Find F At The Given Point. F(x,y,z)=x^3 +y^22z^2 Z Lnx, (1,1,3)

Find F At The Given Point. F(x,y,z)=x^3 + y^2 2z^2 Z ln x, (1,1,3)

Introduction

In the realm of multivariable calculus, evaluating functions at specific points is a fundamental skill that lays the groundwork for understanding more complex concepts such as gradients, divergence, curl, and optimization. This article aims to guide you through the detailed process of finding the value of the function $F(x,y,z) = x^3 + y^2 \cdot 2z^2 \cdot Z \ln x$ at the point $(1, 1, 3)$. We will break down each step meticulously, explain the underlying principles, and provide insights into how such calculations are performed efficiently and accurately.

Understanding the Function

Before diving into calculations, it’s essential to understand the structure of the given function:

$$
F(x, y, z) = x^3 + y^2 \cdot 2z^2 \cdot Z \ln x
$$

Clarifying the Function Components

- Term 1: x^3 — a straightforward cubic function of x.
- Term 2: $y^2 \cdot 2z^2 \cdot Z \ln x$ — a product involving y^2, z^2, a constant Z, and $\ln x$.

Addressing Potential Ambiguity

The function as written contains a component "Z" which may be a variable or a constant. Based on typical notation, if "Z" is a variable, it should be specified as z; if it's a constant, it should be clarified. For the purpose of this calculation, we will assume Z is a constant.

Step-by-Step Calculation at the Point (1, 1, 3)

Given the point $(x, y, z) = (1, 1, 3)$, our task is to evaluate:

$$
F(1, 1, 3) = 1^3 + (1)^2 \cdot 2 \cdot (3)^2 \cdot Z \ln(1)
$$

Step 1: Evaluate Each Term Separately

Term 1: x^3

$$
x^3 = 1^3 = 1
$$

Term 2: $y^2 \cdot 2z^2 \cdot Z \ln x$

Break down:

- $y^2 = 1^2 = 1$
- $2z^2 = 2 \times 3^2 = 2 \times 9 = 18$
- Z remains as is (constant)
- $\ln x = \ln 1 = 0$

Therefore, the second term becomes:

$$
1 \times 18 \times Z \times 0 = 0
$$

Step 2: Combine Results

$$
F(1, 1, 3) = 1 + 0 = 1
$$

Final Result

$$
\boxed{F(1, 1, 3) = 1}
$$

This is the value of the function at the specified point, assuming Z is a constant and the natural logarithm is evaluated at $x=1$.

Additional Insights: The Role of the Constant Z

How does Z affect the function?

- Since Z is a constant multiplying the second term, its value directly influences the magnitude of that component.
- In this particular calculation, because $\ln 1 = 0$, the second term vanishes regardless of Z. However, if x were different (say $x \neq 1$), Z would significantly impact the value.

Example: Evaluating at $(x, y, z) = (2, 1, 3)$

Suppose we evaluate at $(2, 1, 3)$:

$$
F(2, 1, 3) = 2^3 + 1^2 \times 2 \times 3^2 \times Z \times \ln 2
$$

Calculate:

- $2^3 = 8$
- $1^2 = 1$
- $2 \times 3^2 = 2 \times 9 = 18$
- $\ln 2 \approx 0.6931$

So,

$$
F(2, 1, 3) = 8 + 1 \times 18 \times Z \times 0.6931 = 8 + 12.455 \times Z
$$

This illustrates how Z scales the second term when $x \neq 1$.

Understanding the Importance of Logarithmic Evaluation

The natural logarithm function, $\ln x$, plays a crucial role in many calculus applications, especially in growth models, decay processes, and entropy calculations.

Key properties:

- $\ln 1 = 0$, simplifying calculations at $x=1$.
- $\ln x$ is undefined for $x \leq 0$.
- For $x > 0$, $\ln x$ increases monotonically.

Practical implications:

- When evaluating functions at specific points, always consider the domain restrictions imposed by functions like $\ln x$.
- Recognize that at $x=1$, many logarithmic terms simplify, making calculations easier.

Extending to Derivative and Gradient Calculations

While this article focuses on evaluating F at a point, understanding how to compute derivatives and gradients is crucial for optimization and analysis.

Gradient of F:

The gradient vector ∇F contains partial derivatives with respect to x, y, z:

$$
\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right)
$$

Computing partial derivatives:

- $\frac{\partial F}{\partial x}$:

$$
\frac{\partial}{\partial x} \left(x^3 + y^2 \cdot 2z^2 Z \ln x \right) = 3x^2 + y^2 \cdot 2z^2 Z \cdot \frac{1}{x}
$$

- $\frac{\partial F}{\partial y}$:

$$
\frac{\partial}{\partial y} \left(x^3 + y^2 \cdot 2z^2 Z \ln x \right) = 0 + 2y \cdot 2z^2 Z \ln x
$$

- $\frac{\partial F}{\partial z}$:

$$
\frac{\partial}{\partial z} \left(x^3 + y^2 \cdot 2z^2 Z \ln x \right) = 0 + y^2 \cdot 2 \cdot 2z Z \ln x = 4 y^2 z Z \ln x
$$

Summary of Key Steps in Evaluating F

1. Identify the function components and clarify constants or variables.
2. Plug in the point coordinates into each term separately.
3. Evaluate simple functions like powers and logs.
4. Combine the results to find the function value at the point.
5. Interpret the influence of constants like Z and the role of the logarithmic component.

Conclusion

Evaluating a multivariable function at a given point requires careful analysis of each term, especially when involving complex expressions like products and logarithms. In the case of $F(x, y, z) = x^3 + y^2 \cdot 2z^2 Z \ln x$, the calculation simplifies significantly at $(1, 1, 3)$ due to $\ln 1 = 0$. This results in the function value being 1, irrespective of the constant Z. Understanding such evaluations is foundational to further explorations in calculus, such as finding directional derivatives, gradients, and optimization problems.

By mastering the step-by-step approach outlined here, students and practitioners can confidently handle similar problems involving multivariable functions, ensuring accurate analysis and deeper comprehension of multivariable calculus concepts.

References and Further Reading

- Stewart, James. Calculus: Early Transcendentals. Cengage Learning.
- Adams, Robert A., and Christopher Essex. Calculus: A Complete Course. Pearson.
- Khan Academy. Multivariable Calculus. [Online Resource](https://www.khanacademy.org/math/multivariable-calculus)
- Paul's Online Math Notes. Partial Derivatives and Gradients. [Online Resource](https://tutorial.math.lamar.edu/)

Note: Always verify the assumptions about constants and variables in complex functions. Clarify notation before proceeding with calculations.

Frequently Asked Questions

How do I find the value of the function F at the point (1, 1, 3) for $F(x, y, z) = x^3 + y^2 2z^2 + z \ln x$?

To find F(1, 1, 3), substitute x=1, y=1, z=3 into the function: $F(1, 1, 3) = (1)^3 + (1)^2\ 2\ (3)^2 + 3 \ln(1)$. Simplify: 1 + 1 2 9 + 3 0 = 1 + 18 + 0 = 19.

What is the process for evaluating $F(x, y, z) = x^3 + y^2 2z^2 + z \ln x$ at a specific point?

First, substitute the given point coordinates into the function, then compute each term carefully, paying attention to the natural logarithm of x, especially if $x>0$.

Are there any restrictions on the point (1, 1, 3) when evaluating $F(x, y, z) = x^3 + y^2 2z^2 + z \ln x$?

Yes, since the function contains ln x, x must be positive. At (1, 1, 3), x=1, which is valid for the logarithm.

Can I use a calculator to evaluate F at (1, 1, 3)?

Yes, using a calculator makes it easier to evaluate the terms, especially the natural logarithm and the powers of the variables.

What is the significance of evaluating F at a specific point in multivariable calculus?

Evaluating F at a point helps understand the function's value at that point, which is useful for analysis such as finding local maxima, minima, or for use in gradient and tangent plane calculations.

How does the value of F(1, 1, 3) relate to the derivatives or gradients at that point?

While the value itself is a specific function output, the gradient at that point can be computed to understand the function's rate of change. The value provides a baseline, but gradients give directional information.

Additional Resources

Find F At The Given Point. F(x,y,z)=x^3 + y^2 2z^2 + z ln x, (1,1,3)

In the realm of multivariable calculus, understanding how to evaluate and analyze functions at specific points is fundamental. Whether you're a student tackling advanced calculus problems or a professional applying mathematical models, grasping how to find the value of a function at a given coordinate is essential. Today, we delve into a particular function:
$F(x, y, z) = x^3 + y^2 2z^2 + z \ln x$
and explore how to compute its value at the point (1, 1, 3). This process, though seemingly straightforward, involves a careful step-by-step approach that combines substitution, algebraic manipulation, and understanding of the function's components.

Understanding the Function: An Overview

Before diving into calculations, it's important to understand the structure of the function:

$F(x, y, z) = x^3 + y^2 2z^2 + z \ln x$

Breaking it down:

- x^3: A cubic term in x.
- $y^2 2z^2$: The product of y squared and twice z squared.
- $z \ln x$: The product of z and the natural logarithm of x.

Each component contributes differently to the overall value of F at a given point. Recognizing the components helps in systematically evaluating the function.

Step 1: Substituting the Coordinates

Given the point (1, 1, 3), the variables are:

- $x = 1$
- $y = 1$
- $z = 3$

Substituting these into the function:

$F(1, 1, 3) = (1)^3 + (1)^2\ 2\ (3)^2 + (3) \ln(1)$

Step 2: Evaluating Each Term

Now, evaluate each term separately for clarity:

Term 1: x^3

- $(1)^3 = 1$

Term 2: $y^2\ 2z^2$

- $y^2 = (1)^2 = 1$
- $z^2 = (3)^2 = 9$
- $2z^2 = 2\ 9 = 18$
- Therefore, $y^2\ 2z^2 = 1\ 18 = 18$

Term 3: $z \ln x$

- $z = 3$
- $\ln x = \ln 1 = 0$ (since the natural logarithm of 1 is zero)
- So, $z \ln x = 3\ 0 = 0$

Step 3: Summing Up the Components

Adding the evaluated terms:

$F(1, 1, 3) = 1 + 18 + 0 = 19$

Result: The value of the function F at the point (1, 1, 3) is 19.

Deep Dive: Why Is This Important?

While the calculation appears straightforward, understanding why each component behaves as it does offers deeper insight:

- The x^3 term emphasizes the importance of the x-coordinate's cubic growth.
- The $y^2\ 2z^2$ term highlights the combined influence of y and z, especially the quadratic growth of z.
- The $z \ln x$ term introduces a logarithmic element, which for x=1 simplifies to zero, but can be significant at other points.

In more advanced applications, evaluating functions at specific points helps in optimization, modeling physical phenomena, and solving differential equations. Ensuring accuracy in these evaluations is foundational.

Additional Considerations: Domain and Behavior

While we've calculated the function's value at a specific point, it's worth noting:

- The natural logarithm $\ln x$ is only defined for $x > 0$. Since $x=1$ in our case, the function is well-defined.
- For values of x approaching zero from the positive side, $\ln x$ approaches negative infinity, causing the function to behave differently.
- For large positive x and z, the function's value will grow rapidly due to the cubic and quadratic terms.

Understanding the domain and the behavior of each component is crucial when analyzing functions, especially for more complex problems.

Broader Applications and Implications

Evaluating functions at specific points isn't just a mathematical exercise; it's a practical tool across various fields:

- Physics: Calculating potential energy at certain positions.
- Economics: Determining cost or profit functions at specific levels of production.
- Engineering: Assessing system responses at particular operating points.
- Data Science: Computing feature values for data points in multivariable models.

Mastering the process of substitution, simplification, and interpretation enhances one's ability to work with complex functions confidently.

Summary: The Step-by-Step Approach

To summarize, when asked to find the value of a multivariable function at a particular point, follow these steps:

1. Identify the point's coordinates: (x, y, z).
2. Substitute these values into the function: Replace variables with their values.
3. Evaluate each component separately: Simplify terms carefully, especially those involving logs or powers.
4. Sum the results: Add all evaluated terms to find the function's value.
5. Interpret the result: Understand what the value signifies in context.

This systematic approach ensures accuracy and clarity, especially when dealing with more complicated functions.

Final Thoughts

The exercise of evaluating $F(x, y, z) = x^3 + y^2 2z^2 + z \ln x$ at the point (1, 1, 3) is a foundational skill in multivariable calculus. It combines substitution, algebraic manipulation, and a grasp of function behavior. As you advance in mathematical studies or professional applications, such skills form the basis for more complex analyses—such as finding derivatives, gradients, or optimizing functions.

By mastering these fundamental steps, you lay a solid groundwork for exploring the multifaceted world of multivariable functions, enabling more sophisticated problem-solving and a deeper understanding of the mathematical models that describe our world.

Find F At The Given Point F X Y Z X 3 Y 22z 2 Z Lnx 1 1 3

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