Cos = 5/13 And Sin < 0. Identify The Quadrant Of And Find Sin .

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Understanding the relationships between the trigonometric functions is fundamental in mathematics, especially in the context of the unit circle. When given a cosine value and an inequality involving sine, it becomes essential to determine the quadrant in which the angle resides and to find the sine value accordingly. In this article, we will explore how to interpret the given information—cosine value of 5/13 and sine being negative—and how to identify the quadrant and compute the sine of the angle step-by-step.

Understanding the Basics of Trigonometric Functions

What Are Cosine and Sine?

- Cosine (cos) and sine (sin) are fundamental trigonometric functions that relate the angles of a right triangle to the ratios of its sides.
- In the context of the unit circle, which has a radius of 1, these functions correspond to the coordinates of points on the circle:
- Cosine is the x-coordinate of the point.
- Sine is the y-coordinate of the point.

The Unit Circle and Coordinates

- The unit circle allows us to analyze the signs of sine and cosine based on the quadrant:
- Quadrant I: Both sine and cosine are positive.
- Quadrant II: Sine is positive, cosine is negative.
- Quadrant III: Both sine and cosine are negative.
- Quadrant IV: Cosine is positive, sine is negative.

Understanding which quadrant an angle resides in helps determine the signs of its sine and cosine.

Interpreting the Given Data: Cos = 5/13 and Sin < 0

Given the cosine value: cos θ = 5/13

- The value 5/13 is a positive number, less than 1.
- This indicates that the x-coordinate of the point on the unit circle corresponding to angle θ is positive.

Given the inequality: sin θ < 0

- The sine of the angle is negative, meaning the y-coordinate on the unit circle is negative.

Combining the Information

- Since cos θ > 0 and sin θ < 0, the angle θ must be located in the Fourth Quadrant.
- This aligns with the standard signs on the unit circle, where cosine is positive and sine is negative in Quadrant IV.

Identifying the Quadrant of the Angle

Quadrant Sign Patterns

- Quadrant I (0° to 90°): sin > 0, cos > 0
- Quadrant II (90° to 180°): sin > 0, cos < 0
- Quadrant III (180° to 270°): sin < 0, cos < 0
- Quadrant IV (270° to 360°): sin < 0, cos > 0

Since the information given indicates cos θ = 5/13 (> 0) and sin θ < 0, the angle θ resides in Quadrant IV.

Calculating Sin θ Based on Cos θ

The Pythagorean Identity

- The fundamental identity for sine and cosine is:

$$
\sin^2 θ + \cos^2 θ = 1
$$

- Using this identity, we can find sin θ when cos θ is known.

Step-by-Step Calculation

1. Plug in the known cosine value:

$$
\cos \theta = \frac{5}{13}
$$

2. Compute $\cos^2 \theta$:

$$
\left(\frac{5}{13}\right)^2 = \frac{25}{169}
$$

3. Apply the Pythagorean identity:

$$
\sin^2 \theta = 1 - \cos^2 \theta = 1 - \frac{25}{169}
$$

4. Simplify:

$$
\sin^2 \theta = \frac{169}{169} - \frac{25}{169} = \frac{144}{169}
$$

5. Find $\sin \theta$:

$$
\sin \theta = \pm \sqrt{\frac{144}{169}} = \pm \frac{12}{13}
$$

6. Determine the sign of $\sin \theta$:

- Since $\sin \theta < 0$ (given), we select the negative root:

$$
\boxed{
\sin \theta = -\frac{12}{13}
}
$$

Summary of Findings and Final Answer

- The angle θ is located in Quadrant IV because:
- $\cos \theta > 0$
- $\sin \theta < 0$

- The exact value of $\sin \theta$ is:

$$
\boxed{
\sin \theta = -\frac{12}{13}
}
$$

This comprehensive analysis confirms the quadrant and provides the precise sine value based on the given cosine.

Additional Insights and Applications

Real-World Applications of Trigonometric Quadrants

- Navigation and Geography: Determining directions based on angles and signs.
- Physics: Analyzing projectile motion and oscillations.
- Engineering: Signal processing and wave analysis.

Common Mistakes to Avoid

- Ignoring signs: Always verify the signs of sine and cosine based on the quadrant.
- Misapplying identities: Use the Pythagorean identity carefully, especially when dealing with the positive or negative root.
- Assuming default signs: Don’t assume the sine or cosine signs; always analyze based on the given information.

Practice Problems for Mastery

- Given $\sin \theta = 3/5$, find $\cos \theta$ in Quadrant II.
- If $\tan \theta = - \frac{4}{3}$, determine the quadrant and find $\sin \theta$.

Conclusion

In summary, starting from $\cos \theta = 5/13$ and knowing that $\sin \theta < 0$, we identified that the angle θ is in the Fourth Quadrant. Using the Pythagorean identity, we calculated $\sin \theta = -12/13$. Recognizing the quadrant and applying fundamental trigonometric identities are crucial skills that allow us to analyze and solve various problems involving angles and their functions. Whether for academic purposes or practical applications, mastering these concepts enhances comprehension of the interconnected nature of trigonometry.

Frequently Asked Questions

Given that cos θ = 5/13 and sin θ < 0, in which quadrant does θ lie?

θ is in the fourth quadrant because cosine is positive and sine is negative in that quadrant.

How do you find sin θ given cos θ = 5/13 and that sin θ < 0?

Use the Pythagorean identity: $\sin^2 \theta + \cos^2 \theta = 1$. Substitute $\cos \theta = 5/13$: $\sin^2 \theta + (5/13)^2 = 1$, then solve for sin θ, considering sin θ < 0.

What is the value of sin θ when cos θ = 5/13 and sin θ < 0?

sin θ = -12/13, because from the Pythagorean theorem: $\sin \theta = -\sqrt{1 - (5/13)^2} = -12/13$.

Why is sin θ negative when cos θ = 5/13 and θ is in the fourth quadrant?

Because in the fourth quadrant, sine values are negative while cosine values are positive, which aligns with the given information.

What is the Pythagorean identity used to find sin θ in this problem?

The identity is $\sin^2 \theta + \cos^2 \theta = 1$.

Can you determine the exact values of sin θ and cos θ with only the given data?

Yes, cos θ is given as 5/13 and sin θ can be found as -12/13 using the Pythagorean theorem, so both values are determined.

How does knowing the sign of sin θ help in identifying the quadrant?

The sign of sin θ (positive or negative) combined with the sign of cos θ helps pinpoint the exact quadrant where θ is located.

What is the significance of the given cos θ and the inequality for sin θ in solving trigonometric problems?

It helps to determine the specific quadrant and accurately calculate the sine value, ensuring the solution aligns with the given conditions.

Additional Resources

Cos = 5/13 And Sin < 0: Identify The Quadrant Of and Find Sin

Understanding the relationship between cosine and sine values is fundamental in trigonometry, especially when analyzing angles in different quadrants. When given specific values such as Cos = 5/13 and the condition Sin < 0, it is essential to determine the exact quadrant in which the angle resides and then accurately find the value of sine. This guide aims to walk you through the process step-by-step, providing clarity on how to interpret these values and applying trigonometric principles to arrive at the solution.

Introduction to Trigonometric Ratios and Quadrants

Before diving into the problem, let’s review some key concepts:

The Basic Trigonometric Ratios
- Sine (sin): The ratio of the length of the side opposite the angle to the hypotenuse in a right-angled triangle.
- Cosine (cos): The ratio of the length of the side adjacent to the angle to the hypotenuse.

The Unit Circle and Quadrants
- The unit circle is a circle of radius 1 centered at the origin (0,0) on the coordinate plane.
- The angle θ is measured from the positive x-axis.
- The coordinates of a point on the unit circle are (cos θ, sin θ).

Sign of Trigonometric Functions in Different Quadrants
- Quadrant I: Both sin and cos are positive.
- Quadrant II: sin is positive, cos is negative.
- Quadrant III: Both sin and cos are negative.
- Quadrant IV: sin is negative, cos is positive.

Given the data:
- Cos = 5/13 (positive value)
- Sin < 0 (negative value)

From these, we can immediately infer the quadrant in which the angle lies.

Step 1: Determining the Quadrant of the Angle

Since cos θ = 5/13 and sin θ < 0, the signs of the trigonometric functions indicate the following:

- cos θ > 0 → cosine is positive.
- sin θ < 0 → sine is negative.

Referring to the sign chart of the four quadrants:

| Quadrant | sin | cos | Explanation |

I	+	+	Both positive
II	+	-	Sine positive, cosine negative
III	-	-	Both negative
IV	-	+	Sine negative, cosine positive

Conclusion: The angle θ is in Quadrant IV.

Step 2: Visualizing the Problem and the Right Triangle

Since cosine is given as 5/13, we can think of a right triangle (or a point on the unit circle) where:

- The adjacent side to θ is 5 units.
- The hypotenuse is 13 units.

In the context of the unit circle, these represent the x-coordinate (cosine) and the hypotenuse (radius).

Constructing the Triangle
- Adjacent side (x): 5
- Hypotenuse (r): 13

The opposite side (y) can be found using the Pythagorean theorem:

$$ y^2 + x^2 = r^2 $$

$$ y^2 + 5^2 = 13^2 $$

$$ y^2 + 25 = 169 $$

$$ y^2 = 169 - 25 = 144 $$

$$ y = \pm \sqrt{144} = \pm 12 $$

Since the angle θ is in Quadrant IV, where $\sin \theta < 0$, the opposite side (y) should be negative:

$$ y = -12 $$

Step 3: Calculating Sin θ

Now that we have both the adjacent side and the opposite side, we can find $\sin \theta$:

$$
\sin \theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{-12}{13}
$$

Answer: $\boxed{\sin \theta = -\frac{12}{13}}$

This negative value confirms the earlier deduction that θ is in Quadrant IV.

Summary of Key Findings

- The quadrant of the angle is Quadrant IV, based on the signs of cosine and sine.
- The value of sine is $-12/13$, derived from the Pythagorean theorem considering the given cosine value.

Additional Insights and Applications

Verifying the Result
It's always good practice to verify the consistency of your findings:

$$
\sin^2 \theta + \cos^2 \theta = 1
$$

$$
\left(-\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2 = \frac{144}{169} + \frac{25}{169} = \frac{169}{169} = 1
$$

This confirms the correctness of the calculated sine value.

Practical Uses
- Trigonometric identities: Knowing the sine and cosine values allows for solving more complex problems involving tangent, cotangent, and their identities.
- Angular measurements: Helps in fields like physics, engineering, and navigation where precise angle calculations are critical.
- Graphing functions: Understanding the signs of sine and cosine in quadrants is essential for graphing trigonometric functions accurately.

Final Thoughts

Identifying the quadrant of an angle based on the signs of sine and cosine is a foundational skill in trigonometry. When given specific ratios like Cos = 5/13 and conditions such as Sin < 0, you can determine the precise location of the angle in the coordinate plane and compute the sine value confidently. Remember to leverage the Pythagorean theorem and the sign conventions of the quadrants to guide your calculations. This approach not only helps in solving the problem at hand but also builds a deeper understanding of the geometric interpretations of trigonometric functions.

In summary:

- The angle with Cos = 5/13 and Sin < 0 lies in Quadrant IV.
- The sine of the angle is \(-12/13\).
- The process involves identifying the quadrant, constructing a right triangle, applying the Pythagorean theorem, and confirming the signs.

With these steps and insights, you're well-equipped to handle similar problems involving trigonometric ratios and quadrants, ensuring accuracy and confidence in your mathematical reasoning.

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