

Find All Real Values Of X That Satisfy The Equation $x^2 - 7x = 98$.

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Understanding how to solve quadratic equations such as $(x^2 - 7x = 98)$ is fundamental in algebra. This particular equation involves a quadratic expression set equal to a constant, and solving it requires employing methods like factoring, completing the square, or using the quadratic formula. In this article, we will explore these approaches step-by-step, providing a comprehensive understanding of how to find all real solutions to the equation. We will also analyze the solutions, interpret their meaning, and discuss relevant concepts such as the nature of roots and the graph of the quadratic function.

Rearranging the Equation into Standard Quadratic Form

Before applying any solution techniques, it's essential to write the given equation in a standard form. The original equation is:

$$(x^2 - 7x = 98)$$

To do this, we need to move all terms to one side of the equation:

$$(x^2 - 7x - 98 = 0)$$

This is a quadratic equation in the standard form $(ax^2 + bx + c = 0)$, where:

- $(a = 1)$
- $(b = -7)$
- $(c = -98)$

Having the equation in this form allows us to apply various solution methods systematically.

Methods for Solving the Quadratic Equation

There are primarily three methods to solve quadratic equations:

1. Factoring

2. Completing the Square
3. Quadratic Formula

We will explore each method, starting with the most straightforward approach when applicable.

Method 1: Factoring

Factoring involves expressing the quadratic as a product of two binomials:

$$(x + p)(x + q) = 0$$

where p and q are numbers such that:

- $p \times q = c$
- $p + q = b$

Given our quadratic:

$$x^2 - 7x - 98 = 0$$

We seek two numbers p and q such that:

$$\begin{aligned} p \times q &= -98 \\ p + q &= -7 \end{aligned}$$

Step 1: Find factor pairs of (-98) .

The pairs are:

- $(1, -98)$
- $(-1, 98)$
- $(2, -49)$
- $(-2, 49)$
- $(7, -14)$
- $(-7, 14)$

Step 2: Check which pair sums to (-7) .

Calculate the sums:

- $1 + (-98) = -97$
- $-1 + 98 = 97$
- $2 + (-49) = -47$
- $-2 + 49 = 47$
- $7 + (-14) = -7$
- $-7 + 14 = 7$

The pair $(7, -14)$ sums to (-7) , which matches our (b) .

Step 3: Write the factors.

Since $(p = 7)$ and $(q = -14)$, the factored form is:

$$(x + 7)(x - 14) = 0$$

Step 4: Solve for (x) :

$$-(x + 7 = 0 \Rightarrow x = -7)$$

$$-(x - 14 = 0 \Rightarrow x = 14)$$

Result: The solutions are $(x = -7)$ and $(x = 14)$.

Method 2: Completing the Square

Completing the square is a technique to convert a quadratic into a perfect square trinomial, making it easier to solve.

Step 1: Start with the quadratic:

$$x^2 - 7x - 98 = 0$$

Step 2: Isolate the quadratic and linear terms:

$$x^2 - 7x = 98$$

Step 3: Complete the square on the left side.

- Take half of the coefficient of (x) , which is $(-7/2)$.

- Square it: $(\left(-\frac{7}{2}\right)^2 = \frac{49}{4})$.

Add this square to both sides:

$$x^2 - 7x + \frac{49}{4} = 98 + \frac{49}{4}$$

Express the right side with a common denominator:

$$98 = \frac{392}{4}$$

So,

$$\frac{392}{4} + \frac{49}{4} = \frac{441}{4}$$

Step 4: Write the left side as a perfect square:

$$\left(x - \frac{7}{2} \right)^2 = \frac{441}{4}$$

Step 5: Take the square root of both sides:

$$x - \frac{7}{2} = \pm \frac{\sqrt{441}}{2}$$

Since $(\sqrt{441} = 21)$:

$$x - \frac{7}{2} = \pm \frac{21}{2}$$

Step 6: Solve for (x) :

$$-(x = \frac{7}{2} + \frac{21}{2} = \frac{28}{2} = 14)$$

$$-(x = \frac{7}{2} - \frac{21}{2} = -\frac{14}{2} = -7)$$

Result: The solutions are $(x = -7)$ and $(x = 14)$, confirming our previous results.

Method 3: Quadratic Formula

The quadratic formula provides a universal method for solving any quadratic:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Step 1: Identify (a, b, c) :

$$a=1, \quad b=-7, \quad c=-98$$

Step 2: Calculate the discriminant:

$$D = b^2 - 4ac = (-7)^2 - 4 \times 1 \times (-98) = 49 + 392 = 441$$

Since $(D > 0)$, there are two real solutions.

Step 3: Compute the square root of the discriminant:

$$\sqrt{D} = \sqrt{441} = 21$$

Step 4: Plug into the formula:

$$x = \frac{-(-7) \pm 21}{2 \times 1} = \frac{7 \pm 21}{2}$$

Step 5: Find the two solutions:

$$-(x = \frac{7 + 21}{2} = \frac{28}{2} = 14)$$

$$-(x = \frac{7 - 21}{2} = \frac{-14}{2} = -7)$$

Again, solutions are $(x = -7)$ and $(x=14)$.

Analysis of the Solutions

Having obtained the solutions, it is critical to analyze their significance and verify their correctness.

Verification of Solutions

Let's verify each solution by substituting back into the original equation:

- For $(x = -7)$:

$$[(-7)^2 - 7(-7) = 49 + 49 = 98]$$

- For $(x = 14)$:

$$[(14)^2 - 7(14) = 196 - 98 = 98]$$

Both satisfy the original equation, confirming their validity.

Nature of the Roots

The discriminant $(D = 441)$ is positive, indicating two real and distinct roots. The solutions are both real numbers, as expected for a quadratic with real coefficients and a positive discriminant.

Graphical Interpretation

Visualizing the quadratic function:

$$[y = x^2 - 7x - 98]$$

can provide intuition about the solutions.

- The parabola opens upward (since $(a=1 > 0)$).
- The roots are the points where the parabola intersects the x-axis.
- The solutions $(x=-7)$ and $(x=14)$ correspond to these intersection points.

Plotting the parabola would show it crossing the x-axis at these two points, confirming the solutions graphically.

Summary and Key Takeaways

- The original quadratic equation $(x^2 - 7x = 98)$ can be rearranged into standard form $(x^2 - 7x - 98 = 0)$.
- Multiple methods—factoring, completing the square, and quadratic formula—can be employed to find solutions.
- All methods lead to the same solutions: $(x = -7)$ and $(x = 14)$.
- Verifying solutions by substitution confirms their correctness.
- The quadratic's discriminant indicates two distinct real roots, consistent with our solutions.
- Graphing provides a visual confirmation that the parabola intersects the x-axis at the solutions.

Conclusion (0)). The solutions $x = 14$ and $x = -7$ correspond to the x-intercepts of this parabola—points where the graph crosses the x-axis.

Visual Characteristics:

- **Vertex:** Located at $(x = -\frac{b}{2a} = -\frac{-7}{2} = \frac{7}{2} = 3.5)$. This vertex point gives the minimum value of the parabola.
- **X-intercepts:** The solutions we found, $(x = -7)$ and $(x = 14)$, are the roots, where the parabola meets the x-axis.

Plotting this parabola would show two points where the curve intersects the x-axis, confirming the real solutions.

Context and Applications of Quadratic Equations

Quadratic equations like the one discussed are not merely academic exercises; they have diverse real-world applications.

Engineering and Physics

- Calculating projectile trajectories.**
- Analyzing the motion of objects under constant acceleration.**
- Design of parabolic reflectors and antennas.**

Economics and Business

- Modeling profit maximization and cost functions.**
- Analyzing quadratic relationships between variables.**

Computer Science and Data Analysis

- Algorithms involving quadratic time complexity.**
- Curve fitting and regression analysis.**

Understanding how to solve quadratic equations accurately ensures professionals across disciplines can model, analyze, and interpret various phenomena effectively.

Common Pitfalls and Tips for Solving Quadratic Equations

While the process may seem straightforward, students and practitioners should be aware of common mistakes:

- Miscalculating the discriminant: Ensure correct sign handling, especially with negative coefficients.**
- Ignoring extraneous solutions: For equations involving radicals, always verify solutions by substitution.**
- Overlooking complex solutions: When the discriminant is negative, solutions involve complex numbers, which are outside the scope of real solutions.**
- Incorrect factorization: When factoring, check factors thoroughly to avoid errors.**

Tips for Success:

- Always write the quadratic in standard form before applying any method.**
- Use the quadratic formula as a reliable fallback.**
- Visualize the problem graphically when possible.**
- Verify solutions to ensure accuracy.**

Conclusion: Summarizing the Solution Process

The equation $(X^2 - 7x = 98)$ exemplifies the fundamental approach to solving quadratics. By rewriting it into the standard form and applying the quadratic formula, we determined the solutions $(x = 14)$ and $(x = -7)$. These solutions satisfy the original equation, confirmed through substitution and graphical interpretation.

Mastering such methods equips learners and professionals with essential tools for tackling more complex algebraic problems, fostering analytical thinking and problem-solving skills. Quadratic equations are a vital component of mathematical literacy, bridging theoretical

concepts and practical applications across numerous fields.

In essence, solving $X^2 - 7x = 98$ not only yields precise numerical answers but also reinforces foundational algebraic techniques, analytical reasoning, and the importance of verification, all crucial for advanced mathematical proficiency and real-world problem-solving.

Final Thought: Embracing the process of solving quadratic equations enhances understanding of the interplay between algebraic manipulation, geometric visualization, and practical applications—an indispensable skill in the toolkit of scientists, engineers, economists, and mathematicians alike.

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