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Understanding the values of a parameter like K that satisfy certain matrix conditions is essential in linear algebra, particularly when dealing with matrix norms, eigenvalues, and stability analyses. The inquiry here focuses on identifying the specific values of K for which the norm of matrix A equals 1, given the matrix:

$$
A = \begin{bmatrix}
K & 2 & -20 \\
- K & 3 & 1 \\
-4 & 0 & 0
\end{bmatrix}
$$

and the condition $|A| = 1$. This expression likely refers to the matrix norm, which could be the spectral norm, Frobenius norm, or another norm, but most commonly, in the context of eigenvalues and stability, it relates to the spectral norm (the largest singular value).

This comprehensive guide will explore how to determine the values of K that satisfy this condition, delving into matrix norms, eigenvalues, and the step-by-step process to solve the problem systematically.

Understanding the Problem: Matrix Norms and Their Significance

Before we proceed with calculations, it’s crucial to clarify what the expression $|A| = 1$ signifies. In matrix analysis, the notation $|A|$ often refers to:

- **Norm of matrix A**: a measure of the size or length of a matrix, with common norms including the Frobenius norm, spectral norm, operator norm, etc.
- **Determinant of A**: but this is less likely here, as the absolute value of the determinant being 1 is a different condition.

Given the context and common usage, it is most probable that $|A|$ denotes the spectral norm of A, which equals the largest singular value of A.

Why is this important?
The spectral norm provides information about how the matrix scales vectors and is closely related to eigenvalues and stability. If $|A|=1$, it implies that the maximum stretching factor of the matrix is 1, which is a critical condition in stability analyses, especially in systems theory.

Step 1: Clarify the Matrix and the Norm

The matrix A is given as:

$$
A = \begin{bmatrix}
K & 2 & -20 \\
- K & 3 & 1 \\
-4 & 0 & 0
\end{bmatrix}
$$

Our goal is to find the value(s) of K such that:

$$
|A| = 1
$$

Assuming $|A|$ represents the spectral norm, which is equal to the largest singular value of A.

What are singular values?
For a matrix A, singular values are the square roots of the eigenvalues of $A^T A$. The largest singular value corresponds to the spectral norm.

Therefore, the process involves:

1. Computing $A^T A$.
2. Finding the eigenvalues of $A^T A$.
3. Determining the square roots of these eigenvalues.
4. Setting the largest singular value equal to 1 and solving for K.

Step 2: Compute $A^T A$

First, compute the transpose of A:

$$
A^T = \begin{bmatrix}
K & -K & -4 \\
2 & 3 & 0 \\
-20 & 1 & 0
\end{bmatrix}
$$

Now, multiply A^T by A:

$$
A^T A = \begin{bmatrix}
K & -K & -4 \\
2 & 3 & 0 \\
-20 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
K & 2 & -20 \\
- K & 3 & 1 \\
-4 & 0 & 0
\end{bmatrix}
$$

Let’s compute each element of $A^T A$:

Element (1,1):

$$
(K)(K) + (-K)(-K) + (-4)(-4) = K^2 + K^2 + 16 = 2K^2 + 16
$$

Element (1,2):

$$
(K)(2) + (-K)(3) + (-4)(0) = 2K - 3K + 0 = -K
$$

Element (1,3):

$$
(K)(-20) + (-K)(1) + (-4)(0) = -20K - K + 0 = -21K
$$

Element (2,1):

$$
(2)(K) + (3)(-K) + (0)(-4) = 2K - 3K + 0 = -K
$$

Element (2,2):

$$
(2)(2) + (3)(3) + (0)(0) = 4 + 9 + 0 = 13
$$

Element (2,3):

$$
(2)(-20) + (3)(1) + (0)(0) = -40 + 3 + 0 = -37
$$

Element (3,1):

$$
(-20)(K) + (1)(-K) + (0)(-4) = -20K - K + 0 = -21K
$$

Element (3,2):

$$
(-20)(2) + (1)(3) + (0)(0) = -40 + 3 + 0 = -37
$$

Element (3,3):

$$
(-20)(-20) + (1)(1) + (0)(0) = 400 + 1 + 0 = 401
$$

Thus, the matrix $A^T A$ is:

$$
A^T A = \begin{bmatrix}
2K^2 + 16 & -K & -21K \\
- K & 13 & -37 \\
-21K & -37 & 401
\end{bmatrix}
$$

Step 3: Determine Eigenvalues of $A^T A$

The singular values of A are the square roots of the eigenvalues of $A^T A$. To find the largest singular value, we need the largest eigenvalue of $A^T A$.

The eigenvalues λ satisfy:

$$
\det(A^T A - \lambda I) = 0

\]

Where \(I\) is the identity matrix.

Set up the characteristic polynomial:

\[
\det \left(
\begin{bmatrix}
2K^2 + 16 - \lambda & -K & -21K \\
- K & 13 - \lambda & -37 \\
-21K & -37 & 401 - \lambda
\end{bmatrix}
\right) = 0
\]

This determinant expands into a cubic polynomial in \(\lambda\). Solving this polynomial exactly can be algebraically complex, but the key is to find the largest eigenvalue as a function of K, then impose the condition that the largest singular value, which is \(\sqrt{\lambda_{\max}}\), equals 1.

Step 4: Set the Largest Singular Value Equal to 1

Since the spectral norm is the square root of the largest eigenvalue of \(A^T A\), the condition \(|A|=1\) implies:

\[
\sqrt{\lambda_{\max}} = 1 \quad \Rightarrow \quad \lambda_{\max} = 1
\]

Thus, the maximum eigenvalue of \(A^T A\) must be 1.

But the eigenvalues depend on K, so the problem reduces to solving:

\[
\lambda_{\max}(A^T A) = 1
\]

This leads to solving the characteristic polynomial for \(\lambda = 1\):

\[
\det(A^T A - I) = 0
\]

which simplifies to:

\[

\det \left(
\begin{bmatrix}
2K^2 + 16 - 1 & -K & -21K \\
- K & 13 - 1 & -37 \\
-21K & -37 & 401 - 1
\end{bmatrix}
\right) = 0
\]

or,

\[
\det \left(
\begin{bmatrix}
2K^2 + 15 & -K & -21K \\
- K & 12 & -37 \\
-21K & -37 & 400
\end{bmatrix}
\right) = 0
\]

Now, compute this determinant:

\[
D(K) = \begin{bmatrix}
2K^2 + 15 & -K & -21K \\
- K & 12 & -37 \\
-21K & -37 & 400
\end{bmatrix}
\]

Step 5: Calculate the Determin

Frequently Asked Questions

What is the primary goal when solving for values of K in the equation |A| = 1?

The primary goal is to determine the specific

values of K for which the magnitude of matrix A equals 1, satisfying the given matrix equation.

How do you interpret the equation $|A| = 1$ in the context of matrix determinants?

It means that the determinant of matrix A has an absolute value of 1, indicating that A is either volume-preserving or invertible with a scaling factor of 1.

What is the significance of the quadratic equation $K^2 - 20K - 3 = 0$ in solving for K?

This quadratic equation arises from the determinant condition, and solving it yields the specific K values that satisfy the original equation for $|A| = 1$.

How do you solve the quadratic equation $K^2 - 20K - 3 = 0$?

Use the quadratic formula $K = [20 \pm \sqrt{(20^2 - 41(-3))}]/2$ to find the roots $K = [20 \pm \sqrt{(400 + 12)}]/2$.

What are the solutions for K from the quadratic equation $K^2 - 20K - 3 = 0$?

The solutions are $K = (20 + \sqrt{412})/2$ and $K = (20 - \sqrt{412})/2$, which approximately equal 20.103 and -0.103 respectively.

Why are only these K values relevant in the context of the original problem?

Because these K values satisfy the determinant condition $|A| = 1$, making them the valid solutions to the problem.

How does understanding the determinant help in analyzing the properties of matrix A in this problem?

The determinant indicates whether the matrix preserves volume (when $|A|=1$), is invertible, and helps identify the specific K values that ensure this property.

Additional Resources

Determinant Analysis: Exploring the Values of K

for Which |A|| = 1

Understanding the behavior of determinants in relation to matrix parameters is a fundamental aspect of linear algebra, with applications spanning engineering, physics, computer science, and more. In this detailed exploration, we analyze a specific matrix-dependent determinant condition:

$$
|A|| = 1 \quad \text{where} \quad A = \begin{bmatrix} 3 & 1 & -4 \\ 1 & -4 & -K \\ 2 & -20 & -K \end{bmatrix}
$$

Our goal: Determine the values of K for which the determinant of matrix A equals ± 1. This involves calculating the determinant of A as a function of K, and then solving the resulting equations.

Understanding the Problem: Determinants and Parameter Dependencies

Before diving into calculations, it's essential to clarify what the problem entails:

- Matrix A: A 3x3 matrix with entries depending on the parameter K.
- Determinant $|A|$: A scalar quantity representing the volume scaling factor of the linear transformation associated with A. When $|A| = 1$, the transformation preserves volume; when $|A| = -1$, it preserves volume but reverses orientation.
- Goal: Find all real values of K such that $|A| = \pm 1$.

This problem involves two steps:

1. Express $|A|$ explicitly as a function of K.
2. Solve the equations $|A| = 1$ and $|A| = -1$.

Calculating the Determinant of A as a Function of K

Step 1: Write down the matrix

$$
A = \begin{bmatrix}
3 & 1 & -4 \\
1 & -4 & -K \\
2 & -20 & -K
\end{bmatrix}
$$

Step 2: Calculate the determinant $|A|$

The determinant of a 3x3 matrix:

$$
|A| = a_{11}(a_{22}a_{33} - a_{23}a_{32}) - a_{12}(a_{21}a_{33} - a_{23}a_{31}) + a_{13}(a_{21}a_{32} - a_{22}a_{31})
$$

Plugging in:

- $a_{11} = 3$
- $a_{12} = 1$
- $a_{13} = -4$
- $a_{21} = 1$
- $a_{22} = -4$
- $a_{23} = -K$
- $a_{31} = 2$
- $a_{32} = -20$

- $a_{33} = -K$

Compute each term separately:

Term 1:

$$
3 \times [(-4) \times (-K) - (-K) \times (-20)]
= 3 \times [4K - (K \times 20)]
= 3 \times (4K - 20K)
= 3 \times (-16K)
= -48K
$$

Term 2:

$$
-1 \times [1 \times (-K) - (-K) \times 2]
= -1 \times (-K - (-2K))
= -1 \times (-K + 2K)
= -1 \times K
= -K
$$

Term 3:

$$
-4 \times [1 \times (-20) - (-4) \times 2]
= -4 \times (-20 - (-8))
$$

$$
= -4 \times (-20 + 8)
= -4 \times (-12)
= 48
$$

Putting it all together:

$$
|A| = \text{Term 1} + \text{Term 2} + \text{Term 3} = -48K - K + 48
$$

Simplify:

$$
|A| = -49K + 48
$$

Thus, the determinant as a function of K is:

$$
|A| = -49K + 48
$$

Determining the Values of K for $|A| = \pm 1$

Having obtained the explicit form, the next step is to solve for K when the determinant magnitude equals 1:

$$
|A| = \pm 1
$$

which gives two equations:

1. $\quad -49K + 48 = 1$
2. $\quad -49K + 48 = -1$

Equation 1: $|A| = 1$

$$
-49K + 48 = 1
$$
$$
-49K = 1 - 48
$$
$$
-49K = -47
$$
$$
K = \frac{-47}{-49} = \frac{47}{49}
$$

\]

Result: \(\boxed{K = \frac{47}{49}}\)

Equation 2: \(|A| = -1\)

\[
-49K + 48 = -1
\]
\[
-49K = -1 - 48
\]
\[
-49K = -49
\]
\[
K = \frac{-49}{-49} = 1
\]

Result: \(\boxed{K = 1}\)

Summary of Results and Interpretation

Values of K for which $|A| = 1$:

Condition	Solution for K	Explanation
$|A| = 1$	$K = \frac{47}{49}$	The determinant magnitude equals 1 at this specific K value.
$|A| = -1$	$K = 1$	The determinant equals -1 at this K, so $|A| = 1$ as well.

Note: Since the determinant $|A|$ is a linear function of K, the solutions are straightforward and unique for each condition.

Additional considerations:

- **The determinant function is linear, so the set of K satisfying $|A| = 1$ or -1 are isolated points.**
- **These K values could have implications for the invertibility and stability of the matrix A. For instance, at these K, the matrix's volume scaling factor has a magnitude of 1, which suggests volume-preserving transformations.**

Final Remarks and Practical Implications

Understanding the specific K values where $|A| = \pm 1$ is crucial in various applications:

- In Control Systems: These K values might indicate system parameters that preserve certain invariants.
- In Geometry and Physics: Such parameters could correspond to transformations that preserve volume or orientation, important in conservation laws.
- In Numerical Analysis: Knowing when determinants reach these critical values helps in assessing matrix invertibility and conditioning.

In conclusion:

$$
\boxed{
\begin{aligned}
&\text{The matrix } A \text{ has } |\det A| = 1 \text{ at the following } K \text{ values:} \\
&\quad K = \frac{47}{49} \quad \text{and} \quad K = 1
\end{aligned}
}
$$

This analysis exemplifies the power of algebraic manipulation combined with linear algebra

principles to solve parameter-dependent determinant problems, offering insights into the structure and behavior of matrices in various scientific domains.

3 For What Value S Of K Will A 1 K 2 20 K 0 3 1 4

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