A Line With A Slope Of -3/4 That Passes Through The Point (-14,4).

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Understanding how to formulate and analyze lines in coordinate geometry is fundamental for students, educators, and professionals working in fields such as engineering, physics, and mathematics. In particular, lines characterized by their slope and a specific point on the line serve as essential tools for modeling relationships, solving geometric problems, and interpreting data.

This article provides an in-depth exploration of the line with a slope of -3/4 passing through the point (-14, 4). We will delve into the mathematical principles involved, demonstrate step-by-step how to derive the equation of this line, discuss its graphical representation, and explore real-world applications. Whether you're a student preparing for exams or a professional seeking to reinforce your understanding, this comprehensive guide aims to equip you with the knowledge you need.

Understanding the Basics: Slope-Intercept Form and Point-Slope Form

Before diving into the specific line in question, it is essential to familiarize ourselves with the fundamental equations that describe lines in the coordinate plane.

Slope-Intercept Form

The slope-intercept form of a line is given by:
$$ y = mx + b $$
where:
- m is the slope of the line,
- b is the y-intercept, the point where the line crosses the y-axis.

This form is particularly useful when you know the slope and the y-intercept.

Point-Slope Form

Alternatively, when you know a specific point (x_1, y_1) through which the line passes and the slope m, the point-slope form is:
$$ y - y_1 = m(x - x_1) $$

This form is often more versatile for deriving the equation of a line when a point and slope are given.

Deriving the Equation of the Line with Slope -3/4 Passing Through (-14, 4)

Given:
- Slope $m = -\frac{3}{4}$,
- Point $(x_1, y_1) = (-14, 4)$.

Our goal is to find the explicit equation of this line in slope-intercept form for clarity and practical use.

Applying the Point-Slope Formula

Using the point-slope form:
$$ y - y_1 = m(x - x_1) $$
substitute the known values:
$$ y - 4 = -\frac{3}{4}(x - (-14)) $$
which simplifies to:
$$ y - 4 = -\frac{3}{4}(x + 14) $$

Expanding and Simplifying

Distribute the slope:
$$ y - 4 = -\frac{3}{4}x - \frac{3}{4} \times 14 $$
Calculate:
$$ -\frac{3}{4} \times 14 = -\frac{3 \times 14}{4} = -\frac{42}{4} = -\frac{21}{2} $$
Thus, the equation becomes:
$$ y - 4 = -\frac{3}{4}x - \frac{21}{2} $$

Add 4 to both sides to solve for y:
$$ y = -\frac{3}{4}x - \frac{21}{2} + 4 $$

Express 4 as a fraction with denominator 2:
$$ 4 = \frac{8}{2} $$
So:
$$ y = -\frac{3}{4}x - \frac{21}{2} + \frac{8}{2} $$
Combine the constants:
$$ y = -\frac{3}{4}x - \frac{13}{2} $$

Final Equation in Slope-Intercept Form:
$$ y = -\frac{3}{4}x - \frac{13}{2} $$

This is the explicit equation of the line passing through (-14, 4) with a slope of -3/4.

Graphical Representation of the Line

Visualizing the line helps in understanding its behavior and relationship with the coordinate axes.

Key Features of the Graph

- Slope ($m = -\frac{3}{4}$): The line decreases as x increases; for every 4 units increase in x, y decreases by 3 units.
- Y-intercept ($b = -\frac{13}{2} = -6.5$): The line crosses the y-axis at $(0, -6.5)$.

Plotting the Line

To graph the line:
1. Plot the y-intercept at (0, -6.5).
2. Use the slope to find another point:
- From (0, -6.5), move 4 units to the right (increase x by 4):
$$ y = -\frac{3}{4} \times 4 - 6.5 = -3 - 6.5 = -9.5 $$
So, another point is (4, -9.5).
3. Draw a straight line passing through these points.
4. Confirm that the point (-14, 4) lies on the line:
- Substitute $x = -14$:
$$ y = -\frac{3}{4} \times (-14) - 6.5 = \frac{42}{4} - 6.5 = 10.5 - 6.5 = 4 $$
- The calculation confirms the point is on the line.

Applications of the Line with Slope -3/4 and Point (-14, 4)

Lines with known slopes and points are essential across various disciplines. Here are some notable applications:

1. Physics: Velocity and Motion

- The line can represent the position of an object over time when the velocity is constant.
- A slope of -3/4 might indicate a downward trend, such as an object descending at a rate of 0.75 units per time interval.

2. Economics: Cost and Revenue Models

- The line could model the relationship between production quantity and cost or revenue.
- A negative slope indicates decreasing returns or costs as production increases.

3. Engineering: Signal Processing

- The line can represent the change in signal amplitude over time, where the negative slope indicates attenuation.

4. Data Analysis and Regression

- Understanding how to derive the line equation aids in linear regression analysis, predicting outcomes based on existing data points.

5. Algebra and Geometry Education

- Constructing and interpreting such lines enhances comprehension of fundamental concepts in algebra and coordinate geometry.

Extended Concepts and Variations

Understanding a single line opens doors to exploring more complex topics.

1. Parallel and Perpendicular Lines

- Parallel lines share the same slope ($-\frac{3}{4}$) but have different y-intercepts.
- Perpendicular lines have slopes that are negative reciprocals: $\frac{4}{3}$.

2. Line Equations in Different Forms

- Standard form: $Ax + By = C$
- Two-point form: when two points are known.

3. Calculating Distance from a Point to the Line

- Useful in optimization problems and geometric constructions.

4. Line Intersections

- Finding where this line intersects other lines or curves.

Summary and Key Takeaways

- The line with a slope of $-\frac{3}{4}$ passing through the point $(-14, 4)$ can be expressed as:
$$ y = -\frac{3}{4}x - \frac{13}{2} $$
- It has a y-intercept at $(0, -6.5)$ and passes through the point $(-14, 4)$.
- The slope indicates a decline of 0.75 units in y for every 1 unit increase in x.
- Graphical plotting confirms the correctness of the derived equation.

Understanding how to derive and utilize such lines is crucial for solving real-world problems and strengthening foundational mathematical skills.

Additional Resources for Learning

- Interactive graphing tools like Desmos or GeoGebra.
- Algebra textbooks covering linear equations.
- Online tutorials on coordinate geometry.
- Practice problems involving line equations and their applications.

In conclusion, mastering the derivation and interpretation of lines with given slopes and points enhances your analytical capabilities in mathematics and related fields. Whether for academic purposes or practical applications, the principles outlined in this guide serve as a robust foundation for further exploration and problem-solving in coordinate geometry.

Frequently Asked Questions

What is the equation of the line with a slope of -3/4 passing through the point (-14, 4)?

Using point-slope form: y - 4 = -3/4(x + 14). Simplifying, the equation is y = -3/4 x - 3/4 14 + 4 = -3/4 x - 10.5 + 4 = -3/4 x - 6.5.

How do you find the y-intercept of a line given a point and slope?

Use the point-slope form $y - y_1 = m(x - x_1)$, then solve for y to find the y-intercept when x=0.

What is the slope-intercept form of the line passing

through (-14, 4) with a slope of -3/4?

The slope-intercept form is y = -3/4 x - 6.5.

Is the line with slope -3/4 passing through (-14, 4) increasing or decreasing?

Since the slope is negative (-3/4), the line is decreasing.

How can I graph the line with slope -3/4 passing through (-14, 4)?

Plot the point (-14, 4), then use the slope -3/4 to find another point: from (-14, 4), move down 3 units and right 4 units, then draw the line through these points.

What does a slope of -3/4 indicate about the steepness of the line?

It indicates a moderate negative slope, meaning the line descends 3 units vertically for every 4 units it moves horizontally.

Can the line passing through (-14, 4) with slope -3/4 be written in standard form?

Yes. Starting from y = -3/4 x - 6.5, multiply both sides by 4 to eliminate fractions: 4y = -3x - 26, then bring all to one side: 3x + 4y + 26 = 0.

How do I verify that a point lies on the line with slope -3/4 passing through (-14, 4)?

Substitute the point's x and y coordinates into the line's equation; if the equality holds, the point lies on the line.

What is the significance of the point (-14, 4) in defining this line?

It is the specific point through which the line passes, serving as a reference to determine the line's equation.

How does the slope of -3/4 affect the angle of the line relative to the x-axis?

A slope of -3/4 means the line descends at an angle whose tangent is -3/4, indicating a moderate downward incline relative to the horizontal.

Additional Resources

A Line With A Slope Of -3/4 That Passes Through The Point (-14,4): A Comprehensive Review and Analysis

Understanding the fundamentals of linear equations is essential in various fields, from mathematics and engineering to economics and data science. One such fundamental concept involves analyzing lines characterized by their slopes and specific points through which they pass. In particular, the line with a slope of -3/4 passing through the point (-14, 4) offers an excellent case study for exploring the principles of linear equations, their graphical representations, and their practical applications. This article provides an in-depth review of this line, its properties, derivation, and significance, breaking down each aspect for clarity and comprehensive understanding.

Understanding the Basic Concept

Before delving into the specifics of the line in question, it’s important to revisit some foundational concepts related to linear equations and their characteristics.

What Is a Linear Equation?

A linear equation in two variables, typically written as $y = mx + b$, describes a straight line on the Cartesian plane. The variables x and y represent coordinates, m is the slope of the line, and b is the y-intercept—the point where the line crosses the y-axis.

The Significance of Slope and Point

- Slope (m): Indicates the steepness and the direction of the line. A positive slope rises from left to right; a negative slope falls.
- Point (x_1, y_1): A specific coordinate through which the line passes, used to determine the line’s precise position.

Understanding these two components allows us to formulate the line's equation accurately.

Deriving the Equation of the Line

Given the slope m = -3/4 and the point P(-14, 4), we can derive the line’s equation using the point-slope form:

Point-Slope Form:

$$ y - y_1 = m (x - x_1) $$

Plugging in the known values:
$$ y - 4 = -\frac{3}{4} (x + 14) $$

Simplifying the Equation:
$$ y - 4 = -\frac{3}{4}x - \frac{3}{4} \times 14 $$
$$ y - 4 = -\frac{3}{4}x - \frac{3 \times 14}{4} $$
$$ y - 4 = -\frac{3}{4}x - \frac{42}{4} $$
$$ y - 4 = -\frac{3}{4}x - \frac{21}{2} $$

Adding 4 (which is $\frac{8}{2}$) to both sides:
$$ y = -\frac{3}{4}x - \frac{21}{2} + \frac{8}{2} $$
$$ y = -\frac{3}{4}x - \frac{13}{2} $$

Final Equation:
$$ y = -\frac{3}{4}x - \frac{13}{2} $$

This equation succinctly describes the line passing through (-14, 4) with a slope of -3/4.

Graphical Representation

Graphing this line provides visual insight into its behavior and relationship to the coordinate plane.

Plotting the Line

- Y-intercept: At $b = -\frac{13}{2} \approx -6.5$, the line crosses the y-axis.
- Slope: -3/4 indicates that for every 4 units moved horizontally to the right, the line descends 3 units vertically.

Key Points for Plotting:
- Starting point at (0, -6.5)
- Use the slope to find another point:
- From (0, -6.5), move 4 units right to x=4:
- y = -3/4 4 - 13/2 = -3 - 6.5 = -9.5
- Point: (4, -9.5)
- Plot these points and draw a straight line through them.

Visualization:
The line will slope downward from left to right, crossing the y-axis at -6.5, passing through (-14, 4), and extending infinitely in both directions.

Key Features and Properties of the Line

A thorough review of the line’s features reveals its geometric and algebraic properties.

Slope Analysis

- The slope of -3/4 indicates a moderate negative inclination.
- The slope’s fractional form simplifies understanding the rate of change between x and y.

Y-Intercept

- The line crosses the y-axis at approximately -6.5.
- This point is essential in graphing and understanding the line’s position relative to the origin.

Point Through Which the Line Passes

- Confirmed point: (-14, 4)
- Substituting x = -14 into the equation:
- y = -3/4 (-14) - 13/2 = 10.5 - 6.5 = 4
- Validates the line passes through the point.

Line's Behavior

- As x increases, y decreases, consistent with the negative slope.
- For large positive x, y tends toward negative infinity; for large negative x, y tends toward positive infinity.

Applications and Practical Uses

Lines with specific slopes passing through specific points are foundational in many real-world applications.

Engineering and Physics

- Analyzing motion along a straight path where the slope relates to velocity or acceleration rates.
- Designing slopes and inclines where the rate of change is critical.

Economics and Business

- Cost functions, revenue models, and profit margins often follow linear relationships.
- The line’s slope can represent rate of change in costs or revenues relative to quantities.

Mathematics Education

- Serves as an excellent example for teaching concepts like point-slope form, slope-intercept form, and graphing techniques.
- Demonstrates how algebraic equations translate into visual graphs.

Data Science & Analytics

- Linear models are the backbone of regression analysis, helping predict outcomes based on input variables.
- The line’s properties assist in understanding relationships between variables.

Pros and Cons of Analyzing This Line

Understanding the specific line with slope -3/4 passing through (-14, 4) offers various benefits and some limitations.

Pros:
- Clear Geometric Interpretation: The slope and point provide an intuitive understanding of the line’s position and inclination.
- Analytical Precision: Deriving the equation allows for exact calculations and predictions.
- Versatility: Applicable across multiple disciplines, from physics to economics.
- Visualization: Easy to plot and interpret graphically, aiding comprehension.

Cons:
- Limited to Linear Relationships: Cannot model non-linear phenomena or more complex relationships.
- Simplistic Assumption: Assumes constant rate of change, which may not reflect real-world complexities.
- Dependence on Accurate Data: Small errors in the point or slope can significantly alter the line’s equation and interpretation.

Extensions and Further Study

This line serves as a basis for more advanced topics in mathematics and related fields.

Parallel and Perpendicular Lines

- Lines parallel to this one share the same slope (-3/4).
- Lines perpendicular have slopes that are negative reciprocals (4/3).

Transformations

- Shifting the line vertically or horizontally involves adjusting the intercept or the point used.
- Rotations and reflections can explore the line's symmetry.

Higher-Dimensional Analogues

- Extending concepts into three-dimensional space involves planes with similar slope and point parameters.

Conclusion

The line with a slope of -3/4 passing through the point (-14, 4) exemplifies the elegance and utility of linear equations. Its derivation from fundamental principles, graphical representation, and practical applications showcase the power of algebraic tools in understanding the world around us. Whether used in academic settings or real-world problem-solving, mastering the analysis of such lines enhances mathematical literacy and analytical skills. The detailed examination of this line underscores the importance of foundational concepts, while also opening pathways for further exploration into more complex mathematical structures and their applications.

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