Find The Angle Between The Planes - 4x + 2y 4z = 6 And -5x 2y +

Find The Angle Between The Planes - 4x + 2y + 4z = 6 And -5x + 2y + ...

Understanding how to find the angle between two planes is a fundamental concept in three-dimensional geometry. This process involves analyzing the normal vectors of the planes, since the angle between the planes is directly related to the angle between their normal vectors. In this guide, we'll explore the detailed steps to calculate the angle between the given planes, including the necessary mathematical principles, formulas, and illustrative examples.

Understanding the Geometry of Planes and Their Normals

Planes in Three-Dimensional Space

- A plane in 3D space can be represented by a linear equation of the form:

$$ ax + by + cz = d $$

- Here, a, b, and c are coefficients that define the orientation of the plane, and d is a constant.

Normal Vectors to Planes

- The coefficients a, b, and c in the plane equation form the normal vector $\vec{n}$.

$$ \vec{n} = \langle a, b, c \rangle $$

- The normal vector is perpendicular (orthogonal) to the plane.

Importance of Normal Vectors in Finding the Angle

- The angle between two planes is the same as the angle between their respective normal vectors.
- Therefore, the problem reduces to calculating the angle between two vectors.

Mathematical Approach to Find the Angle Between Two Planes

Step 1: Extract Normal Vectors from the Plane Equations

- For the two planes given, identify their normal vectors:

1. Plane 1: $4x + 2y + 4z = 6$
Normal vector: $\vec{n}_1 = \langle 4, 2, 4 \rangle$

2. Plane 2: $-5x + 2y + c z = d$ (Note: the original problem seems incomplete, but assuming the general form is similar, e.g., $-5x + 2y + mz = n$...)
Normal vector: $\vec{n}_2 = \langle -5, 2, m \rangle$

- In case the second plane's equation is incomplete, ensure to identify the coefficients correctly from the problem statement.

Step 2: Use the Dot Product Formula for Vectors

- The angle θ between two vectors $\vec{a}$ and $\vec{b}$ is given by:

$$
\cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}
$$

- Where:
- $\vec{a} \cdot \vec{b}$ is the dot product.
- $|\vec{a}|$ and $|\vec{b}|$ are the magnitudes (lengths) of the vectors.

Step 3: Calculate the Dot Product and Magnitudes

- Dot product:

$$
\vec{n}_1 \cdot \vec{n}_2 = (a_1)(a_2) + (b_1)(b_2) + (c_1)(c_2)
$$

- Magnitudes:

$$
|\vec{n}_i| = \sqrt{a_i^2 + b_i^2 + c_i^2}
$$

- Substitute the specific values from the normal vectors.

Step 4: Find the Angle θ

- Calculate $\cos \theta$:

$$
\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|}
$$

- Then, determine θ:

$$
\theta = \arccos \left(\frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|} \right)
$$

- Ensure the angle is in degrees or radians as needed.

Worked Example: Calculating the Angle Between the Given Planes

Given Planes

- Plane 1: $4x + 2y + 4z = 6$
- Plane 2: $-5x + 2y + m z = n$ (Assuming a specific value for m and n for illustration, say $m=3$ and $n=8$)

So, Plane 2: $-5x + 2y + 3z = 8$

Step 1: Extract Normal Vectors

- $\vec{n}_1 = \langle 4, 2, 4 \rangle$
- $\vec{n}_2 = \langle -5, 2, 3 \rangle$

Step 2: Compute Dot Product

$$
\vec{n}_1 \cdot \vec{n}_2 = (4)(-5) + (2)(2) + (4)(3) = -20 + 4 + 12 = -4
$$

Step 3: Compute Magnitudes

$$
|\vec{n}_1| = \sqrt{4^2 + 2^2 + 4^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6
$$

$$
|\vec{n}_2| = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{25 + 4 + 9} = \sqrt{38} \approx 6.164
$$

Step 4: Calculate $\cos \theta$

$$
\cos \theta = \frac{-4}{6 \times 6.164} \approx \frac{-4}{36.984} \approx -0.108
$$

Step 5: Find θ

$$
\theta = \arccos(-0.108) \approx 96.2^\circ
$$

Result: The angle between the two planes is approximately 96.2 degrees, indicating they are nearly perpendicular but slightly obtuse.

Additional Tips for Accurate Calculation

1. **Double-check the coefficients:** Ensure the plane equations are correctly written and coefficients are correctly identified.

2. **Use a calculator or software:** For inverse cosine calculations, utilize scientific calculators or computational tools for precision.

3. **Consider the range of the angle:** Since the dot product can be negative, resulting in an angle greater than 90°, verify whether you are interested in the acute or obtuse angle between the planes.

4. **Normalize vectors:** When in doubt, normalize the normal vectors before calculating the dot product, but this is optional if using the dot product formula directly.

Applications of Finding the Angle Between Planes

- Engineering and Architecture: Determining the angle between structural components.
- Computer Graphics: Calculating surface orientations for rendering.
- Robotics: Understanding the relative positioning of planes in space for navigation.
- Mathematical Research: Analyzing geometric properties and relationships.

Conclusion

Finding the angle between two planes involves a straightforward process rooted in vector mathematics. By identifying the normal vectors from the plane equations, applying the dot product formula, and computing the inverse cosine, you can determine the precise angle between the planes. This process is essential in various fields that require spatial understanding and geometric analysis. Always ensure the coefficients are accurately extracted, and double-check calculations to maintain precision. With practice, calculating the angle between planes becomes an intuitive and valuable skill in three-dimensional geometry.

Remember: The key to mastering this topic lies in understanding the connection between the planes' equations and their normal vectors, and how vector operations facilitate geometric insights in 3D space.

Frequently Asked Questions

How do you find the angle between two planes given their equations?

To find the angle between two planes, you need to determine the angle between their normal vectors. First, extract the normal vectors from each plane's equation, then use the dot product formula: $\cos\theta = (n_1 \cdot n_2) / (|n_1| |n_2|)$. Finally, take the arccosine to find the angle θ.

What are the normal vectors of the given planes?

For the plane $4x + 2y + 4z = 6$, the normal vector is $n_1 = (4, 2, 4)$. For the plane $-5x - 2y + \ldots$ (assuming the full equation is provided), the normal vector is $n_2 = (-5, -2, \ldots)$. You need the complete second equation to determine its normal vector accurately.

How do I compute the dot product of the normal vectors in this problem?

The dot product of the normal vectors $n_1 = (4, 2, 4)$ and $n_2 = (-5, -2, c)$ (assuming the z-component c is given) is calculated as: $(4)(-5) + (2)(-2) + (4)(c)$. Plug in the value of c from the second plane's equation to compute the dot product.

What is the formula for calculating the angle between two planes once the normal vectors are known?

The angle θ between the two planes is given by: $\cos\theta = (n_1 \cdot n_2) / (|n_1| |n_2|)$, where n_1 and n_2 are the normal vectors of the planes. Use this to find θ = arccos of that value.

Can the angle between two planes be greater than 90 degrees?

No, the angle between two planes is defined as the acute angle between their normal vectors, which ranges from 0° to 90°. If the calculated angle exceeds 90°, subtract it from 180° to find the smaller, acute angle between the planes.

Additional Resources

Find The Angle Between The Planes - 4x + 2y + 4z = 6 And -5x + 2y + ...

Understanding the spatial relationship between planes is a fundamental aspect of vector calculus and analytical geometry, with applications spanning engineering, physics, and computer graphics. The problem of determining the angle between two planes is both conceptually intriguing and mathematically rich. In this article, we delve deeply into the methodology for finding the angle between two planes—specifically focusing on the planes defined by the equations $4x + 2y + 4z = 6$ and $-5x + 2y + \ldots$ (note: the second plane's equation appears incomplete, so for the purposes of this analysis, we will assume a typical form, say, $-5x + 2y + 3z = c$, where c is a constant). Our goal is to elucidate the principles, calculations, and interpretations involved in such a problem.

Understanding Planes in 3D Space

What Is a Plane?

In three-dimensional space, a plane is a flat, two-dimensional surface extending infinitely in all directions within its dimension. Mathematically, a plane can be represented by a linear equation of the form:

$$ Ax + By + Cz + D = 0 $$

where A, B, and C are the coefficients that form the normal vector to the plane, and D is a constant.

The Role of the Normal Vector

The normal vector $\vec{n}$ of a plane is perpendicular to the surface of the plane. For the general equation $Ax + By + Cz + D = 0$, the normal vector is $\vec{n} = \langle A, B, C \rangle$.

This normal vector is crucial because the angle between two planes is directly related to the angle between their normal vectors. Specifically, the angle between the planes is the same as the angle between their normal vectors.

Mathematical Foundation for Finding the Angle Between Planes

Angles Between Normal Vectors

Given two planes with normal vectors $\vec{n}_1$ and $\vec{n}_2$, the angle θ between the planes is given by the angle between these vectors:

$$
\cos \theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1| |\vec{n}_2|}
$$

where:

- $\vec{n}_1 \cdot \vec{n}_2$ is the dot product of the two vectors.
- $|\vec{n}_1|$ and $|\vec{n}_2|$ are the magnitudes (lengths) of the vectors.

Important: The angle θ obtained from this formula is the acute angle (between 0° and 90°). If the angle exceeds 90°, the supplementary angle can be considered, but for most applications, the principal value suffices.

Computing the Dot Product and Magnitudes

Given vectors:

$$
\vec{n}_1 = \langle A_1, B_1, C_1 \rangle
$$
$$
\vec{n}_2 = \langle A_2, B_2, C_2 \rangle
$$

the dot product is:

$$
\vec{n}_1 \cdot \vec{n}_2 = A_1A_2 + B_1B_2 + C_1C_2
$$

and the magnitude of each vector is:

$$
|\vec{n}_i| = \sqrt{A_i^2 + B_i^2 + C_i^2}
$$

\]

Applying the Method to the Given Planes

Equation of the First Plane

The first plane is given as:

$$
4x + 2y + 4z = 6
$$

Rearranged in standard form:

$$
4x + 2y + 4z - 6 = 0
$$

From this, the normal vector of the first plane is:

$$
\vec{n}_1 = \langle 4, 2, 4 \rangle
$$

Equation of the Second Plane

The second plane appears to be incomplete in the prompt: "-5x 2y +". Assuming a standard form similar to the first, a reasonable assumption is that the full plane equation is:

$$
-5x + 2y + 3z = c
$$

or some similar form. For illustration, let's proceed with:

$$
-5x + 2y + 3z = d
$$

which rearranges to:

$$
-5x + 2y + 3z - d = 0
$$

The normal vector of the second plane is:

$$
\vec{n}_2 = \langle -5, 2, 3 \rangle
$$

\]

Note: The constant \(d\) does not influence the angle between the planes, as only the normal vectors matter.

Calculating the Angle Between the Planes

Step 1: Compute the Dot Product

\[
\vec{n}_1 \cdot \vec{n}_2 = (4)(-5) + (2)(2) + (4)(3) = -20 + 4 + 12 = -4
\]

Step 2: Compute the Magnitudes

\[
|\vec{n}_1| = \sqrt{4^2 + 2^2 + 4^2} = \sqrt{16 + 4 + 16} = \sqrt{36} = 6
\]

\[
|\vec{n}_2| = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{25 + 4 + 9} = \sqrt{38} \approx 6.1644
\]

Step 3: Determine the Cosine of the Angle

\[
\cos \theta = \frac{-4}{6 \times 6.1644} \approx \frac{-4}{36.9864} \approx -0.1082
\]

Step 4: Find the Angle \(\theta\)

\[
\theta = \arccos(-0.1082) \approx 95.8^\circ
\]

Since the cosine of the angle is negative and close to \(-0.1082\), the actual physical angle between the planes is:

\[
\text{angle} = 180^\circ - 95.8^\circ = 84.2^\circ
\]

or, more straightforwardly, taking the absolute value:

\[
\boxed{
\theta \approx 95.8^\circ

}
\]

but conventionally, the angle between planes is considered the smaller angle, so approximately 84.2°.

Interpretation of the Result

The calculated angle (~84.2°) indicates that the two planes are neither parallel nor perpendicular but intersect at an oblique angle. If the normal vectors were parallel or antiparallel, the planes would be parallel, corresponding to a 0° or 180° angle. If the normal vectors were orthogonal, the planes would intersect at a right angle (90°).

In this case, the angle being close to 84° suggests that the planes are somewhat close to intersecting at a right angle but are slightly skewed. Such information is valuable in fields like structural engineering, where understanding the inclination between different structural elements is critical, or in computer graphics for rendering and shading calculations.

Special Cases and Considerations

Parallel Planes

Planes are parallel if their normal vectors are scalar multiples. For example, if $\vec{n}_1 = k \vec{n}_2$, then the planes are parallel, and the angle between them is 0° (or 180°, depending on orientation).

Perpendicular Planes

Planes are perpendicular if the dot product of their normal vectors is zero:

$$
\vec{n}_1 \cdot \vec{n}_2 = 0
$$

which corresponds to a 90° angle between the planes.

Implications of the Constant Terms

While the constants D in the plane equations influence the position but not the orientation, understanding their role is important in related problems like finding the shortest distance between two planes or points.

Conclusion

Determining the angle between two planes is a process rooted in vector analysis. By extracting the normal vectors from the plane equations, calculating their dot product and magnitudes, and then applying the arccosine function, we obtain the measure of the angle at which the two planes intersect. This approach not only simplifies the problem but also offers intuitive geometric insights.

In our specific case, assuming the second plane's equation is $-5x + 2y + 3z = c$, the approximate angle

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