3. Solve Y"+y=2t, Y(/4)= /2, Y'=2-2 By Using Laplace Transform.

3. Solve Y"+y=2t, Y(/4)= /2, Y'=2-2 By Using Laplace Transform.

Introduction

Differential equations are fundamental in modeling various real-world phenomena, from physics and engineering to economics and biology. Among the most effective methods for solving linear ordinary differential equations with constant coefficients is the Laplace transform. This technique simplifies the process by transforming differential equations into algebraic equations, making solutions more straightforward to obtain.

In this article, we will explore how to solve the second-order differential equation:

$$
Y'' + y = 2t
$$

subject to the initial conditions:

$$
Y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}, \quad Y' = 2 - 2
$$

using the Laplace transform method. Although the initial conditions seem somewhat unusual or potentially misprinted, we will interpret and clarify them within the context of solving the problem.

Understanding the Differential Equation

Before applying the Laplace transform, it’s essential to understand the structure of the differential equation:

$$
Y'' + y = 2t
$$

Key points:
- It is a second-order linear differential equation.
- The equation involves the function $y(t)$ and its second derivative $Y''(t)$.
- The right-hand side is a non-homogeneous term, $2t$.

Clarification:
In the notation, assuming Y is the unknown function of t, and the notation Y'' indicates

the second derivative of Y with respect to t. The equation appears to have a typo, as it involves both Y'' and y. Typically, such equations are written as:

$$
Y'' + aY' + bY = \text{some function of } t
$$

Alternatively, if the notation intends to represent a simpler form, such as:

$$
Y'' + Y = 2t
$$

then the problem becomes more standard. For clarity, we will interpret the differential equation as:

$$
Y'' + Y = 2t
$$

which is a common second-order linear non-homogeneous differential equation.

Step 1: Writing the Differential Equation in Standard Form

Assuming the corrected form:

$$
Y'' + Y = 2t
$$

with initial conditions:

$$
Y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}
$$
and
$$
Y'(\pi/4) = 2 - 2 = 0
$$

Note: The initial condition $Y' = 2 - 2$ might imply $Y'(\pi/4) = 0$. We will proceed with this interpretation.

Step 2: Applying the Laplace Transform

2.1. Recall Laplace Transform Definitions

The Laplace transform of a function $f(t)$ is:

$$
\mathcal{L}\{f(t)\} = F(s) = \int_0^{\infty} e^{-st}f(t) dt
$$

Important properties for differential equations:

- $\mathcal{L}\{Y'(t)\} = sY(s) - Y(0)$
- $\mathcal{L}\{Y''(t)\} = s^2Y(s) - sY(0) - Y'(0)$

2.2. Transform the Differential Equation

Applying Laplace to both sides:

$$
\mathcal{L}\{Y'' + Y\} = \mathcal{L}\{2t\}
$$

which becomes:

$$
s^2Y(s) - sY(0) - Y'(0) + Y(s) = \frac{2}{s^2}
$$

since the Laplace transform of $2t$ is $2 / s^2$.

Step 3: Incorporating Initial Conditions

From the problem, initial conditions are specified at $t = \pi/4$, but Laplace transforms are typically taken from $t=0$. To proceed, we need initial conditions at $t=0$:

- $Y(0) = y_0$
- $Y'(0) = y_1$

If initial data at $t=0$ are not provided, the problem becomes a bit more complex. However, for demonstration purposes, assume:

$$
Y(0) = y_0, \quad Y'(0) = y_1
$$

We can later relate these to the given initial conditions at $t = \pi/4$ via the solution.

Note:
If the initial conditions are only given at $t = \pi/4$, then the Laplace method requires transforming the problem into an initial value problem at $t=0$. Alternatively, one can shift the problem, but that is beyond this scope.

Step 4: Solving the Algebraic Equation in Laplace Domain

The transformed differential equation:

$$
s^2Y(s) - s y_0 - y_1 + Y(s) = \frac{2}{s^2}
$$

Rearranged as:

$$
(s^2 + 1)Y(s) = s y_0 + y_1 + \frac{2}{s^2}
$$

Thus,

$$
Y(s) = \frac{s y_0 + y_1}{s^2 + 1} + \frac{2}{s^2 (s^2 + 1)}
$$

Step 5: Inverse Laplace Transform

5.1. Break down the inverse transform

The total solution $Y(t)$ is the sum of:

- The inverse transform of $\frac{s y_0 + y_1}{s^2 + 1}$
- The inverse transform of $\frac{2}{s^2 (s^2 + 1)}$

5.2. Inverse of the first term

$$
\mathcal{L}^{-1}\left\{\frac{s y_0 + y_1}{s^2 + 1}\right\} = y_0 \cos t + y_1 \sin t
$$

5.3. Inverse of the second term

We focus on:

$$
\frac{2}{s^2 (s^2 + 1)}
$$

Using partial fractions:

$$
\frac{2}{s^2 (s^2 + 1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2 + 1}
$$

Multiplying both sides by $s^2 (s^2 + 1)$:

$$
2 = A s (s^2 + 1) + B (s^2 + 1) + (Cs + D) s^2
$$

Determine coefficients A, B, C, D by equating coefficients for powers of s:

- For s^3: $A = 0$
- For s^2: $B + D = 0$
- For s: $A + C = 0 \Rightarrow C = 0$
- Constant term: $B = 2$

From above:

- $B=2$
- $B + D = 0 \Rightarrow D = -2$
- $A=0$
- $C=0$

So partial fractions:

$$
\frac{2}{s^2 (s^2 + 1)} = \frac{2}{s^2} - \frac{2}{s^2 + 1}
$$

5.4. Inverse transforms:

- $\mathcal{L}^{-1}\left\{\frac{2}{s^2}\right\} = 2t$
- $\mathcal{L}^{-1}\left\{\frac{2}{s^2 + 1}\right\} = 2 \sin t$

Final General Solution:

$$
Y(t) = y_0 \cos t + y_1 \sin t + 2t - 2 \sin t
$$

Step 6: Applying Initial Conditions

Given initial conditions at $t = \pi/4$:

$$
Y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}
$$
$$
Y'\left(\frac{\pi}{4}\right) = 0
$$

Express $Y(t)$:

$$
Y(t) = y_0 \cos t + y_1 \sin t + 2t - 2 \sin t
$$

Compute $Y'(t)$:

$$
Y'(t) = - y_0 \sin t + y_1 \cos t + 2 - 2 \cos t
$$

Apply at $t = \pi/4$:

$$
Y(\pi/4) = y_0 \cos (\pi/4) + y_1 \sin (\pi/4) + 2 \times \frac{\pi}{4} - 2 \sin (\pi/4) =
$$

Frequently Asked Questions

How do you set up the Laplace transform for the differential equation Y'' + y = 2t?

Apply the Laplace transform to both sides, using $L\{Y''\} = s^2Y(s) - sY(0) - Y'(0)$ and $L\{Y\} = Y(s)$, converting the differential equation into an algebraic equation in terms of Y(s).

What initial conditions are used in solving Y'' + y = 2t with Laplace transforms?

Given initial conditions are $Y(\frac{1}{4}) = \frac{1}{2}$ and Y' = 2 - 2B, but since the problem involves $Y(\frac{1}{4})$, you need to convert this to Y(0) and Y'(0) if possible, or use the given point as boundary conditions accordingly.

How do you incorporate the initial condition $Y(\frac{1}{4}) = \frac{1}{2}$ into the Laplace transform solution?

Laplace transforms are typically initial value problems at t=0, so to incorporate $Y(\frac{1}{4}) = \frac{1}{2}$, you may need to use the Laplace transform solution to find Y(s), then invert to evaluate at $t=\frac{1}{4}$, or adjust the problem to initial conditions at t=0.

How is the nonhomogeneous term 2t handled in the Laplace transform method?

The Laplace transform of 2t is $2 / s^2$, which is substituted into the algebraic equation, allowing you to solve for Y(s).

What is the general approach to solve the differential equation using Laplace transforms in this case?

Transform the differential equation into an algebraic equation in Y(s), apply initial conditions, solve for Y(s), and then perform the inverse Laplace transform to find Y(t).

How do you interpret the boundary condition Y' = 2 - 2B in the context of the Laplace transform solution?

Since Y' is given in terms of B, it suggests a parameter or boundary condition that must be specified; for the Laplace method, initial derivatives are needed at t=0, so clarify or convert these conditions accordingly to proceed.

Additional Resources

Solve Y'' + y = 2t, Y(π/4) = π/2, Y' = 2 - 2b Using Laplace Transform — this is a common type of differential equation problem encountered in advanced mathematics, especially when dealing with linear ordinary differential equations with initial conditions. The Laplace Transform is a powerful technique for solving such equations because it transforms differential equations into algebraic equations, which are often easier to manipulate and solve. In this guide, we'll walk through a comprehensive, step-by-step process to solve the differential equation Y'' + y = 2t with the initial conditions Y(π/4) = π/2 and Y' = 2 - 2b using the Laplace Transform method.

Understanding the Problem

Before diving into the solution, it’s essential to understand the problem's components:

- Differential Equation:

$$
Y'' + y = 2t
$$

This is a second-order linear nonhomogeneous differential equation.

- Initial Conditions:

$$
Y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}
$$

$$
Y' = 2 - 2b
$$

Note: The initial condition for the derivative appears incomplete or possibly miswritten. Typically, initial conditions specify the value of the function and its derivative at a particular point. It's likely intended to be something like $Y'(\pi/4) = 2 - 2b$. For consistency, we'll assume the initial conditions are:

$$
Y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}
$$

$$
Y'\left(\frac{\pi}{4}\right) = 2 - 2b
$$

- Objective:

Find the solution $Y(t)$ that satisfies the differential equation with the given initial conditions using the Laplace Transform.

Step 1: Recognize the Type of Differential Equation

The differential equation is:

$$
Y'' + y = 2t
$$

It's a second-order linear differential equation with variable coefficients if Y depends on t. However, the notation suggests that the independent variable is t, and the dependent variable is $Y(t)$.

Note: There seems to be a possible typo or notation inconsistency. The equation appears to be:

$$
Y'' + y = 2t
$$

But in typical notation, it might be:

$$
Y'' + Y = 2t
$$

or similar. For clarity, and based on standard form, we'll interpret the equation as:

$$
Y'' + Y = 2t
$$

which is a standard second-order linear ODE with constant coefficients.

If the original problem intended for Y to be the dependent variable, then the equation:

$$
Y'' + Y = 2t
$$

is more typical and manageable.

Step 2: Take the Laplace Transform of Both Sides

Applying the Laplace Transform to the differential equation:

$$
\mathcal{L}\{Y'' + Y\} = \mathcal{L}\{2t\}
$$

Recall the Laplace Transform properties:

- $\mathcal{L}\{Y''(t)\} = s^2 Y(s) - s Y(0) - Y'(0)$
- $\mathcal{L}\{Y(t)\} = Y(s)$
- $\mathcal{L}\{t\} = \frac{1}{s^2}$

Let's denote:

- $Y(s) = \mathcal{L}\{Y(t)\}$
- Initial conditions: $Y(0) = y_0$, $Y'(0) = y_1$.

Given the initial conditions are at $t = \pi/4$, but Laplace transforms are taken at $t=0$, so we need $Y(0)$ and $Y'(0)$. Since the problem states initial conditions at $t = \pi/4$, this suggests a shift or the need to clarify initial data.

Assumption: For the purpose of this guide, assume initial conditions are at $t=0$:

$$
Y(0) = y_0
$$
$$
Y'(0) = y_1
$$

If initial conditions are only given at $t=\pi/4$, then a change of variables or the use of the Laplace transform with shifted initial data would be necessary. For simplicity, proceed with initial conditions at zero.

Step 3: Write the Transformed Equation

Applying the Laplace Transform:

$$
s^2 Y(s) - s Y(0) - Y'(0) + Y(s) = \frac{2}{s^2}
$$

Rearranged:

$$
(s^2 + 1) Y(s) = s Y(0) + Y'(0) + \frac{2}{s^2}
$$

Step 4: Solve for $Y(s)$

$$
Y(s) = \frac{s Y(0) + Y'(0)}{s^2 + 1} + \frac{2}{s^2 (s^2 + 1)}
$$

This expression combines the homogeneous solution (related to initial conditions) and the particular solution (related to the nonhomogeneous term $2t$).

Step 5: Find the Inverse Laplace Transform

To find $Y(t)$, apply inverse Laplace Transform to each term separately.

Part A: Homogeneous solution

$$
Y_h(s) = \frac{s Y(0) + Y'(0)}{s^2 + 1}
$$

Inverse Laplace:

$$
Y_h(t) = Y(0) \cos t + Y'(0) \sin t
$$

Part B: Particular solution

$$
Y_p(s) = \frac{2}{s^2 (s^2 + 1)}
$$

We need to find the inverse Laplace of this expression. Use partial fraction decomposition:

$$
\frac{2}{s^2 (s^2 + 1)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs + D}{s^2 + 1}
$$

Multiply both sides by $s^2 (s^2 + 1)$:

$$
2 = A s (s^2 + 1) + B (s^2 + 1) + (C s + D) s^2
$$

Expand:

$$
2 = A s^3 + A s + B s^2 + B + C s^3 + D s^2
$$

Group like terms:

$$
2 = (A + C) s^3 + (B + D) s^2 + A s + B
$$

Matching coefficients:

- For s^3: $A + C = 0$
- For s^2: $B + D = 0$
- For s^1: $A = 0$
- For constant term: $B = 2$

From $A=0$, then $A + C=0 \Rightarrow C=0$.

From $B=2$, then $B + D=0 \Rightarrow D=-2$.

Now, the partial fractions are:

$$
\frac{2}{s^2 (s^2 + 1)} = \frac{0}{s} + \frac{2}{s^2} + \frac{0 \cdot s - 2}{s^2 + 1}
$$

which simplifies to:

$$
Y_p(s) = \frac{2}{s^2} - \frac{2}{s^2 + 1}
$$

Inverse Laplace transforms:

$$
\mathcal{L}^{-1}\left\{\frac{2}{s^2}\right\} = 2 t
$$

$$
\mathcal{L}^{-1}\left\{\frac{2}{s^2 + 1}\right\} = 2 \sin t
$$

Therefore:

$$
Y_p(t) = 2t - 2 \sin t
$$

Final Solution:

$$
Y(t) = Y(0) \cos t + Y'(0) \sin t + 2 t - 2 \sin t
$$

If initial conditions are known at $t=0$, this expression gives the complete solution.

Incorporating Initial Conditions

Suppose initial conditions are:

$$
Y(0) = y_0
$$
$$
Y'(0) = y_1
$$

then the full solution is:

$$
\boxed{
Y(t) = y_0 \cos t + y_1 \sin t + 2 t - 2 \sin t
}
$$

Addressing the Given Initial Conditions at $t=\pi/4$

If the problem specifies:

$$
Y\left(\frac{\pi}{4}\right) = \frac{\pi}{2}
$$
\[
Y'\left(\frac{\pi}{4

3 Solve Y Y 2t Y 4 2 Y 2 2 By Using Laplace Transform

3 Solve Y Y 2t Y 4 2 Y 2 2 By Using Laplace Transform

Related Articles

- What Is The Definition Of Slope Of A Ramp Using Trigonometry ?
- Find Out The Distance Moved By A Car In 4h At A Speed Of 17km/h
- 2 Triangles Measure 55 And 20 What Is The Measure Of The Third Angle

Back to Home