Complete The Point-slope Equation Of The Line Through -8,1 And -6,5

Complete The Point-slope Equation Of The Line Through -8,1 And -6,5

When working with linear equations, understanding how to formulate the point-slope equation is an essential skill in algebra. The point-slope form provides a straightforward way to express the equation of a line when you know a point on the line and its slope. In this article, we will focus on how to complete the point-slope equation of the line passing through the points (-8, 1) and (-6, 5). This process involves calculating the slope of the line, selecting an appropriate point, and then writing the equation in the point-slope form.

Understanding the Point-Slope Equation of a Line

What Is the Point-Slope Form?

The point-slope form of a linear equation is given by:

$$y - y_1 = m(x - x_1)$$

where:
- (x_1, y_1) is a point on the line.
- m is the slope of the line.

This form is particularly useful when you know one point on the line and the slope but do not yet have the slope-intercept form ($y = mx + b$).

Why Use the Point-Slope Form?

- It simplifies the process of writing the line's equation when a point and slope are known.
- It makes it easier to convert to other forms, such as slope-intercept or standard form.
- It emphasizes the relationship between a specific point and the slope, providing clarity in geometric interpretations.

Finding the Slope of the Line Through Two Points

Given Points

The points provided are:
- Point 1: (-8, 1)
- Point 2: (-6, 5)

Calculating the Slope (m)

The slope formula between two points (x_1, y_1) and (x_2, y_2) is:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

Applying the given points:

- $x_1 = -8$, $y_1 = 1$
- $x_2 = -6$, $y_2 = 5$

Calculating the numerator:

$(5 - 1) = 4$

Calculating the denominator:

$(-6 - (-8)) = (-6 + 8) = 2$

Thus:

$$m = 4 / 2 = 2$$

The slope of the line is 2.

Formulating the Point-Slope Equation

Selecting a Point for the Equation

You can choose either of the given points to write the equation. Typically, selecting the point with smaller coordinates or the one easiest to work with is common. Here, we'll use (-8, 1).

Writing the Equation in Point-Slope Form

Using the point (-8, 1) and the slope m = 2:

$$y - 1 = 2(x - (-8))$$

Simplify:

$$y - 1 = 2(x + 8)$$

This is the complete point-slope form of the line.

Converting the Point-Slope Equation to Slope-Intercept Form

Why Convert?

The slope-intercept form (y = mx + b) is often more familiar and easier to interpret because it explicitly shows the slope and the y-intercept.

Steps to Convert

Starting from:

$$y - 1 = 2(x + 8)$$

Distribute the 2:

$$y - 1 = 2x + 16$$

Add 1 to both sides:

$$y = 2x + 17$$

Thus, the slope-intercept form of the line is y = 2x + 17.

Verifying the Line with the Second Point

To ensure accuracy, verify that the second point (-6, 5) satisfies the slope-intercept form $y = 2x + 17$.

Substitute $x = -6$:

$y = 2(-6) + 17 = -12 + 17 = 5$

Since $y = 5$ matches the y-coordinate of the second point, the equation is confirmed.

Additional Methods to Write the Line's Equation

Standard Form

Rearranging $y = 2x + 17$ into standard form:

- Subtract 2x from both sides:

$$-2x + y = 17$$

Or multiply through by -1 to make the x coefficient positive:

$$2x - y = -17$$

Graphing the Line

- Plot the point (-8, 1).
- Use the slope (rise over run): from the point, move up 2 units and right 1 unit repeatedly.
- Confirm the line passes through the point (-6, 5).

Summary of the Process

To sum up, here are the essential steps to complete the point-slope equation of a line through two points:

1. Identify the points: (-8, 1) and (-6, 5).
2. Calculate the slope:

- Use the formula $m = (y_2 - y_1) / (x_2 - x_1)$.
- For the given points, $m = 2$.

3. Choose a point: (-8, 1).

4. Write the point-slope form: y - 1 = 2(x + 8).
5. Simplify or convert to other forms as needed (e.g., slope-intercept or standard form).

Conclusion

Understanding how to complete the point-slope equation of a line through two points is a fundamental skill in algebra and coordinate geometry. By carefully calculating the slope and selecting a point, you can quickly write the line's equation in various forms to suit different needs. The line passing through (-8, 1) and (-6, 5) has a slope of 2, and its point-slope equation is y - 1 = 2(x + 8). Converting this into slope-intercept form yields y = 2x + 17, which clearly shows the slope and the y-intercept. Mastery of these steps enables you to analyze and graph lines efficiently, providing a strong foundation for more advanced mathematical topics.

Frequently Asked Questions

What is the point-slope form of the line passing through points (-8, 1) and (-6, 5)?

The point-slope form is $y - y_1 = m(x - x_1)$. First, find the slope m: m = (5 - 1) / (-6 - (-8)) = 4 / 2 = 2. Using point (-8, 1), the equation is y - 1 = 2(x + 8).

How do you calculate the slope of a line passing through two points?

The slope m is calculated as $(y_2 - y_1) / (x_2 - x_1)$. For points (-8, 1) and (-6, 5), m = (5 - 1) / (-6 - (-8)) = 4 / 2 = 2.

What is the complete point-slope equation of the line through (-8, 1) with slope 2?

Using point (-8, 1): y - 1 = 2(x + 8).

Can the point-slope form be converted to slope-intercept form? How?

Yes. Starting from y - 1 = 2(x + 8), expand to y - 1 = 2x + 16, then add 1 to both sides to get y = 2x + 17.

What is the significance of the point (-6, 5) in the line's equation?

It is a point on the line used to verify or rewrite the equation, ensuring the line passes through (-6, 5).

How do you verify that the point (-6, 5) lies on the line y = 2x + 17?

Substitute x = -6 into y = 2x + 17: y = 2(-6) + 17 = -12 + 17 = 5. Since y = 5 matches the point's y-coordinate, the point lies on the line.

What is the final equation of the line passing through (-8, 1) and (-6, 5) in slope-intercept form?

The final equation is y = 2x + 17.

Additional Resources

Complete The Point-Slope Equation Of The Line Through (-8, 1) And (-6, 5)

Understanding the equation of a line is fundamental in algebra and coordinate geometry. Among the various forms of linear equations, the point-slope form provides an efficient way to express the line when you know a point on the line and the slope. Here, we will delve into how to find the complete point-slope equation of the line passing through the points (-8, 1) and (-6, 5), exploring every critical aspect to deepen your understanding of the process.

Understanding the Foundations: Coordinates and the Equation of a Line

Before diving into the derivation, let's review some essential concepts:

- Coordinates: Points in the plane are represented as ordered pairs (x, y). For example, (-8, 1) has x = -8 and y = 1.
- Line Equation: The general forms include slope-intercept (y = mx + b), point-slope, and two-point forms. The point-slope form is especially useful when a point and slope are known.

Step 1: Identifying the Given Points

The problem provides two points:

1. Point A: (-8, 1)
2. Point B: (-6, 5)

These points are essential because they serve as the basis for calculating the slope and subsequently deriving the line's equation.

Step 2: Calculating the Slope (m)

The slope of a line (m) quantifies its steepness and is calculated as the change in y over the change in x between two points:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Applying the points:

- $x_1 = -8$, $y_1 = 1$
- $x_2 = -6$, $y_2 = 5$

Calculating:

$$
m = \frac{5 - 1}{-6 - (-8)} = \frac{4}{-6 + 8} = \frac{4}{2} = 2
$$

Result: The slope of the line is 2.

Note: The slope being positive indicates the line rises as x increases.

Step 3: Understanding the Point-Slope Form

The point-slope form of a line's equation is:

$$
y - y_1 = m(x - x_1)
$$

where:

- (x_1, y_1) is a known point on the line
- m is the slope

This form is particularly useful because it directly incorporates known points and the slope, making it straightforward to write the equation once these are known.

Step 4: Writing the Complete Point-Slope Equation

Using the point (-8, 1) and the slope $m = 2$:

$$
y - 1 = 2(x - (-8))
$$

Simplify:

$$
y - 1 = 2(x + 8)
$$

This is the complete point-slope equation of the line passing through the two points.

Step 5: Expanding and Simplifying the Equation (Optional)

While the point-slope form is complete, sometimes it's helpful to express the equation in slope-intercept form, especially for graphing or interpretation. To convert:

$$
y - 1 = 2(x + 8)
$$

Distribute:

\[

$$
y - 1 = 2x + 16
$$

Add 1 to both sides:

$$
y = 2x + 17
$$

This is the slope-intercept form of the same line, indicating a y-intercept at 17.

Note: The point-slope form remains more flexible for many purposes, especially when multiple points are involved or when working with calculus.

Deep Dive: Verifying the Equation

It's essential to verify that the derived equation correctly represents the line through the given points.

- Check with Point (-8, 1):

$$
y - 1 = 2(x + 8)
$$
Plug in x = -8:

$$
1 - 1 = 2(-8 + 8) \Rightarrow 0 = 2(0) \Rightarrow 0 = 0
$$

True.

- Check with Point (-6, 5):

$$
y - 1 = 2(x + 8)
$$
Plug in x = -6, y = 5:

$$
5 - 1 = 2(-6 + 8) \Rightarrow 4 = 2(2) \Rightarrow 4 = 4
$$

True.

Thus, the equation accurately represents the line passing through both points.

Additional Aspects and Insights

1. Significance of the Slope

- The slope $m = 2$ indicates that for every unit increase in x, y increases by 2.
- It reflects the line's inclination and helps in understanding its behavior visually.

2. Using the Equation for Graphing

- The point-slope form allows easy plotting: start at the known point (-8, 1), then use the slope to determine additional points.
- For example, moving 1 unit right (x increases by 1), y increases by 2, leading to the point (-7, 3).

3. Variations of the Line Equation

- Slope-intercept form: $y = 2x + 17$
- Standard form: $Ax + By + C = 0$, which can be derived as:

$$
y = 2x + 17 \Rightarrow 2x - y + 17 = 0
$$

Applications and Real-World Relevance

Understanding how to derive the complete point-slope equation has practical implications:

- Data Analysis: Fitting linear models to data points.
- Engineering: Designing structures or components with linear relationships.
- Physics: Calculating motion equations where relationships are linear.
- Economics: Modeling cost, revenue, or supply-demand relationships.

Summary of the Process

To encapsulate the entire process succinctly:

1. Identify the points: (-8, 1) and (-6, 5).
2. Calculate the slope:

$$
m = \frac{5 - 1}{-6 - (-8)} = 2
$$

3. Select a point (say, (-8, 1)) to use in the point-slope form.
4. Write the equation:

$$
y - 1 = 2(x + 8)
$$

5. Optional conversion to slope-intercept form for clarity or graphing.

Conclusion

The complete point-slope equation of the line passing through the points (-8, 1) and (-6, 5) is:

$$
\boxed{ y - 1 = 2(x + 8) }
$$

This form encapsulates all the necessary information about the line, including its slope and a specific point through which it passes. Mastery of deriving and manipulating such equations is essential for higher-level mathematics, physics, engineering, and numerous applied sciences. It demonstrates how simple calculations and algebraic manipulations come together to describe the geometry of the world around us precisely.

[Complete The Point Slope Equation Of The Line Through 8 1 And 6 5]

Complete The Point Slope Equation Of The Line Through 8 1 And 6 5

Related Articles

- Choose The Correct Description For Each Phase Of Mitosis.Label A
- What Is The First Species To Colonize New Or Undisturbed Land?
- What Do The Particles Of Water Look Like At A And B? A. B. C. D.

Back to Home