(3[tex]\sqrt{7} \\[/tex]+7[tex]\sqrt{3}[/tex])/[tex]\sqrt{21}[/tex]

(3\sqrt{7} + 7\sqrt{3}) / \sqrt{21}

Understanding and simplifying complex algebraic expressions can often seem daunting, especially when they involve nested radicals. One such expression that often appears in algebraic exercises and mathematical discussions is:

$$
\frac{3\sqrt{7} + 7\sqrt{3}}{\sqrt{21}}
$$

In this comprehensive article, we will explore this expression in depth. We will analyze its structure, provide step-by-step methods for simplification, discuss related algebraic concepts, and demonstrate practical applications. Whether you're a student seeking to improve your algebra skills or a teacher preparing instructional material, this guide aims to provide clarity and insight.

Understanding the Expression

Before diving into the simplification process, it's essential to understand the components of the expression:

$$
\frac{3\sqrt{7} + 7\sqrt{3}}{\sqrt{21}}
$$

- Numerator: $3\sqrt{7} + 7\sqrt{3}$

- Denominator: $\sqrt{21}$

Both numerator and denominator contain square roots, some of which are composite, i.e., $\sqrt{21}$ is the radical of a product of primes 3 and 7.

Breaking Down the Components

Prime Factorization of Radicals

- $\sqrt{7}$: The square root of 7, a prime number.

- $\sqrt{3}$: The square root of 3, also prime.

- $\sqrt{21}$: Since $21 = 3 \times 7$, $\sqrt{21} = \sqrt{3 \times 7}$.

This prime factorization is crucial because it allows us to express radicals in a form that can be combined or simplified.

Expressing Radicals in a Common Basis

Note that $\sqrt{21} = \sqrt{3 \times 7}$. Recognizing this, we can attempt to rewrite the numerator terms to relate to the denominator:

$$
\sqrt{7} = \frac{\sqrt{7} \times \sqrt{3}}{\sqrt{3}} = \frac{\sqrt{21}}{\sqrt{3}}
$$
$$
\sqrt{3} = \frac{\sqrt{3} \times \sqrt{7}}{\sqrt{7}} = \frac{\sqrt{21}}{\sqrt{7}}
$$

This insight can be used to rationalize the expression or to express numerator terms in terms of $\sqrt{21}$.

Step-by-Step Simplification Process

Let's proceed carefully to simplify the expression:

$$
\frac{3\sqrt{7} + 7\sqrt{3}}{\sqrt{21}}
$$

Step 1: Express numerator terms to relate to the denominator.

Recall that:

$$
\sqrt{7} = \frac{\sqrt{21}}{\sqrt{3}} \quad \text{and} \quad \sqrt{3} = \frac{\sqrt{21}}{\sqrt{7}}
$$

So,

$$
3 \sqrt{7} = 3 \times \frac{\sqrt{21}}{\sqrt{3}} = \frac{3 \sqrt{21}}{\sqrt{3}}
$$

$$
7 \sqrt{3} = 7 \times \frac{\sqrt{21}}{\sqrt{7}} = \frac{7 \sqrt{21}}{\sqrt{7}}
$$

Step 2: Rewrite the numerator as a single expression.

$$
\text{Numerator} = \frac{3 \sqrt{21}}{\sqrt{3}} + \frac{7 \sqrt{21}}{\sqrt{7}}
$$

Step 3: Simplify the denominators inside the numerator.

Note that:

$$
\sqrt{3} = \sqrt{3}
$$
$$
\sqrt{7} = \sqrt{7}
$$

But to combine these terms, it would be better to rationalize the denominators:

$$
\frac{3 \sqrt{21}}{\sqrt{3}} = 3 \sqrt{21} \times \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = 3 \sqrt{21} \times \frac{\sqrt{3}}{3} = \sqrt{21} \times \sqrt{3} = \sqrt{21 \times 3} = \sqrt{63}
$$

Similarly,

$$
\frac{7 \sqrt{21}}{\sqrt{7}} = 7 \sqrt{21} \times \frac{\sqrt{7}}{\sqrt{7} \times \sqrt{7}} = 7 \sqrt{21} \times \frac{\sqrt{7}}{7} = \sqrt{21} \times \sqrt{7} = \sqrt{21 \times 7} = \sqrt{147}
$$

Step 4: Rewrite numerator as sum of simplified radicals:

$$
\text{Numerator} = \sqrt{63} + \sqrt{147}
$$

Further Simplification of Radicals

Let's simplify $\sqrt{63}$ and $\sqrt{147}$:

- $\sqrt{63} = \sqrt{9 \times 7} = \sqrt{9} \times \sqrt{7} = 3 \sqrt{7}$

- $\sqrt{147} = \sqrt{7 \times 21} = \sqrt{7} \times \sqrt{21}$

But $\sqrt{21}$ can be expressed as $\sqrt{3 \times 7}$, so:

$$
\sqrt{147} = \sqrt{7} \times \sqrt{3 \times 7} = \sqrt{7} \times \sqrt{3} \times \sqrt{7} = \sqrt{7} \times \sqrt{3} \times \sqrt{7}
$$

Note that:

$$
\sqrt{7} \times \sqrt{7} = 7
$$

Thus,

$$
\sqrt{147} = 7 \times \sqrt{3}
$$

Summary:

$$
\text{Numerator} = 3 \sqrt{7} + 7 \sqrt{3}
$$

Which interestingly, is the original numerator. This confirms that our earlier radical manipulations are consistent.

Alternative Approach: Rationalizing the Denominator

Another common technique in simplifying radicals is to rationalize the denominator:

$$
\frac{3 \sqrt{7} + 7 \sqrt{3}}{\sqrt{21}}
$$

Since $\sqrt{21}$ is irrational, rationalizing involves multiplying numerator and denominator by $\sqrt{21}$:

$$
\frac{(3 \sqrt{7} + 7 \sqrt{3}) \times \sqrt{21}}{\sqrt{21} \times \sqrt{21}}
= \frac{(3 \sqrt{7} + 7 \sqrt{3}) \times \sqrt{21}}{21}
$$

Now, evaluate numerator:

$$
(3 \sqrt{7}) \times \sqrt{21} + (7 \sqrt{3}) \times \sqrt{21}
$$

Recall:

$$
\sqrt{7} \times \sqrt{21} = \sqrt{7 \times 21} = \sqrt{147} = 7 \sqrt{3}
$$

Similarly:

$$
\sqrt{3} \times \sqrt{21} = \sqrt{3 \times 21} = \sqrt{63} = 3 \sqrt{7}
$$

So,

$$
(3 \sqrt{7}) \times \sqrt{21} = 3 \times 7 \sqrt{3} = 21 \sqrt{3}
$$
$$
(7 \sqrt{3}) \times \sqrt{21} = 7 \times 3 \sqrt{7} = 21 \sqrt{7}
$$

Total numerator:

$$
21 \sqrt{3} + 21 \sqrt{7} = 21 (\sqrt{3} + \sqrt{7})
$$

\]

Putting this all together:

\[
\frac{21 (\sqrt{3} + \sqrt{7})}{21} = \sqrt{3} + \sqrt{7}
\]

Final Simplified Result

After the rationalization process, the original complex radical expression simplifies neatly to:

\[
\boxed{\sqrt{3} + \sqrt{7}}
\]

This is a significant simplification—a radical sum that appears complicated reduces to the sum of two simpler radicals.

Key Takeaways and Learning Points

- Rationalization is a powerful technique for simplifying radicals, especially when dealing with denominators involving radicals.

- Recognizing prime factorizations aids in breaking down radicals into simpler components.

- Expressing radicals in terms of common factors can reveal opportunities for simplification.

- Sometimes, multiplying numerator and denominator by a radical expression can dramatically simplify the entire expression.

- The original expression:

\[
\frac{3\sqrt{7} + 7\sqrt{3}}{\sqrt{21

Frequently Asked Questions

How can I simplify the expression (3√7 + 7√3) / √21?

To simplify, first note that √21 = √(3×7). You can express the numerator as 3√7 + 7√3 and look for common factors or rationalize if needed. Alternatively, split the fraction into two parts: (3√7)/√21 + (7√3)/√21, then simplify each separately.

What is the simplified form of (3√7 + 7√3) / √21?

The simplified form is (3√7 + 7√3) / √21 = √3 + √7, after rationalizing the denominator and simplifying the radicals.

How do I rationalize the denominator in (3√7 + 7√3) / √21?

Multiply numerator and denominator by √21 to rationalize: [(3√7 + 7√3) × √21] / 21. Simplify numerator and denominator accordingly to get a rationalized form.

Is the value of (3√7 + 7√3) / √21 irrational or rational?

The value is irrational because it involves square roots of prime numbers that cannot be simplified to a rational number.

Can the expression (3√7 + 7√3) / √21 be approximated numerically?

Yes. Approximate √7 ≈ 2.6458, √3 ≈ 1.7321, and √21 ≈ 4.5826. Plugging these in gives an approximate value of (3×2.6458 + 7×1.7321) / 4.5826 ≈ (7.9374 + 12.1247) / 4.5826 ≈ 20.0621 / 4.5826 ≈ 4.38.

Are there any special identities that can help simplify (3√7 + 7√3) / √21?

Yes. Recognizing that √21 = √3 × √7 allows you to split the fraction and use properties of radicals to simplify, leading to expressions involving √3 and √7.

What is the significance of the expression (3√7 + 7√3) / √21 in mathematics?

While not a standard formula, it's a good example of combining radicals and rationalizing denominators, illustrating techniques in radical simplification common in algebra.

Can I write $(3\sqrt{7} + 7\sqrt{3}) / \sqrt{21}$ as a sum of two simpler radicals?

Yes. The simplified form can be expressed as $\sqrt{3} + \sqrt{7}$, making it easier to interpret and work with in calculations.

Additional Resources

Mathematical Expression Evaluation

Introduction

Mathematics often presents us with intriguing expressions that challenge our understanding and require careful analysis to simplify. Among these, expressions involving roots and rationalization stand out due to their complexity and elegance. One such expression is:

$$
\frac{3 \sqrt{7} + 7 \sqrt{3}}{\sqrt{21}}
$$

At first glance, this appears to be a straightforward algebraic fraction, but a deeper exploration reveals rich insights into the properties of radicals, simplification techniques, and algebraic manipulation. In this article, we will delve into this expression comprehensively, breaking down each component, understanding the underlying principles, and arriving at a simplified form with clarity.

The Significance of the Expression

Before diving into the calculations, it’s crucial to recognize why such an expression warrants detailed examination. Expressions involving multiple radicals are common in advanced mathematics, physics, and engineering, especially when dealing with measurements, wave functions, or geometric computations. Simplifying these expressions can reveal underlying relationships, facilitate computation, and enhance conceptual understanding.

Moreover, this specific example provides an excellent opportunity to review strategies for working with radicals—such as rationalizing denominators, combining radicals, and exploiting algebraic identities—making it an ideal candidate for a detailed, step-by-step analysis.

Understanding the Components

The Numerator: $3 \sqrt{7} + 7 \sqrt{3}$

This part of the expression features two terms, each a product of a coefficient and a radical. These terms are:

- $3 \sqrt{7}$: Three times the square root of 7
- $7 \sqrt{3}$: Seven times the square root of 3

These radicals are irrational numbers, meaning they cannot be expressed as a simple fraction. Each radical is in its simplest form because 7 and 3 are prime numbers, and their square roots are in their simplest radical form.

The Denominator: $\sqrt{21}$

The denominator is the square root of 21, which can be factored into primes:

$$
21 = 3 \times 7
$$

This factorization is pivotal because it allows us to explore relationships between the numerator and denominator involving common factors or radicals.

Step-by-Step Simplification

Step 1: Express the entire fraction clearly

$$
\frac{3 \sqrt{7} + 7 \sqrt{3}}{\sqrt{21}}
$$

Our goal is to simplify this expression as much as possible, ideally expressing it as a sum (or difference) of simpler radicals or rational numbers.

Step 2: Break down the radical in the denominator

Since $\sqrt{21} = \sqrt{3 \times 7}$, and both radicals in the numerator involve 3 and 7, this suggests the possibility of expressing the numerator terms to relate directly to $\sqrt{21}$.

Step 3: Rationalize the denominator

To simplify, a common approach is to rationalize the denominator—multiplying numerator and denominator by a conjugate or an appropriate radical to eliminate radicals from the denominator.

However, in this case, since the denominator involves $\sqrt{21}$, we can consider expressing the numerator terms to incorporate $\sqrt{21}$, or split the fraction into parts:

$$
\frac{3 \sqrt{7}}{\sqrt{21}} + \frac{7 \sqrt{3}}{\sqrt{21}}
$$

This allows us to analyze each term separately.

Detailed Term-by-Term Analysis

Term 1: $\frac{3 \sqrt{7}}{\sqrt{21}}$

Observe that:

$$
\frac{\sqrt{7}}{\sqrt{21}} = \sqrt{\frac{7}{21}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}
$$

Therefore:

$$
\frac{3 \sqrt{7}}{\sqrt{21}} = 3 \times \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}}
$$

Simplify numerator and denominator:

$$
\frac{3}{\sqrt{3}} = \frac{3}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{3 \sqrt{3}}{3} = \sqrt{3}
$$

Result for Term 1:

$$
\boxed{\sqrt{3}}
$$

Term 2: $\frac{7 \sqrt{3}}{\sqrt{21}}$

Similarly, analyze:

$$
\frac{\sqrt{3}}{\sqrt{21}} = \sqrt{\frac{3}{21}} = \sqrt{\frac{1}{7}} =

\frac{1}{\sqrt{7}}
\]

Hence:

\[
\frac{7 \sqrt{3}}{\sqrt{21}} = 7 \times \frac{1}{\sqrt{7}} =
\frac{7}{\sqrt{7}}
\]

Simplify:

\[
\frac{7}{\sqrt{7}} = \frac{7}{\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}} =
\frac{7 \sqrt{7}}{7} = \sqrt{7}
\]

Result for Term 2:

\[
\boxed{\sqrt{7}}
\]

Final Simplified Expression

Adding the two simplified parts:

\[
\sqrt{3} + \sqrt{7}
\]

Therefore:

\[
\frac{3 \sqrt{7} + 7 \sqrt{3}}{\sqrt{21}} = \boxed{\sqrt{3} + \sqrt{7}}
\]

This is a remarkably elegant result, reducing a seemingly complex radical fraction into a simple sum of two radicals.

Additional Insights and Implications

Algebraic Verification

To ensure the validity of our simplification, we can verify by cross-multiplied expansion.

Starting from the simplified form:

$$
\sqrt{3} + \sqrt{7}
$$

Squaring both sides:

$$
(\sqrt{3} + \sqrt{7})^2 = 3 + 7 + 2 \sqrt{3 \times 7} = 10 + 2 \sqrt{21}
$$

Now, consider the original expression multiplied by $\sqrt{21}$:

$$
(3 \sqrt{7} + 7 \sqrt{3}) = \sqrt{21} \times (\sqrt{3} + \sqrt{7})
$$

Multiplying the right side:

$$
\sqrt{21} \times (\sqrt{3} + \sqrt{7}) = \sqrt{21} \times \sqrt{3} + \sqrt{21} \times \sqrt{7} = \sqrt{63} + \sqrt{147}
$$

Simplify radicals:

- $\sqrt{63} = \sqrt{9 \times 7} = 3 \sqrt{7}$
- $\sqrt{147} = \sqrt{49 \times 3} = 7 \sqrt{3}$

Thus:

$$
\sqrt{21} \times (\sqrt{3} + \sqrt{7}) = 3 \sqrt{7} + 7 \sqrt{3}
$$

which matches the numerator perfectly. This confirms that:

$$
\frac{3 \sqrt{7} + 7 \sqrt{3}}{\sqrt{21}} = \sqrt{3} + \sqrt{7}
$$

Broader Context and Applications

In Mathematics

This example highlights the power of radical manipulation and algebraic identities. Recognizing how to relate radicals to their products simplifies many complex expressions, especially in fields like algebra, calculus, and number theory.

In Physics and Engineering

Expressions involving radicals, especially those in denominators, are common in wave mechanics, quantum physics, and electrical engineering. Simplifying such expressions can streamline calculations, improve numerical stability, and enhance conceptual clarity.

Teaching and Learning

From an educational perspective, this problem demonstrates the importance of radical properties and algebraic techniques. It encourages learners to look for relationships and factorization opportunities rather than accepting complex forms at face value.

Summary

- The original expression:

$$
\frac{3 \sqrt{7} + 7 \sqrt{3}}{\sqrt{21}}
$$

can be elegantly simplified through radical properties.

- By splitting into two parts and rationalizing, each term reduces to a simple radical:

$$
\frac{3 \sqrt{7}}{\sqrt{21}} = \sqrt{3}
$$

$$
\frac{7 \sqrt{3}}{\sqrt{21}} = \sqrt{7}
$$

- The combined simplified form:

$$
\boxed{\sqrt{3} + \sqrt{7}}
$$

- Verified through algebraic identities, confirming the correctness of the simplification.

Final Thoughts

Expressions involving radicals may initially appear complex, but with careful application of algebraic identities, prime factorizations, and radical properties, they can often be simplified into beautiful, elegant forms. Understanding these techniques not only enhances problem-solving skills but also deepens appreciation for the inherent harmony in mathematical structures.

Whether used in theoretical mathematics, applied sciences, or educational contexts, mastering radical simplification is a valuable skill that bridges abstract concepts with practical computation.

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