What Is The Value Of (a,7) And (1,a) And A Slope Of -5/9

What Is The Value Of (a,7) And (1,a) And A Slope Of -5/9

Understanding the relationships between points on a coordinate plane and the concept of slope is fundamental in algebra and coordinate geometry. When given specific points such as (a, 7) and (1, a), along with a known slope of -5/9, it becomes essential to determine the value of the variable 'a'. This process involves applying the slope formula, algebraic manipulation, and understanding the properties of linear equations. This article offers a comprehensive guide to solving for 'a' under these conditions, exploring the underlying concepts, step-by-step solutions, and practical applications. Whether you're a student preparing for exams or a math enthusiast seeking clarity, this detailed exploration will enhance your understanding of coordinate points and slope calculations.

Understanding Coordinates and Slope

What Are Coordinates?

Coordinates are pairs of numbers that specify the position of a point in a two-dimensional plane. The format (x, y) indicates the point's horizontal (x) and vertical (y) positions relative to the origin (0,0):

- x-coordinate: Determines the position along the horizontal axis.
- y-coordinate: Determines the position along the vertical axis.

In our case:
- The point (a, 7) has an x-coordinate of 'a' and a y-coordinate of 7.
- The point (1, a) has an x-coordinate of 1 and a y-coordinate of 'a'.

What Is Slope?

Slope measures the steepness or incline of a line. It indicates how much y changes for a unit increase in x. The slope between two points, (x_1, y_1) and (x_2, y_2), is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Where:
- m is the slope.
- (x_1, y_1) and (x_2, y_2) are the coordinates of the two points.

In this context, the slope is given as -5/9, meaning the line descends as x increases.

Setting Up the Problem

Given:
- Two points: (a, 7) and (1, a)
- Slope: $m = -\frac{5}{9}$

Our goal is to find the value of 'a' that satisfies these conditions.

Applying the Slope Formula

Step 1: Write the slope formula with the given points

Using the points:
$$
m = \frac{a - 7}{1 - a}
$$
or
$$
m = \frac{7 - a}{a - 1}
$$

Note: Whether to use $\frac{a - 7}{1 - a}$ or $\frac{7 - a}{a - 1}$ depends on the order of points, but both are valid as long as the slope formula is applied consistently.

Step 2: Set the slope equal to -5/9

$$
\frac{a - 7}{1 - a} = -\frac{5}{9}
$$

Solving for 'a'

Step 3: Cross-multiplied Equation

Cross-multiplied form:

$$
9(a - 7) = -5(1 - a)
$$

Step 4: Expand both sides

$$
9a - 63 = -5 + 5a
$$

Step 5: Rearrange terms to isolate 'a'

Subtract 5a from both sides:

$$
9a - 5a - 63 = -5
$$

$$
4a - 63 = -5
$$

Add 63 to both sides:

$$
4a = 58
$$

Divide both sides by 4:

$$
a = \frac{58}{4} = \frac{29}{2} = 14.5
$$

Result: The value of 'a' is $\boxed{\frac{29}{2}}$ or 14.5.

Verification of the Solution

Check the calculated 'a' in the slope formula

Substitute $a = \frac{29}{2}$ into the slope formula:

$$
\frac{a - 7}{1 - a} = \frac{\frac{29}{2} - 7}{1 - \frac{29}{2}}
$$

Calculate numerator:

$$
\frac{29}{2} - 7 = \frac{29}{2} - \frac{14}{2} = \frac{15}{2}
$$

Calculate denominator:

$$
1 - \frac{29}{2} = \frac{2}{2} - \frac{29}{2} = -\frac{27}{2}
$$

Compute the fraction:

$$
\frac{\frac{15}{2}}{-\frac{27}{2}} = \frac{15/2}{-27/2} = \frac{15/2} \times \frac{2}{-27} = \frac{15}{-27} = -\frac{5}{9}
$$

This matches the given slope, confirming that a = 14.5 is correct.

Additional Scenarios and Variations

1. If the points are reversed

- The calculation remains similar, but the order of points might affect the sign.
- Always verify by plugging the value back into the slope formula.

2. When points are given with different coordinates

- The same approach applies; just substitute the given points into the slope formula.

3. Solving for 'a' with different slopes

- Adjust the slope value in the equation and solve accordingly.

Practical Applications of Finding 'a'

Understanding how to find the value of 'a' based on points and slope has broad applications, including:

- Graphing linear equations: Determining unknown points for accurate plotting.
- Analyzing data trends: Calculating missing data points based on known relationships.
- Engineering and physics: Calculating variables when modeling linear relationships.
- Economics: Understanding rate changes and relationships between variables.

Summary and Key Takeaways

- To find the value of 'a' given points and slope, start with the slope formula.
- Carefully substitute the points into the formula, ensuring correct order.
- Cross-multiplied algebra simplifies the equation to isolate 'a'.
- Always verify the solution by substituting back into the original formula.
- The value of 'a' in this specific problem is \(\boxed{\frac{29}{2}} \) or 14.5, which satisfies the given conditions.

Conclusion

Determining the value of 'a' from points like (a, 7) and (1, a) when given a specific slope is a fundamental skill in algebra. The process involves understanding coordinate geometry, applying the slope formula correctly, and performing algebraic manipulations. This example demonstrates how algebra can be used to solve real-world problems involving linear relationships. Mastery of these concepts enables students and professionals to analyze and interpret data effectively, enhancing their mathematical reasoning and problem-solving skills.

Meta Keywords: coordinate geometry, slope calculation, find 'a' in points, algebra problems, linear equations, slope formula, coordinate points, algebra tutorial, math problem solving, algebraic equations.

Frequently Asked Questions

How do I find the value of 'a' given the points (a, 7) and (1, a) with a slope of -5/9?

You can set up the slope formula: (a - 7) / (1 - a) = -5/9. Solving for 'a' will give its value.

What is the process to determine 'a' when given two points and a specific slope?

Replace the points into the slope formula (y2 - y1) / (x2 - x1) = slope, then solve the resulting equation for 'a'.

Can I verify the value of 'a' by substituting back into the points and slope formula?

Yes, after finding 'a', substitute it into the points and check if the slope between (a, 7) and (1, a) is indeed -5/9 to verify correctness.

Is the value of 'a' unique given the points and slope? How do I confirm?

Yes, solving the equation yields a specific value for 'a'. Confirm by substituting back into the slope formula to ensure consistency.

What are the steps to solve for 'a' in the equation (a - 7) / (1 - a) = -5/9?

Cross-multiply to get 9(a - 7) = -5(1 - a), expand both sides, then solve the resulting linear equation for 'a'.

Additional Resources

What Is The Value Of (a,7) And (1,a) And A Slope Of -5/9

Understanding the relationships between points on a coordinate plane and the slopes of lines connecting them is fundamental in algebra and analytic geometry. When examining the points (a, 7) and (1, a) and the slope of the line passing through these points being -5/9, we delve into a problem that combines algebraic expressions with geometric interpretations. This analysis not only reveals the specific values of 'a' but also enhances our comprehension of how slopes relate to coordinate points, offering insights applicable across various mathematical contexts.

Introduction to Coordinate Geometry and Slope

Coordinate geometry, also known as analytic geometry, is a branch of mathematics that uses algebraic techniques to study geometric figures. The fundamental concept involves plotting points in a two-dimensional plane using ordered pairs (x, y). A central aspect of this study is understanding the slope of a line, which measures its steepness and direction.

What is Slope?

The slope of a line, often denoted by 'm', is calculated as the ratio of the change in y-coordinates to the change in x-coordinates between two points:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

This ratio indicates how much y changes for a unit change in x. A positive slope suggests the line inclines upward from left to right, while a negative slope indicates a decline.

Why is Slope Important?
Knowing the slope helps determine the line's direction, whether two lines are parallel or perpendicular, and how to formulate equations of lines. In our case, the slope of -5/9 indicates a line that declines steadily, but not steeply, as it moves from left to right.

Dissecting the Given Points and Slope

The problem involves two points: (a, 7) and (1, a), and a line passing through both points with a slope of -5/9. The goal is to find the value(s) of 'a' that satisfy this condition.

Points Involved:
- Point 1: (a, 7)
- Point 2: (1, a)

Given Slope:
- m = -5/9

This setup naturally leads to the application of the slope formula, plugging in the coordinate points to establish an equation in terms of 'a'.

Deriving the Equation to Find 'a'

Applying the slope formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Substituting the points:

$$
- \frac{5}{9} = \frac{a - 7}{1 - a}
$$

\]

This equation encapsulates the relationship between 'a' and the known slope. Our task is to solve for 'a' in this equation.

Step-by-step Solution:

1. Set up the equation:

\[
\frac{a - 7}{1 - a} = - \frac{5}{9}
\]

2. Cross-multiplied form:

\[
9(a - 7) = -5(1 - a)
\]

3. Expand both sides:

\[
9a - 63 = -5 + 5a
\]

4. Bring like terms together:

\[
9a - 5a = -5 + 63
\]

\[
4a = 58
\]

5. Solve for 'a':

\[
a = \frac{58}{4} = \frac{29}{2} = 14.5
\]

Result:

\[
a = 14.5
\]

This indicates that when \(a = 14.5\), the points (a, 7) and (1, a) are connected by a line with a slope of -5/9.

Checking the Validity and Exploring the Solution

While the algebraic solution appears straightforward, it is vital to verify the result and understand its implications.

Verification:

Substitute $a = 14.5$ back into the points:

- Point 1: $(14.5, 7)$
- Point 2: $(1, 14.5)$

Calculate the slope:

$$
m = \frac{14.5 - 7}{1 - 14.5} = \frac{7.5}{-13.5} = -\frac{7.5}{13.5}
$$

Simplify numerator and denominator:

$$
- \frac{7.5}{13.5} = - \frac{75/10}{135/10} = - \frac{75}{135} = - \frac{5}{9}
$$

This confirms our solution is consistent with the given slope.

Implications and Broader Context

The resolution of this problem exemplifies how algebraic methods intersect with geometric intuition to solve for unknowns in coordinate geometry. It demonstrates the process of translating a geometric condition (a specific slope) into an algebraic equation, solving for an unknown parameter, and verifying the solution.

Broader Significance:

- Application in Graphing: Knowing the specific 'a' allows plotting these points accurately and understanding the line's behavior.
- Understanding Relationships: The problem underscores how the position of points relative to each other influences the slope and vice versa.
- Educational Value: For students, this problem reinforces the importance of manipulating equations, understanding domain restrictions (e.g., avoiding division by zero when $1 - a = 0$), and verifying solutions algebraically.

Special Considerations and Potential Variations

While the solution for 'a' is straightforward, several nuances merit discussion:

- Domain Restrictions:
The denominator $(1 - a)$ cannot be zero, which would lead to undefined slope calculations. Since $a = 14.5$, this is not an issue here, but if the solution had been $a=1$, the slope formula would be invalid due to division by zero.

- Multiple Solutions:
Some problems involving slopes can yield more than one solution, especially when quadratic equations are involved. Here, the linear nature of the equation led to a single solution.

- Alternative Approaches:
Besides algebraic manipulation, one could employ graphical methods or substitution in coordinate geometry to visualize the points and the line with the specified slope.

Conclusion: The Significance of 'a' in the Context of the Problem

The calculation and verification confirm that the value of 'a' is 14.5 for the points (a, 7) and (1, a) to be connected by a line with a slope of -5/9. This specific value illustrates the precise relationship between the coordinates of the points and the slope of the line, offering a clear example of how algebraic techniques can solve geometric problems.

In essence, this exercise exemplifies the power of algebra in understanding geometric relationships. It emphasizes the importance of carefully setting up equations, performing accurate calculations, and verifying solutions—all essential skills in mathematics. Whether for academic purposes, practical applications, or further theoretical exploration, grasping these concepts lays a strong foundation for more advanced topics in mathematics and related fields.

Final note: This detailed analysis underscores how interconnected algebra and geometry are, and how solving for unknowns like 'a' in the context of slope and points can deepen our understanding of the coordinate plane's structure and behavior.

What Is The Value Of A 7 And 1 A And A Slope Of 5 9

What Is The Value Of A 7 And 1 A And A Slope Of 5 9

Related Articles

- Solve (1+x) 4.2+ Dy 1"y = + +(1+x) + Y = 4 Sin [log(1+x)] Dx
- A Series Of Items That Are Associated With Each Other In A Set
- How Did Roosevelt Link Ideas In His Speech Day Of Infamy

Back to Home