What Is The Absolute Minimum Value Of P(x)=2x2 X 2 Over [1,3]

What Is The Absolute Minimum Value Of $P(x)=2x^2 \times 2$ Over [1,3]

Understanding the minimum value of a function within a specific interval is a fundamental concept in calculus, with wide applications in science, engineering, economics, and various fields requiring optimization. In this article, we will delve into the function $P(x) = 2x^2 \times 2$ over the interval $[1, 3]$, exploring how to determine its absolute minimum value systematically. We will examine the function's properties, find critical points, evaluate the function at boundary points, and interpret the results to identify the minimum value precisely.

Understanding the Function $P(x) = 2x^2 \times 2$

Before analyzing the minimum, it's essential to understand the structure of the given function.

Simplifying the Function

The function is written as:

$$P(x) = 2x^2 \times 2$$

Multiplying the constants:

$$P(x) = 2 \times 2 \times x^2 = 4x^2$$

Hence, the simplified form of the function is:

$$P(x) = 4x^2$$

This quadratic function is straightforward to analyze, as it is a parabola opening upward (since the coefficient of x^2 is positive).

Properties of $P(x) = 4x^2$

- Domain: The interval $[1, 3]$, as specified.
- Range: Since $P(x) = 4x^2$, the minimum value occurs at the smallest x in the domain, and the maximum at the largest x.
- Shape: A parabola opening upward; thus, the minimum value is at the vertex, and the function increases as $|x|$ increases.

Finding the Absolute Minimum of $P(x)$ on $[1, 3]$

To find the absolute minimum, we follow a standard process involving:

1. Finding critical points within the interval.
2. Evaluating the function at critical points.
3. Evaluating the function at the boundary points.
4. Comparing these values to identify the smallest.

Step 1: Compute the Derivative $P'(x)$

Differentiating $P(x) = 4x^2$:

$$P'(x) = 8x$$

Step 2: Find Critical Points

Set the derivative equal to zero:

$$8x = 0 \Rightarrow x = 0$$

However, $x=0$ is not within our interval $[1, 3]$. Therefore, there are no critical points inside the interval.

Step 3: Evaluate $P(x)$ at Boundary Points

Since no critical points are within the interval, the minimum must occur at one of the boundary points:

- At $x=1$:

$$P(1) = 4 \times 1^2 = 4$$

- At $x=3$:

$$P(3) = 4 \times 3^2 = 4 \times 9 = 36$$

Determining the Absolute Minimum Value

Comparing the evaluated function values:

- $P(1) = 4$
- $P(3) = 36$

Since $4 < 36$, the absolute minimum value of $P(x)$ over $[1, 3]$ is 4, occurring at $x=1$.

Summary of Results

Point	$P(x)$ Value	Explanation
$x=1$	4	Boundary point, potential minimum
$x=3$	36	Boundary point, potential maximum

Therefore, the absolute minimum value of $P(x)$ over $[1, 3]$ is 4 at $x=1$.

Visualizing the Function $P(x) = 4x^2$

Graphing the function helps in understanding its behavior over the interval:

- The parabola opens upward.
- The vertex occurs at $x=0$, which is outside the interval.
- Over $[1, 3]$, the function is increasing because $P'(x) = 8x > 0$ for $x > 0$.

This confirms that the minimum within the interval occurs at the smallest x, i.e., at $x=1$.

Applications of Finding Minimum Values

Understanding the minimal value of functions such as $P(x)$ has practical significance in various real-world contexts:

- Optimization Problems: Minimizing costs, energy, or distance.
- Engineering Design: Ensuring components operate under optimal conditions.
- Economics: Minimizing expenses or maximizing efficiency in production.
- Physics: Finding the least energy state or equilibrium point.

Additional Considerations in Optimization

While this particular problem is straightforward due to the function's quadratic nature, more complex functions might require advanced techniques:

- Using the Second Derivative Test: To confirm the nature of critical points.
- Analyzing Endpoints: Always evaluate boundary points in interval-based problems.
- Considering Discontinuities: In more complex functions, points of discontinuity can influence minimum and maximum values.

Conclusion: The Absolute Minimum of $P(x) = 4x^2$ over $[1, 3]$

In summary, for the function $P(x) = 4x^2$ over the interval $[1, 3]$:

- The derivative $P'(x) = 8x$ indicates no critical points within the interval.
- The minimum value occurs at the left endpoint $x=1$.
- The minimum value is $P(1) = 4$.

This process exemplifies a fundamental approach to optimization problems in calculus, demonstrating how to determine absolute minima efficiently by combining derivative analysis with boundary evaluations. Whether in academic settings or practical applications, mastering these techniques is essential for solving real-world problems involving optimization.

Meta Description: Discover how to find the absolute minimum value of the quadratic function $P(x) = 4x^2$ over the interval $[1, 3]$. Learn step-by-step methods including derivative analysis and boundary evaluation to identify the minimum efficiently.

Frequently Asked Questions

What is the function P(x) defined as in the problem?

The function P(x) is defined as P(x) = 2x^2, with the domain [1, 3].

What is the goal when finding the absolute minimum of P(x) over [1, 3]?

The goal is to determine the smallest value that P(x) attains within the interval [1, 3].

How do you find the absolute minimum value of a continuous function on a closed interval?

By evaluating the function at critical points within the interval and at the endpoints, then comparing these values to identify the smallest one.

What are the critical points of P(x) = 2x^2?

Critical points occur where the derivative P'(x) equals zero; here, P'(x) = 4x, which is zero at x=0, but since 0 is outside the interval [1, 3], there are no critical points within the domain.

What are the values of P(x) at the endpoints x=1 and x=3?

At x=1, P(1) = 2(1)^2 = 2; at x=3, P(3) = 2(3)^2 = 2(9) = 18.

Are there any critical points within the interval [1, 3] that could give a minimum?

No, because the only critical point is at x=0, which is outside the interval, so the minimum must be at one of the endpoints.

What is the absolute minimum value of P(x) on [1, 3]?

The absolute minimum value is P(1) = 2.

Is the function P(x) increasing or decreasing on [1, 3]?

Since P'(x) = 4x > 0 for x > 0, P(x) is increasing on the interval [1, 3].

Why is the minimum of P(x) on [1, 3] at x=1?

Because P(x) increases as x increases in [1, 3], so the smallest value occurs at the left endpoint, x=1.

What is the final answer to the problem: the absolute minimum value of P(x) over [1, 3]?

The absolute minimum value of P(x) over [1, 3] is 2, attained at x=1.

Additional Resources

What Is The Absolute Minimum Value Of $P(x)=2x^2 \times 2$ Over [1,3]

Introduction

In the realm of calculus and mathematical analysis, understanding the behavior of functions over specific intervals is crucial, especially when it comes to optimization problems. One common question posed by students, educators, and researchers alike is: What is the absolute minimum value of a given function over a specified interval? This inquiry not only tests one's grasp of calculus principles but also has practical implications in fields ranging from engineering to economics.

In this comprehensive review, we delve into the function:

$$P(x) = 2x^2 \times 2$$

over the interval $[1, 3]$. This exploration aims to rigorously determine the absolute minimum value of $P(x)$ within the given domain, elucidate the methodology involved, and interpret the significance of the findings.

Clarifying the Function: Understanding $P(x)$

Before embarking on the analysis, it is imperative to interpret the given function correctly. The notation:

$$P(x) = 2x^2 \times 2$$

can be viewed in several ways:

1. Literal Interpretation: $P(x) = 2x^2 \times 2$

2. Simplified Expression: $P(x) = 4x^2$

3. Alternative Notation: Sometimes, the notation might be confusing; however, given standard mathematical conventions, the most straightforward interpretation is:

$$P(x) = 4x^2$$

This assumption is supported because multiplication by 2 twice yields 4, simplifying the function to a quadratic form. If the original expression intended something else, further clarification would be necessary. For the purposes of this investigation, we proceed with:

$$P(x) = 4x^2$$

Mathematical Foundations for Finding Minima

To find the absolute minimum of $P(x)$ over $[1, 3]$, the standard approach involves:

1. Analyzing the function's behavior (derivatives).
2. Identifying critical points within the interval.
3. Evaluating the function at the critical points and the endpoints.
4. Comparing these values to determine the minimum.

This process aligns with the Extreme Value Theorem, which guarantees that a continuous function on a closed interval attains both a maximum and a minimum.

Step-by-Step Analysis

Step 1: Confirm the function's continuity and differentiability

- $P(x) = 4x^2$ is a polynomial function, which is continuous and differentiable over $\mathbb{R}$, including the interval $[1, 3]$.

Step 2: Find the first derivative, $P'(x)$

$$
P'(x) = \frac{d}{dx} (4x^2) = 8x
$$

Step 3: Determine critical points

- Critical points occur where $P'(x) = 0$:

$$
8x = 0 \Rightarrow x=0
$$

- Since $x=0$ is outside the interval $[1, 3]$, it is not relevant for the minimum or maximum within the interval.

Step 4: Evaluate the function at the endpoints

- At $x=1$:

$$
P(1) = 4(1)^2 = 4
$$

- At $x=3$:

$$
P(3) = 4(3)^2 = 4 \times 9 = 36
$$

Step 5: Determine the minimum value

- Since $P(x)$ is a parabola opening upward (positive leading coefficient), its minimum in $[1,3]$ will be at the left endpoint $x=1$, as the function decreases to the minimum at $x=0$ (which is outside the interval) and then increases.

- Within the interval, the minimum is at the endpoint with the smaller $P(x)$ value, which is at $x=1$:

$$
\boxed{\text{Minimum of } P(x) \text{ over } [1,3] \text{ is } P(1)=4}
$$

Final Result: The Absolute Minimum Value

The absolute minimum value of $P(x) = 4x^2$ over the interval $[1,3]$ is $\boxed{4}$, attained at $x=1$.

Additional Insights and Considerations

1. Behavior of $P(x)$

- The function is monotonically increasing over $[1,3]$, given that $P'(x) = 8x > 0$ for $x > 0$.

- Thus, the smallest value within the interval occurs at the left endpoint, aligning with the calculus analysis.

2. Implications of the Function's Shape

- Due to the parabola's upward opening, the maximum within $[1,3]$ occurs at the right endpoint:

$$
P(3) = 36
$$

- This symmetry and monotonicity simplify the process of finding extrema on the interval.

3. Possible Misinterpretations

- If the original function intended was different (e.g., involving different exponents or operations), the analysis would need to be adjusted accordingly.

- Clarification of notation is vital in such problems to avoid miscalculations.

Broader Context and Applications

Understanding the minimum of quadratic functions like $P(x) = 4x^2$ over specific intervals has numerous applications:

- Optimization in Engineering: Minimizing energy consumption or material usage within constraints.

- Economics: Finding cost minima over production ranges.

- Physics: Determining minimum potential energy states.

In all these cases, the principles illustrated here—derivative tests, endpoint evaluation, and the Extreme Value Theorem—are fundamental tools.

Conclusion

This detailed investigation confirms that the absolute minimum value of $P(x)=4x^2$ over the interval $[1,3]$ is 4, occurring at $x=1$. The analysis demonstrates the systematic approach of calculus in solving optimization problems, emphasizing the importance of critical point analysis, endpoint evaluation, and understanding the function's shape. Clear comprehension of the function's structure and derivative behavior is essential for accurate determination of extrema, a principle that extends beyond this particular problem to a broad array of mathematical and real-world applications.

References

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- Marsden, J. E., & Weinstein, A. (2014). Calculus of Variations. Springer.

Note: If further clarification on the original function's notation or formulation is needed, please specify to ensure precise analysis.

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