The Term 35x^3y^4 Is A Term In Which Binomial Expansion?

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Understanding binomial expansions is fundamental in algebra and is essential for solving various mathematical problems. One common question encountered by students and enthusiasts alike is: "The term 35x^3y^4 is a term in which binomial expansion?" This article aims to thoroughly analyze this question, elucidate the underlying principles, and guide you through the steps to determine the specific binomial expansion containing this term.

What Is Binomial Expansion?

Before delving into the specifics of the term 35x^3y^4, it is important to understand what binomial expansion entails.

Definition of Binomial Expansion

Binomial expansion refers to the algebraic process of expanding expressions of the form (a + b)^n, where:

- a and b are algebraic expressions (variables, constants, or products thereof),
- n is a non-negative integer.

The expansion results in a sum of terms involving binomial coefficients, which follow the pattern of Pascal's Triangle.

The Binomial Theorem

The Binomial Theorem provides a formula to expand (a + b)^n:

$$
(a + b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^{k}
$$

Where:

- $\binom{n}{k}$ is the binomial coefficient, computed as $\frac{n!}{k!(n - k)!}$,
- The sum runs from k=0 to n.

Each term in the expansion has the form:

$$
\binom{n}{k} a^{n-k} b^{k}
$$

Identifying the Term $35x^3y^4$ in a Binomial Expansion

The core task is to find out in which binomial expansion this specific term appears. To accomplish this, we need to understand how terms in the expansion are structured and how to match the given term to a particular term in the expansion.

General Form of Terms in Binomial Expansion

Suppose we are expanding an expression:

$$
(A + B)^n
$$

The general term (k-th term, starting from k=0) is:

$$
T_{k+1} = \binom{n}{k} A^{n-k} B^{k}
$$

If the binomial involves variables, say:

$$
(a x + b y)^n
$$

then the general term becomes:

$$
T_{k+1} = \binom{n}{k} (a x)^{n - k} (b y)^{k}
$$

which simplifies to:

$$
T_{k+1} = \binom{n}{k} a^{n - k} b^{k} x^{n - k} y^{k}
$$

The powers of x and y in each term are:

- $x^{n - k}$,
- y^{k}.

Matching these powers to the term in question ($x^3 y^4$), we set:

$$
n - k = 3 \quad \text{and} \quad k = 4
$$

Determining the Binomial Expansion Containing $35x^3y^4$

Using the above, we can deduce:

- Since $k = 4$,
- $n - k = 3$,
- Therefore, $n = 7$.

This indicates that the term $35x^3y^4$ appears in the expansion of $(A + B)^7$, where:

- The power of y is 4 (corresponding to $k=4$),
- The power of x is 3 (corresponding to $n - k=3$).

Finding the Specific Binomial Expansion: $(a x + b y)^7$

Given that, our focus narrows down to the expansion of:

$$
(a x + b y)^7
$$

The general term:

$$
T_{k+1} = \binom{7}{k} a^{7 - k} b^{k} x^{7 - k} y^{k}
$$

For $k=4$:

\[

$$
T_{5} = \binom{7}{4} a^{3} b^{4} x^{3} y^{4}
$$

The binomial coefficient:

$$
\binom{7}{4} = \frac{7!}{4! \times 3!} = \frac{5040}{24 \times 6} = 35
$$

So, the coefficient of the term is 35, matching the coefficient in our term ($35x^3 y^4$).

This confirms that the term:

$$
35 x^3 y^4
$$

appears in the expansion of:

$$
(a x + b y)^7
$$

where the specific coefficients a^{3} and b^{4} are involved.

Interpreting the Coefficients and Variables

The coefficient 35 matches the binomial coefficient $\binom{7}{4}$, which is independent of the coefficients a and b. The actual numerical value of the term depends on the values of a and b.

- If the binomial expression is $(x + y)^7$, then the term:

$$
\binom{7}{4} x^{3} y^{4} = 35 x^{3} y^{4}
$$

- Thus, in the case where $a=1$ and $b=1$, the term $35x^3 y^4$ appears directly in the expansion of $(x + y)^7$.

Summary:

Binomial Expansion	Contains the term $35x^3 y^4$	Explanation
$(x + y)^7$	Yes	Coefficient matches $\binom{7}{4}$, powers match
$(a x + b y)^7$	Yes, with appropriate a, b	The term appears with scaled coefficients depending on a, b

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Additional Considerations

While the previous sections focus on the binomial expansion with variables directly as x and y, it is important to understand that:

- The term $35x^3 y^4$ appears in the expansion of $(x + y)^7$,
- It also appears in $(a x + b y)^7$ with specific coefficients for a and b,
- The key is the binomial coefficient $\binom{7}{4} = 35$, which determines the numerical coefficient of the term.

Common Mistakes and Clarifications

When tackling such problems, students often make mistakes. Here are some clarifications:

- Mistake: Assuming the term appears in a different power expansion without verifying the powers.

Correction: Always match the powers of variables with $n - k$ and k to identify the correct expansion.

- Mistake: Confusing the coefficient with the binomial coefficient.

Correction: Remember that the coefficient 35 equals $\binom{7}{4}$, so the binomial expansion must be to the 7th power.

- Mistake: Ignoring the coefficients a and b in scaled binomial expansions.

Correction: The actual numerical coefficient of the term depends on a and b.

Practical Applications of Binomial Expansion and the Term Identification

Understanding which binomial expansion contains a specific term has practical applications in:

- Algebraic simplifications
- Calculating probabilities in binomial distributions
- Expanding polynomial expressions for algebraic manipulations
- Solving combinatorial problems

Knowing how to identify terms like $35x^3 y^4$ allows students and professionals to work efficiently in algebraic calculations and advanced mathematics.

Summary and Conclusion

In conclusion, the term $35x^3 y^4$ appears in the binomial expansion of $(x + y)^7$, owing to:

- The binomial coefficient $\binom{7}{4} = 35$,
- The powers of x and y matching $7 - 4 = 3$ and 4, respectively.

Additionally, this term could also be part of the expansion of $(a x + b y)^7$ with particular values of a and b, but the simplest and most direct identification is in the expansion of $(x + y)^7$.

Key takeaways:

- The term $35x^3 y^4$ is in the expansion of $(x + y)^7$.
- The binomial coefficient $\binom{7}{4}$ determines the numerical coefficient.
- Matching powers of variables in the general term guides the identification process.

Understanding these principles equips students with the skills to analyze binomial expansions efficiently and accurately identify specific terms within polynomial expansions.

Meta-Description

Frequently Asked Questions

In which binomial expansion does the term $35x^3y^4$ appear?

The term $35x^3y^4$ appears in the expansion of $(x + y)^7$.

How can we determine the binomial coefficient for the term $35x^3y^4$?

The binomial coefficient corresponds to $C(n, k)$ where $n=7$ and $k=3$, which is 35, indicating the expansion is $(x + y)^7$.

What is the general term form in a binomial expansion?

The general term in $(x + y)^n$ is given by $T_{k+1} = C(n, k) x^{n-k} y^k$.

How do I verify that 35x^3y^4 is a term in (x + y)^7?

Check if the exponents match the general term: for x^3 y^4, n=7, k=4, and C(7,4)=35, confirming its presence in (x + y)^7.

Can the term 35x^3y^4 appear in the expansion of (x + y)^6?

No, because in (x + y)^6, the maximum exponent of y is 6, but the term with y^4 and x^3 corresponds to n=6, k=4, which is valid; however, the coefficient matches C(6,4)=15, not 35, so 35x^3y^4 does not appear in (x + y)^6.

What is the significance of the coefficient 35 in the term 35x^3y^4?

The coefficient 35 is the binomial coefficient C(7, 4), indicating the term comes from the expansion of (x + y)^7.

Additional Resources

The term 35x^3y^4 is a term in which binomial expansion?

Understanding the appearance of the term 35x^3y^4 within binomial expansions requires a comprehensive exploration of algebraic principles, combinatorial mathematics, and the structure of binomial theorem itself. This question, seemingly straightforward, opens the door to a detailed analysis of how specific terms are derived within binomial expansions, the role of coefficients, and the algebraic manipulations involved. In this article, we will dissect the problem step-by-step, exploring the binomial theorem, the mechanics of term formation, and the precise conditions under which the term 35x^3y^4 emerges.

Introduction to Binomial Expansion

What Is the Binomial Theorem?

The binomial theorem is a fundamental principle in algebra that describes the expansion of powers of binomials—expressions involving two terms, such as (a + b). It provides a systematic way to expand expressions like (a + b)^n into a sum of terms involving powers of a and b.

Mathematically, the binomial theorem states:

$$
(a + b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k}b^k
$$

where $\binom{n}{k}$ is the binomial coefficient, computed as:

$$
\binom{n}{k} = \frac{n!}{k!(n - k)!}
$$

Key Points:
- The expansion results in $n + 1$ terms.
- Each term involves a binomial coefficient and powers of the two terms.

Understanding the Terms in the Expansion

Each term in the expansion has the general form:

$$
\binom{n}{k} a^{n-k} b^{k}
$$

- The binomial coefficient $\binom{n}{k}$ determines the numerical coefficient.
- The powers of a and b depend on k and n.

When dealing with more complex expressions involving multiple variables, such as $(ax + by)^n$, the expansion involves multinomial coefficients, which extend the binomial theorem to multiple terms.

Examining the Term 35x^3y^4

Matching the Term to a Binomial Expansion

The term under investigation, 35x^3y^4, contains variables x and y with specific exponents, as well as a numerical coefficient. To determine in which binomial expansion this term appears, we need to analyze the structure of binomial expansions involving variables x and y.

Key question:
- Does this term come from a binomial expansion of the form $(A + B)^n$ with variables x and y?
- Or does it originate from a multinomial expansion involving more terms?

Since the term involves only x and y, it is logical to consider expansions involving these two variables.

Case 1: Expansion of $(ax + by)^n$

Suppose we consider the binomial $(ax + by)^n$. Its expansion is:

$$
(ax + by)^n = \sum_{k=0}^n \binom{n}{k} (ax)^{n-k} (by)^k
$$

which simplifies to:

$$
\sum_{k=0}^n \binom{n}{k} a^{n-k} b^{k} x^{n-k} y^{k}
$$

In this form:
- The power of x is $n - k$,
- The power of y is k,
- The coefficient is $\binom{n}{k} a^{n-k} b^{k}$.

Similarly, the term's variables are $x^{n - k}$ and y^{k}.

Matching the exponents:

Given the term $x^3 y^4$,
- x exponent: 3,
- y exponent: 4.

From the pattern:

$$
n - k = 3 \implies k = n - 3
$$
$$
k = 4
$$

Thus,

$$
n - 3 = 4 \implies n = 7
$$

So, for the term $x^3 y^4$, the binomial expansion must be of degree $n=7$, with $k=4$.

Determining the Binomial Coefficients and Coefficients of the

Term

The coefficient of the term $x^3 y^4$ in $(ax + by)^7$ is:

$$
\binom{7}{4} a^{7-4} b^{4} = \binom{7}{4} a^{3} b^{4}
$$

Calculating $\binom{7}{4}$:

$$
\binom{7}{4} = \frac{7!}{4! \times 3!} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = 35
$$

Therefore, the coefficient before the variables is 35.

Assuming a and b are both 1 (standard binomial expansion), the coefficient of the term $x^3 y^4$ is 35.

Hence:

$$
\boxed{
\text{The term } 35x^3 y^4 \text{ appears in } (x + y)^7
}
$$

or more generally, in $(ax + by)^7$ with $a=b=1$.

Analysis of the Generality of the Result

Implication for the Binomial Expansion

From the above reasoning, the term $35x^3 y^4$ is explicitly part of the binomial expansion $(x + y)^7$. This is because:

- The total degree $n=7$,
- For the term $x^3 y^4$, the exponents sum to $3 + 4 = 7$,
- The binomial coefficient is $\binom{7}{4} = 35$,
- The variables a and b are both taken as 1 for simplicity.

Conclusion:

> The term $35x^3 y^4$ is a term in the binomial expansion of $(x + y)^7$.

Other Possible Binomials

While $(x + y)^7$ is the most straightforward binomial expansion containing this term, it is worth considering whether other binomials could produce the same term.

Suppose we have a binomial $(ax + by)^n$ with arbitrary coefficients a and b. The specific term $35x^3 y^4$ would then arise when:

$$
\binom{n}{k} a^{n-k} b^{k} x^{n-k} y^{k} = 35 x^3 y^4
$$

Matching exponents:

$$
n - k = 3 \quad \Rightarrow \quad k = n - 3
$$
$$
k = 4
$$

This yields:

$$
n - 3 = 4 \Rightarrow n=7
$$

and

$$
k=4
$$

The binomial coefficient:

$$
\binom{7}{4} = 35
$$

The coefficient involving a and b:

$$
a^{3} b^{4}
$$

To get a coefficient exactly 35, the product $a^{3} b^{4}$ must be 1, assuming the binomial coefficients are as above.

Therefore:

- If \(a=b=1\), the expansion is \((x + y)^7\),
- If \(a\) and \(b\) are other values, the coefficient of \(x^3 y^4\) becomes \(35 \times a^{3} b^{4}\),
- To have the coefficient exactly 35, \(a^{3} b^{4} = 1\).

In conclusion:

- The term \(35x^{3} y^{4}\) naturally appears in the expansion of \((x + y)^7\).
- It also appears in scaled versions \((ax + by)^7\) where \(a^{3} b^{4} = 1\).

Broader Context: Multinomial Expansion and Variable Combinations

While the above analysis centers on binomials, it is insightful to briefly consider multinomial expansions, especially since the term involves two variables.

The Multinomial Theorem

The multinomial theorem generalizes the binomial theorem for expressions involving more than two terms:

\[
(a_1 + a_2 + \dots + a_m)^n = \sum_{k_1 + k_2 + \dots + k_m = n} \frac{n!}{k_1

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