Use The Equations To Find Z/x And Z/y.x2 + 2y2 + 9z2 = 1

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Understanding how to manipulate and interpret equations involving multiple variables is fundamental in multivariable calculus and analytical geometry. The given equations, Z/x and $Z/y \cdot x^2 + 2y^2 + 9z^2 = 1$, provide an excellent opportunity to explore the relationships between variables, solve for specific derivatives, and analyze the geometric shapes they represent. This article delves deeply into the methods to find the derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$, as well as understanding the geometric significance of the second equation.

Understanding the Given Equations

Before embarking on the process of finding derivatives, it's crucial to interpret the equations correctly and understand what they represent.

The Equations at a Glance

The two equations provided are:

1. $\frac{Z}{x}$ (which appears to be a ratio involving Z and x)
2. $x^2 + 2y^2 + 9z^2 = 1$

However, it seems there might be a slight typographical ambiguity. Based on common forms in multivariable calculus, it’s likely that the intended equations are:

- Equation 1: $\frac{\partial Z}{\partial x}$ (or perhaps the ratio Z/x)
- Equation 2: $x^2 + 2y^2 + 9z^2 = 1$

Given the context, we'll interpret the second as the equation of an ellipsoid, and the first as involving the partial derivatives of Z with respect to x and y, perhaps in the context of an implicit function.

Interpreting the Second Equation: The Ellipsoid

Equation of the Ellipsoid

The second equation:

$$ x^2 + 2y^2 + 9z^2 = 1 $$

is a standard form of an ellipsoid centered at the origin. It describes a three-dimensional surface where the sum of the scaled squares of x, y, and z equals 1.

Geometric significance:

- This surface is symmetric about the axes.
- The semi-axes lengths can be deduced from the coefficients:
- Along the x-axis: $a = 1$
- Along the y-axis: $b = \frac{1}{\sqrt{2}}$
- Along the z-axis: $c = \frac{1}{3}$

Implication for derivatives:

- When solving for Z as a function of x and y, the equation can be rearranged.

Expressing Z as a Function of x and y

Since the second equation relates x, y, and z, and assuming z is a function of x and y, i.e., $z = Z(x,y)$, we can write:

$$
x^2 + 2y^2 + 9Z^2 = 1
$$

Rearranged to express Z:

$$
9Z^2 = 1 - x^2 - 2y^2
$$

$$
Z^2 = \frac{1 - x^2 - 2y^2}{9}
$$

Assuming Z is positive (or choosing a branch), we have:

$$
Z(x, y) = \pm \frac{\sqrt{1 - x^2 - 2y^2}}{3}
$$

This explicit form allows us to compute the partial derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$.

Calculating the Partial Derivatives of Z

Derivative with Respect to x

Given:

$$
Z(x, y) = \pm \frac{\sqrt{1 - x^2 - 2y^2}}{3}
$$

The derivative $\frac{\partial Z}{\partial x}$ is:

$$
\frac{\partial Z}{\partial x} = \pm \frac{1}{3} \cdot \frac{1}{2} \cdot \left(1 - x^2 - 2y^2 \right)^{-\frac{1}{2}} \cdot (-2x)
$$

Simplifying:

$$
\frac{\partial Z}{\partial x} = \pm \frac{1}{3} \times \frac{-x}{\sqrt{1 - x^2 - 2y^2}}
$$

$$
\boxed{
\frac{\partial Z}{\partial x} = - \frac{x}{3 \sqrt{1 - x^2 - 2y^2}}
}
$$

Derivative with Respect to y

Similarly, for $\frac{\partial Z}{\partial y}$:

$$
\frac{\partial Z}{\partial y} = \pm \frac{1}{3} \times \frac{1}{2} \times \left(1 - x^2 - 2y^2 \right)^{-\frac{1}{2}} \times (-4y)
$$

Simplify:

$$
\frac{\partial Z}{\partial y} = \pm \frac{-2y}{3 \sqrt{1 - x^2 - 2y^2}}
$$

or

$$
\boxed{
\frac{\partial Z}{\partial y} = - \frac{2y}{3 \sqrt{1 - x^2 - 2y^2}}
}
$$

Note: The choice of positive or negative root depends on the specific branch of the square root relevant to the problem.

Interpreting Z/x in Context

The expression Z/x can be viewed as the ratio of Z to x. Using the explicit form of Z:

$$
Z = \pm \frac{\sqrt{1 - x^2 - 2 y^2}}{3}
$$

we get:

$$
\frac{Z}{x} = \pm \frac{\sqrt{1 - x^2 - 2 y^2}}{3x}
$$

This ratio provides insights into the relative change of Z with respect to x, especially near points where $x \neq 0$. As x approaches zero, the ratio's behavior depends critically on the numerator. For example:

- When $x \rightarrow 0$, $Z \rightarrow \pm \frac{\sqrt{1 - 2 y^2}}{3}$
- The behavior of Z/x near $x=0$ indicates the slope or rate of change of Z relative to x.

Geometric and Analytical Significance

Understanding Surface Behavior

The derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ describe the slopes of the surface $Z(x,y)$ at any point. These are essential for:

- Computing tangent planes
- Analyzing surface curvature
- Understanding how the surface responds to changes in x and y

Implications for the Shape of the Surface

The negative signs in the derivatives indicate that as x or y increase, Z generally decreases, assuming the positive branch of the square root. The magnitude of the derivatives depends on the position on the surface, especially how close $x^2 + 2 y^2$ is to 1.

Summary of Key Findings

- The surface described by $x^2 + 2 y^2 + 9 z^2 = 1$ is an ellipsoid centered at the origin.
- Expressing Z explicitly as a function of x and y allows for straightforward calculation of partial derivatives.
- The derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ are rational functions involving the square root of $1 - x^2 - 2 y^2$.
- The ratio Z/x can be expressed explicitly, providing insights into the relative change of Z with respect to x.
- Understanding these derivatives helps in analyzing the geometry and calculus of the surface, including tangent planes and curvature.

Conclusion

The process of finding Z/x and the derivatives $\frac{\partial Z}{\partial x}$ and $\frac{\partial Z}{\partial y}$ hinges on expressing the surface explicitly and applying standard calculus rules

Frequently Asked Questions

How do I find Z/x and Z/y from the equation $x^2 + 2y^2 + 9z^2 = 1$?

To find Z/x and Z/y, differentiate the implicit equation with respect to x and y respectively, treating z as a function of x and y, then solve for Z/x and Z/y.

What method should I use to compute the partial derivatives Z/x and Z/y for the given ellipsoid?

Use implicit differentiation on the equation $x^2 + 2y^2 + 9z^2 = 1$, differentiating both sides with respect to x and y to find the partial derivatives Z/x and Z/y.

Can you show the step-by-step process to find Z/x for

the equation $x^2 + 2y^2 + 9z^2 = 1$?

Yes. Differentiate both sides with respect to x: 2x + 0 + 18z(Z/x) = 0. Then, solve for Z/x: Z/x = -x / (9z).

How do I find Z/y from the equation $x^2 + 2y^2 + 9z^2 = 1$?

Differentiate both sides with respect to y: 0 + 4y + 18z(Z/y) = 0. Solve for Z/y: Z/y = -2y / (9z).

What is the significance of Z/x and Z/y in the context of this equation?

Z/x and Z/y represent the partial derivatives of z with respect to x and y, indicating how z changes as x or y vary on the surface defined by the equation.

Are the derivatives Z/x and Z/y applicable for finding slopes or tangents on the surface?

Yes, Z/x and Z/y are used to determine the slopes of the tangent lines to the surface at a given point in the x and y directions.

What is the final expression for Z/x and Z/y for the surface $x^2 + 2y^2 + 9z^2 = 1$?

The derivatives are Z/x = -x / (9z) and Z/y = -2y / (9z), assuming $z \neq 0$.

Additional Resources

Use The Equations To Find Z/x And Z/y. $x^2 + 2y^2 + 9z^2 = 1$

Understanding how to manipulate and differentiate equations involving multiple variables is a cornerstone of multivariable calculus. In particular, the problem of finding \(\frac{\partial z}{\partial x}\) and \(\frac{\partial z}{\partial y}\) from an implicit equation such as \(x^2 + 2y^2 + 9z^2 = 1\) is a fundamental skill. This involves applying implicit differentiation, a technique that allows us to find derivatives of one variable with respect to another when the variables are intertwined in an equation. In this guide, we'll explore how to use the equations to find \(Z/x\) and \(Z/y\), providing a clear, step-by-step approach suitable for students and professionals alike.

Understanding the Problem

Before diving into differentiation, it's important to understand the structure of the given equation:

\[x^2 + 2y^2 + 9z^2 = 1 \]

This is an implicit equation defining a surface in three-dimensional space. Our goal is to find the partial derivatives of \(z\) with respect to \(x\)

and y:

- $\frac{\partial z}{\partial x}$ (often written as Z_x)
- $\frac{\partial z}{\partial y}$ (often written as Z_y)

These derivatives tell us how the surface changes as we move along the x and y directions, respectively.

Step 1: Recognize the Implicit Relationship

The key insight is that z is implicitly defined by the equation:

$$ f(x, y, z) = x^2 + 2y^2 + 9z^2 - 1 = 0 $$

Because z is not isolated, we need to differentiate this relation with respect to x and y, treating z as a function of these variables.

Step 2: Applying Implicit Differentiation

Differentiation with respect to x:

Treat y as constant and differentiate both sides of the equation:

$$
\frac{\partial}{\partial x} (x^2 + 2y^2 + 9z^2) = \frac{\partial}{\partial x} (1)
$$

Since y is constant:

$$
2x + 0 + 18z \frac{\partial z}{\partial x} = 0
$$

which simplifies to:

$$
2x + 18z \frac{\partial z}{\partial x} = 0
$$

Solve for $\frac{\partial z}{\partial x}$:

$$
\frac{\partial z}{\partial x} = Z_x = - \frac{2x}{18z} = - \frac{x}{9z}
$$

Differentiation with respect to y:

Treat x as constant and differentiate:

$$
0 + 4y + 18z \frac{\partial z}{\partial y} = 0
$$

which simplifies to:

$$
4y + 18z \frac{\partial z}{\partial y} = 0
$$

Solve for $\frac{\partial z}{\partial y}$:

$$
\frac{\partial z}{\partial y} = Z_y = - \frac{4y}{18z} = - \frac{2y}{9z}
$$

Step 3: Summarize the Results

The partial derivatives of z with respect to x and y are:

$$
Z_x = - \frac{x}{9z}
$$

$$
Z_y = - \frac{2y}{9z}
$$

These expressions provide the slopes of the surface in the directions of x and y at any point (x, y, z) satisfying the original equation.

Step 4: Interpreting the Results

Geometric Perspective:

- Z_x indicates how the height z changes as you move along the x-direction on the surface. A negative value suggests that increasing x tends to decrease z when $z > 0$.
- Z_y shows how z varies with y. Similarly, its sign indicates the direction of change.

Practical applications:

- In physics and engineering, such derivatives help analyze surface slopes, rate of change, and stability.
- In optimization, understanding these derivatives assists in locating maxima and minima on the surface.

Additional Considerations

When is the derivative undefined?

- If $z = 0$, the derivatives become undefined because of division by zero. This indicates potential singular points or points where the surface's slope is vertical.

How to find the actual value of z?

- For specific (x, y), you can solve the original equation for z:

$$
9z^2 = 1 - x^2 - 2y^2
$$

$$
z = \pm \frac{\sqrt{1 - x^2 - 2y^2}}{3}
$$

- The choice of positive or negative root depends on the context.

Practical Example

Suppose we want to compute Z_x and Z_y at the point $(x, y) = (0.5, 0.4)$, assuming z is positive:

1. Calculate z:

$$
z = \frac{\sqrt{1 - (0.5)^2 - 2(0.4)^2}}{3} = \frac{\sqrt{1 - 0.25 - 2 \times 0.16}}{3} = \frac{\sqrt{1 - 0.25 - 0.32}}{3} = \frac{\sqrt{0.43}}{3}
$$

$$
z \approx \frac{0.6557}{3} \approx 0.2186
$$

2. Compute derivatives:

$$
Z_x = - \frac{0.5}{9 \times 0.2186} \approx - \frac{0.5}{1.967} \approx -0.254
$$

$$
Z_y = - \frac{2 \times 0.4}{9 \times 0.2186} \approx - \frac{0.8}{1.967} \approx -0.406
$$

This quantifies how the surface slopes in the x and y directions at that specific point.

Conclusion

Using the equations to find Z/x and Z/y through implicit differentiation offers a powerful method for analyzing complex surfaces defined by multivariable equations. The process involves recognizing the implicit relationship, differentiating with respect to each variable while treating others as constants, and then solving for the derivatives. The results not only provide geometric insights but also serve as foundational tools in advanced calculus, physics, engineering, and data analysis. Mastery of implicit differentiation ensures that you can confidently analyze and interpret surfaces and their behaviors in multi-dimensional spaces.

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