5| 16 is a common problem in algebra that combines the concepts of absolute value inequalities and graphing solutions on a coordinate plane. Understanding how to approach such inequalities not only enhances your algebraic skills but also sharpens your ability to interpret and visualize solutions visually. In this comprehensive guide, we will break down the steps involved in solving this inequality and demonstrate how to accurately graph the solution set, making the process accessible whether you're a student brushing up on algebra or a teacher preparing lesson materials.

Understanding the Inequality

Before diving into the solution process, let's clarify what the inequality 4 |3x + 5| 16 entails. The key component here is the absolute value expression |3x + 5|, which measures the distance of 3x + 5 from zero on the number line, regardless of direction.

The inequality involves a comparison between 4 times the absolute value and the number 16. Depending on whether the inequality is less than, greater than, or equal to, the solution set will differ accordingly.

> Note: The inequality in the problem is written as 4 |3x + 5| 16. Since the comparison operator isn't specified, common interpretations are:

- 4 |3x + 5| < 16 (less than)
- 4 |3x + 5| > 16 (greater than)
- 4 |3x + 5| ≤ 16 (less than or equal to)
- 4 |3x + 5| ≥ 16 (greater than or equal to)

For this guide, we'll consider the most typical case: $4 |3x + 5| \leq 16$. If your problem differs, you can adapt the steps similarly.

Step 1: Set Up the Inequality Properly

Given the inequality:

> $4 |3x + 5| \leq 16$

The first step is to isolate the absolute value expression:

- Divide both sides by 4 to simplify:

$$
|3x + 5| \leq \frac{16}{4} = 4
$$

Now the inequality becomes:

> $|3x + 5| \leq 4$

This form is much easier to work with because it sets a boundary on the magnitude of $3x + 5$.

Step 2: Understand Absolute Value Inequalities

Absolute value inequalities of the form:

$$
|A| \leq B
$$

where $B \geq 0$, translate into a double inequality:

$$
-B \leq A \leq B
$$

Similarly, for $|A| < B$, the solution is:

$$
-B < A < B
$$

And for $|A| \geq B$, the solution breaks into two parts:

- $A \leq -B$ or $A \geq B$

In our case, since $|3x + 5| \leq 4$, the solution is:

$$
-4 \leq 3x + 5 \leq 4
$$

Step 3: Solve the Compound Inequality

Now, let's solve:

$$
-4 \leq 3x + 5 \leq 4
$$

Step 3.1: Subtract 5 from all parts

$$
-4 - 5 \leq 3x \leq 4 - 5
$$

which simplifies to:

$$
-9 \leq 3x \leq -1
$$

Step 3.2: Divide all parts by 3

Since 3 is positive, the inequality signs remain the same:

$$
\frac{-9}{3} \leq x \leq \frac{-1}{3}
$$

which simplifies to:

$$
-3 \leq x \leq -\frac{1}{3}
$$

Result of the Solution

The solution set for the inequality $4 |3x + 5| \leq 16$ is:

$$
x \in [-3, -\frac{1}{3}]
$$

This means x can be any real number between -3 and -1/3, inclusive.

Graphing the Solution Set

Visualizing the solution on a number line helps solidify understanding. Here’s how to do it step-by-step.

Step 1: Draw a Number Line

Create a horizontal line with labeled points at -3 and -1/3. Mark these points clearly.

Step 2: Indicate the Interval

Since the solution includes -3 and -1/3 (the inequalities are ≤), you should:

- Draw a solid circle at -3 and -1/3 to indicate these points are included.
- Shade the region between -3 and -1/3 to represent all the solutions within this interval.

Step 3: Confirm the Graph

Your graph should show a solid line (or shading) covering from -3 to -1/3, inclusive of both endpoints.

Additional Considerations: Other Inequality Types

If the original inequality was $4 |3x + 5| > 16$, the solution would involve two separate inequalities:

$$
|3x + 5| > 4
$$

which translates to:

$$
3x + 5 > 4 \quad \text{or} \quad 3x + 5 < -4
$$

and solving each separately:

- For $3x + 5 > 4$:

$$
3x > -1 \Rightarrow x > -\frac{1}{3}
$$

- For $3x + 5 < -4$:

$$
3x < -9 \Rightarrow x < -3
$$

The solution set is:

$$
x < -3 \quad \text{or} \quad x > -\frac{1}{3}
$$

Graphically, this would be represented as two rays extending to infinity, excluding the interval between -3 and -1/3.

Summary of Key Steps

Here's a quick checklist to approach similar absolute value inequalities:

1. Isolate the absolute value by dividing or simplifying as needed.
2. Translate the inequality into a double inequality (for ≤ or ≥) or split into two inequalities (for > or <).
3. Solve each inequality separately.
4. Combine the solutions to find the overall solution set.
5. Graph the solution on a number line, using solid or open circles based on whether endpoints are included, and shade the appropriate region.

Final Thoughts

Solving inequalities involving absolute value can initially seem intimidating, but breaking them down systematically makes the process straightforward. Remember that absolute value expressions define a distance, which leads naturally to double inequalities. Graphing these solutions provides a visual confirmation and helps in understanding the range of possible solutions.

Whether you're tackling a homework problem or preparing for an exam, mastering these techniques will significantly improve your algebraic problem-solving skills and your ability to interpret inequalities graphically.

Happy solving!

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