Find The Area Inside The Loop Of The Limacon Given By R=714sin

Find The Area Inside The Loop Of The Limacon Given By R=714sin is a captivating problem in polar coordinate geometry that combines the elegance of curves with the precision of calculus. The limacon, a well-known family of polar curves, exhibits fascinating shapes, especially when it contains an inner loop. Determining the area enclosed within this loop involves a blend of understanding the curve's properties and applying integral calculus techniques. Whether you're a student tackling calculus problems or a math enthusiast exploring the beauty of polar graphs, this guide will walk you through the process step-by-step to find the area inside the loop of the limacon described by the equation R = 714 sin θ.

Understanding the Limacon and Its Equation

What Is a Limacon?

A limacon is a type of polar curve characterized by its distinctive shape, which can include dimpled or looped features depending on its parameters. Its general form is given by the equation:

$$ r = a + b \cos \theta \quad \text{or} \quad r = a + b \sin \theta $$

where a and b are constants. The shape of the limacon depends on the ratio $\frac{a}{b}$:

- If $|a| < |b|$, the limacon has an inner loop.
- If $|a| = |b|$, it forms a cardioid.
- If $|a| > |b|$, it is dimpled or convex without a loop.

In our case, the equation is:

$$ r = 714 \sin \theta $$

which is a special case where $a=0$ and $b=714$.

Analyzing the Equation R=714sinθ

The equation $r = 714 \sin \theta$ describes a limacon that is symmetric about the vertical axis. Since $a=0$, this is a limacon with a loop, specifically a lemniscate-like shape, with the following properties:

- When $\theta = 0$, $r=0$
- When $\theta = \pi/2$, $r=714$
- When $\theta = \pi$, $r=0$
- When $\theta = 3\pi/2$, $r=-714$

Note that negative values of r indicate the point is in the opposite direction of the angle, which is crucial for understanding the curve's full shape.

Identifying the Loop and Its Bounds

Where Does the Loop Occur?

To find the area inside the loop, first, we need to determine the range of θ where the loop exists. For the given equation:

$$ r = 714 \sin \theta $$

- The curve passes through the origin at $\theta=0$ and $\theta=\pi$.
- The maximum value of r occurs at $\theta = \pi/2$, where $r=714$.
- The negative values of r occur in the interval $\pi < \theta < 2\pi$, meaning the curve loops back through the origin, creating an inner loop.

The key is to find the θ values where the loop forms, which are typically where r switches from positive to negative or vice versa.

Determining the Bounds of Integration

Since the curve exhibits symmetry, the loop is contained within the θ interval:

$$ 0 \leq \theta \leq \pi $$

but more specifically, the loop exists where r is negative, i.e., where:

$$ \sin \theta < 0 \quad \Rightarrow \quad \pi < \theta < 2\pi $$

Within this range, the negative r values give the inner loop. To find the exact bounds, observe:

- At $\theta = \pi$, $r=0$
- At $\theta = 3\pi/2$, $r=-714$
- At $\theta=2\pi$, $r=0$

Thus, the loop is traced out from $\theta = \pi$ to $\theta=2\pi$.

Calculating the Area Inside the Loop

Area in Polar Coordinates

The general formula for the area enclosed by a polar curve $r = r(\theta)$ between $\theta = \alpha$ and $\theta = \beta$ is:

$$A = \frac{1}{2} \int_{\alpha}^{\beta} r^2 \, d\theta$$

Since our curve has a loop where r is negative, and the loop is traced as θ goes from π to 2π, we will integrate over this interval.

Setting Up the Integral

Because the negative r values correspond to points in the opposite direction, the actual area of the loop can be found by integrating over the range where r is negative, taking the absolute value of r or directly squaring r.

The area A_{loop} inside the loop is:

$$A_{loop} = \frac{1}{2} \int_{\pi}^{2\pi} r^2 \, d\theta$$

Substituting $r = 714 \sin \theta$:

$$A_{loop} = \frac{1}{2} \int_{\pi}^{2\pi} (714 \sin \theta)^2 \, d\theta$$

$$A_{loop} = \frac{1}{2} \times 714^2 \int_{\pi}^{2\pi} \sin^2 \theta \, d\theta$$

Calculating this integral will give the area of the inner loop.

Performing the Integration

Calculating the Constant Factor

First, compute 714^2:

$$714^2 = (700 + 14)^2 = 700^2 + 2 \times 700 \times 14 + 14^2 = 490000 +$$

$$19600 + 196 = 490000 + 19896 = 509896$$

So,

$$A_{loop} = \frac{1}{2} \times 509896 \int_{\pi}^{2\pi} \sin^2 \theta \, d\theta$$

which simplifies to:

$$A_{loop} = 254948 \int_{\pi}^{2\pi} \sin^2 \theta \, d\theta$$

Evaluating $\int \sin^2 \theta \, d\theta$

Recall the identity:

$$\sin^2 \theta = \frac{1 - \cos 2\theta}{2}$$

Thus,

$$\int \sin^2 \theta \, d\theta = \frac{1}{2} \int (1 - \cos 2\theta) \, d\theta = \frac{1}{2} \left(\theta - \frac{1}{2} \sin 2\theta \right) + C$$

Applying the limits $\theta = \pi$ to $\theta = 2\pi$:

$$\int_{\pi}^{2\pi} \sin^2 \theta \, d\theta = \frac{1}{2} \left[(2\pi - \frac{1}{2} \sin 4\pi) - (\pi - \frac{1}{2} \sin 2\pi) \right]$$

Since $\sin 4\pi = 0$ and $\sin 2\pi=0$:

$$= \frac{1}{2} (2\pi - 0 - \pi + 0) = \frac{1}{2} (\pi) = \frac{\pi}{2}$$

Therefore,

$$A_{loop} = 254,948 \times \frac{\pi}{2}$$

Final Calculation of the Area

Multiplying through:

$$A_{loop} = 254,948 \times \frac{\pi}{2}$$

$$A_{loop} = 127,474 \times \pi$$

Using \(\pi \approx 3.1416\):

\[A_{loop} \approx 127,474 \times 3.1416 \approx 400,403.7 \]

Thus, the area inside the loop of the limacon given by \(R=714 \sin \theta \) is approximately 400,404 square units.

Summary and Key Takeaways

- The limacon \(R = 714 \sin \theta \) exhibits a loop because \(a=0\) and \(b=714\).
- The inner loop corresponds to the \(\theta\) interval \(\pi\) to \(2\pi\), where \(r\) is negative.
- Using the polar area formula \(A = \frac{1}{2} \int r^2 d

Frequently Asked Questions

What is the general form of the limacon given by $R=714\sin\theta$?

The limacon is expressed in polar form as $R=714\sin\theta$, which describes a curve centered around the pole with a loop when the coefficient exceeds the constant.

How do you determine the area enclosed inside the loop of the limacon $R=714\sin\theta$?

The area inside the loop can be found by integrating (1/2) R^2 with respect to θ over the interval where the loop exists, typically between the θ-values where $R=0$.

What are the limits of integration for calculating the area of the loop of $R=714\sin\theta$?

The limits are $\theta=0$ to $\theta=\pi$, since $R=714\sin\theta$ is positive in this range, and the loop is enclosed within these bounds.

What is the formula to compute the area inside the loop of the limacon $R=714\sin\theta$?

The area is given by $A = (1/2) \int$ from $\theta=a$ to $\theta=b$ of $[R(\theta)]^2\, d\theta$, specifically from $\theta=0$ to $\theta=\pi$ for this limacon.

Can the area inside the loop be found without integration for R=714sinθ?

No, the area generally requires integration; however, recognizing symmetry can simplify the calculation, but an integral is necessary for an exact value.

What is the computed area inside the loop of the limacon R=714sinθ?

The area is approximately 2,260,000 square units, obtained by evaluating the integral $A = (1/2) \int_0^\pi (714\sin\theta)^2 \, d\theta$.

How does the coefficient 714 influence the size of the loop in the limacon R=714sinθ?

Since the limacon's R depends linearly on 714, increasing this coefficient enlarges the loop, directly scaling the area by the square of the coefficient.

Additional Resources

Find The Area Inside The Loop Of The Limacon Given By R=714sin

The limacon, a fascinating family of polar curves, has intrigued mathematicians and enthusiasts alike for centuries. Its distinctive shape, often characterized by a prominent loop, offers a rich avenue for exploration in calculus and geometry. Today, we delve into a specific limacon defined by the polar equation $R = 714 \sin \theta$, aiming to determine the area enclosed within its loop. This problem not only underscores the beauty of polar coordinate calculus but also exemplifies the practical techniques used to analyze complex curves.

Understanding the Limacon and Its Equation

Before embarking on the calculation of the area, it’s essential to understand the nature of the given limacon $R = 714 \sin \theta$.

What is a Limacon?

A limacon is a type of plane curve generated in the polar coordinate system, characterized by its distinctive "loop" or dimple, depending on its parameters. The general form of a limacon is:

$$ R = a + b \sin \theta \quad \text{or} \quad R = a + b \cos \theta $$

where a and b are real constants. The shape varies based on the ratio $\frac{a}{b}$:

- If $|a| < |b|$, the limacon has an inner loop.
- If $|a| = |b|$, it forms a cardioid.
- If $|a| > |b|$, it is dimpled or convex without a loop.

Specifics of $R = 714 \sin \theta$

Given the equation:

$$ R = 714 \sin \theta $$

- Here, $a = 0$ and $b = 714$.
- Since $a = 0$, the curve is symmetric about the vertical axis.
- Because $|a| < |b|$, the limacon features an inner loop.

This inner loop appears because, for certain values of θ, R becomes negative, which in polar coordinates indicates points traced in the opposite direction, creating the loop.

Identifying the Loop: Critical Angles and Behavior

To find the area enclosed within the inner loop, the first step is understanding where the loop exists and its extent.

When Does the Loop Occur?

The loop appears where the radius R becomes negative, meaning:

$$ R = 714 \sin \theta < 0 $$

which simplifies to:

$$ \sin \theta < 0 $$

Thus, the loop exists in the range:

$$ \pi < \theta < 2\pi $$

Within this interval, the curve traces the loop.

Critical Points

- Maximum radius: \(R_{max} = 714 \) at \(\theta = \frac{\{\pi}{2\}} \).
- Zero radius: \(R = 0 \) at \(\theta = 0, \pi \).

The key points for the inner loop are at the angles where \(R = 0 \):

\[714 \sin \theta = 0 \Rightarrow \sin \theta = 0 \Rightarrow \theta = 0, \pi \]

Between these angles, the curve forms the loop.

Calculating the Area Inside the Loop

The core of the problem is to compute the area enclosed by the inner loop of the limacon. In polar coordinates, the area \(A \) enclosed by a curve \(R = R(\theta) \) from \(\theta = \alpha \) to \(\theta = \beta \) is given by:

\[A = \frac{1}{2} \int_{\alpha}^{\beta} R^2 \, d\theta \]

Setting Up the Integral

Since the inner loop occurs where \(R = 714 \sin \theta < 0 \), the loop is traced as \(\theta \) goes from \(\pi \) to \(2\pi \). However, because \(R \) is negative in that interval, the curve is traced in the opposite direction, effectively creating the loop.

To accurately compute the area of the loop, we should consider the range where \(R \) is negative, but use the positive value of \(R \) since the negative indicates the point is traced in the opposite direction.

The standard approach involves integrating over the angles where the loop exists, which are:

\[\theta = \pi \quad \text{to} \quad 2\pi \]

But because \(R \) is negative in this range, the area is the same as integrating \(R^2 \) over this range, noting that the negative sign of \(R \) is accounted for by the square.

Computing the Integral

The area inside the loop is thus:

\[A_{loop} = \frac{1}{2} \int_{\pi}^{2\pi} R^2 \, d\theta \]

Substituting \(R = 714 \sin \theta \):

\[A_{loop} = \frac{1}{2} \int_{\pi}^{2\pi} (714 \sin \theta)^2 \, d\theta \]

Simplify the integrand:

\[A_{loop} = \frac{1}{2} \times 714^2 \int_{\pi}^{2\pi} \sin^2 \theta \, d\theta \]

Calculate constants:

\[714^2 = 509,796 \]

Thus,

\[A_{loop} = \frac{1}{2} \times 509,796 \int_{\pi}^{2\pi} \sin^2 \theta \, d\theta \]

which simplifies to:

\[A_{loop} = 254,898 \int_{\pi}^{2\pi} \sin^2 \theta \, d\theta \]

Evaluating the Integral of \(\sin^2 \theta\)

The integral:

\[\int \sin^2 \theta \, d\theta \]

can be evaluated using the power-reduction formula:

\[\sin^2 \theta = \frac{1 - \cos 2\theta}{2} \]

Applying this:

\[\int \sin^2 \theta \, d\theta = \int \frac{1 - \cos 2\theta}{2} \, d\theta = \frac{1}{2} \int 1 \, d\theta - \frac{1}{2} \int \cos 2\theta \, d\theta \]

Computing the integrals:

\[= \frac{1}{2} \theta - \frac{1}{2} \times \frac{\sin 2\theta}{2} + C = \frac{\theta}{2} - \frac{\sin 2\theta}{4} + C \]

Now, evaluate between the limits \(\pi \) and \(2\pi \):

\[
\begin{aligned}
A_{loop} &= 254,898 \left[\left(\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right) \Big|_{\pi}^{2\pi} \right] \\
&= 254,898 \left[\left(\frac{2\pi}{2} - \frac{\sin 4\pi}{4} \right) -

\left(\frac{\pi}{2} - \frac{\sin 2\pi}{4} \right) \right]
\end{aligned}
\]

Simplify each term:

- \(\frac{2\pi}{2} = \pi \)
- \(\sin 4\pi = 0 \)
- \(\frac{\pi}{2} \)
- \(\sin 2\pi = 0 \)

Plugging in:

\[
A_{loop} = 254,898 \left[\pi - 0 - \left(\frac{\pi}{2} - 0 \right) \right]
= 254,898 \times \left(\pi - \frac{\pi}{2} \right) = 254,898 \times
\frac{\pi}{2}
\]

Final calculation:

\[
A_{loop} = 254,898 \times \frac{\pi}{2} = 127,449 \times \pi
\]

Final Result and Interpretation

The area enclosed within the inner loop of the limacon \(R = 714 \sin \theta \) is:

\[
\boxed{
A_{loop} = 127,449 \pi
}
\]

or approximately:

\[
A_{loop} \approx 127,449 \times 3.1416 \approx 399,954.7
\]

square units (depending on the units used for the radius).

This significant size reflects the scale of the curve, given the large coefficient 714.

Significance and Applications

Understanding the area inside the loop of a limacon like \(R = 714 \sin \theta \) is not just a theoretical exercise. It has practical implications in fields like physics, engineering, and even computer graphics, where complex curves are used to model real-world phenomena or design intricate patterns.

- Physics: Analyzing the

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