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Understanding how to find the derivative of a function is fundamental in calculus, as it allows us to determine the rate at which a function changes at any given point. In this article, we will walk through the process of differentiating the function $F(x) = 17x^{5/2} + 3x^{1.8}$. By the end, you'll be able to confidently compute derivatives of similar functions, which are essential in fields such as physics, engineering, economics, and beyond.

Understanding the Function Components

Before diving into differentiation, it's important to analyze the given function:

$$
F(x) = 17x^{5/2} + 3x^{1.8}
$$

This is a sum of two terms, each involving a constant multiplied by a power of x. The key to differentiating such functions lies in understanding the power rule and how to apply it to each term.

The Power Rule for Differentiation

What is the Power Rule?

The power rule states that if you have a function of the form $f(x) = ax^n$, where a is a constant and n is any real number, then its derivative is:

$$
f'(x) = a \cdot n \cdot x^{n-1}
$$

This rule simplifies the process of differentiation for polynomial and power functions.

Applying the Power Rule to Our Function

For our function, each term involves a constant multiplied by a power of x:

- First term: $17x^{5/2}$
- Second term: $3x^{1.8}$

Applying the power rule to each:

$$
\frac{d}{dx} [17x^{5/2}] = 17 \times \frac{5}{2} \times x^{(5/2) - 1}
$$

$$
\frac{d}{dx} [3x^{1.8}] = 3 \times 1.8 \times x^{1.8 - 1}
$$

Now, let's explore these steps in detail.

Step-by-Step Differentiation Process

Differentiate Each Term Individually

First Term: $17x^{5/2}$

1. Multiply the constant coefficient by the exponent:

$$
17 \times \frac{5}{2} = 17 \times 2.5 = 42.5
$$

2. Subtract 1 from the exponent:

$$
\frac{5}{2} - 1 = \frac{5}{2} - \frac{2}{2} = \frac{3}{2}
$$

3. Write the derivative of the first term:

$$
42.5x^{3/2}
$$

Second Term: $3x^{1.8}$

1. Multiply the constant coefficient by the exponent:

$$
3 \times 1.8 = 5.4
$$

2. Subtract 1 from the exponent:

$$
1.8 - 1 = 0.8
$$

3. Write the derivative of the second term:

$$
5.4x^{0.8}
$$

Combine the Results

Adding both derivatives, the derivative of the entire function $F(x)$ is:

$$
F'(x) = 42.5x^{3/2} + 5.4x^{0.8}
$$

This is the simplified form of the derivative of the given function.

Expressing the Derivative in Different Forms

Depending on the context or preference, you might want to express the derivatives in different forms.

Using Radical Notation

Recall that fractional exponents can be rewritten as radicals:

- $x^{3/2} = x^{1 + 1/2} = x \times x^{1/2} = x \sqrt{x}$
- $x^{0.8} = x^{4/5}$

Thus, the derivative can be written as:

$$
F'(x) = 42.5 \times x \sqrt{x} + 5.4 x^{4/5}
$$

Final Expression

The derivative of the function, expressed in radical notation, is:

$$
\boxed{
F'(x) = 42.5x \sqrt{x} + 5.4 x^{4/5}
}
$$

This form may be more intuitive in certain applications, especially when graphing or solving equations.

Practical Applications of Derivatives

Understanding the derivative of a function like $F(x)$ allows you to analyze its behavior:

- Find critical points where $F'(x) = 0$ to identify local maxima and minima.
- Determine intervals of increasing or decreasing by analyzing the sign of $F'(x)$.
- Calculate the rate of change at a specific point to understand the function's sensitivity.
- Apply in optimization problems such as maximizing profit or minimizing cost.

How to Use the Derivative in Real-World Contexts

Let's explore some practical scenarios where this derivative plays a crucial role.

Physics: Velocity and Acceleration

If $F(x)$ represents a position function over time, then $F'(x)$ represents velocity, indicating how fast the position is changing at any moment.

Economics: Marginal Cost or Revenue

In economics, the derivative of a cost or revenue function provides the marginal cost or revenue, helping businesses make informed decisions.

Engineering: Rate of Change of Systems

Engineers analyze derivatives to understand how systems respond to changes, such as stress, temperature, or other variables.

Common Mistakes to Avoid When Calculating Derivatives

While differentiating functions like $F(x) = 17x^{5/2} + 3x^{1.8}$, be cautious of the following pitfalls:

- Forgetting to apply the power rule correctly, especially with fractional exponents.
- Neglecting to multiply the coefficient by the exponent.
- Incorrectly subtracting 1 from the exponent, which can lead to wrong derivatives.
- Mixing radical notation with exponential notation, which can cause confusion.

Summary of the Differentiation Steps

To summarize, here are the key steps to differentiate functions involving power terms:

1. Identify each term as a constant multiplied by a power of x.
2. Apply the power rule to each term:
- Multiply the coefficient by the exponent.
- Subtract 1 from the exponent.
3. Simplify the resulting expression.
4. Express the derivative in preferred notation, either exponential or radical.

Final Expression of the Derivative

Putting it all together, the derivative of the function $F(x) = 17x^{5/2} + 3x^{1.8}$ is:

\[
\boxed{

F'(x) = 42.5x^{3/2} + 5.4x^{0.8}
}
\]

Or, equivalently:

\[
F'(x) = 42.5x \sqrt{x} + 5.4 x^{4/5}
\]

This derivative provides insight into how the function changes at any point \(x\), which is vital for analysis and decision-making in various scientific and engineering contexts.

Conclusion

Calculating the derivative of a function involving fractional and decimal exponents may seem challenging at first, but with a solid understanding of the power rule and careful application, it becomes straightforward. Remember to always analyze each term individually, apply the power rule diligently, and simplify your results for clarity. Mastering these techniques enables you to approach more complex functions with confidence and is an essential skill in calculus.

If you want to deepen your understanding, practice differentiating similar functions with different exponents and coefficients, and explore how derivatives influence the behavior of functions in real-world applications.

Frequently Asked Questions

What is the derivative of the function F(x) = 17x^(5/2) + 3x^1.8?

F'(x) = 17 (5/2) x^(3/2) + 3 1.8 x^0.8 = (85/2) x^(3/2) + 5.4 x^{0.8}.

How do you differentiate a function with fractional exponents like F(x) = 17x^(5/2) + 3x^1.8?

Use the power rule for derivatives: d/dx [x^n] = n x^{n-1}. For each term, multiply the coefficient by the exponent and decrease the exponent by 1.

What is the derivative of 17x^(5/2)?

The derivative is 17 (5/2) x^{(5/2) - 1} = (85/2) x^{3/2}.

What is the derivative of $3x^{1.8}$?

The derivative is $3 \cdot 1.8 x^{1.8 - 1} = 5.4 x^{0.8}$.

Can the derivative of $F(x) = 17x^{5/2} + 3x^{1.8}$ be expressed in simplified form?

Yes, $F'(x) = (85/2) x^{3/2} + 5.4 x^{0.8}$.

What is the significance of exponents like 5/2 and 1.8 in differentiation?

They determine the power rule application, where derivatives involve multiplying by the exponent and reducing the exponent by 1.

How do fractional exponents affect the derivative calculation?

They require careful application of the power rule, converting fractional exponents into derivatives, resulting in coefficients and fractional exponents in the answer.

Is the derivative of a sum of functions equal to the sum of their derivatives?

Yes, the derivative of $F(x) = f(x) + g(x)$ is $F'(x) = f'(x) + g'(x)$.

How do you evaluate the derivative at a specific point x = a?

Substitute $x = a$ into the derivative expression to compute the slope at that point.

What tools can assist in differentiating functions with fractional exponents?

Symbolic differentiation tools like calculus calculators, WolframAlpha, or software such as MATLAB or Wolfram Mathematica can be used for accurate derivatives.

Additional Resources

Find The Derivative Of The Function: $F(x) = (17x^5)/2 + 3x^{1.8}$

Understanding how to find the derivative of a function is a fundamental skill in calculus, enabling us to analyze the behavior of functions, determine rates of change, and solve a wide array of real-world problems. In this guide, we will thoroughly explore the process of differentiating the function $F(x) = \frac{17x^5}{2} + 3x^{1.8}$. By breaking down each step, providing explanations of key concepts, and illustrating common techniques, you'll gain a clear and comprehensive understanding of how to approach similar problems confidently.

Introduction to Derivatives and Their Significance

Before diving into the specific function, it's important to grasp what derivatives are and why they matter.

What Is a Derivative?

The derivative of a function $F(x)$, denoted as $F'(x)$ or $\frac{dF}{dx}$, measures the instantaneous rate of change of the function at any given point. Geometrically, it corresponds to the slope of the tangent line to the graph of the function at that point.

Why Find Derivatives?

- Determine increasing or decreasing behavior: Sign of the derivative indicates whether the function is rising or falling.
- Find local maxima and minima: Critical points where the derivative is zero or undefined can indicate peaks or troughs.
- Solve applied problems: Rate of change in physics, economics, biology, etc.
- Optimize functions: Maximize profits, minimize costs, etc.

Breakdown of the Function $F(x) = \frac{17x^5}{2} + 3x^{1.8}$

Let's analyze the structure of our specific function:

- Term 1: $\frac{17x^5}{2}$ – a polynomial term with degree 5.
- Term 2: $3x^{1.8}$ – a power function with a fractional exponent.

Understanding how to differentiate each term independently, then combining the results, is the key approach here.

Step-by-Step Guide to Differentiating $F(x)$

Step 1: Rewrite the Function (if necessary)

While the given form is clear, sometimes rewriting helps to clarify differentiation:

$$
F(x) = \frac{17}{2} x^5 + 3 x^{1.8}
$$

This makes it easier to identify coefficients and exponents.

Step 2: Apply Differentiation Rules

The main rules involved are:

- Constant Multiple Rule: $\frac{d}{dx}[c \cdot g(x)] = c \cdot g'(x)$
- Power Rule: $\frac{d}{dx}[x^n] = n x^{n-1}$

Step 3: Differentiate Each Term

Term 1: $\frac{17}{2} x^5$

- Coefficient: $\frac{17}{2}$
- Applying the power rule:

$$
\frac{d}{dx} \left[\frac{17}{2} x^5 \right] = \frac{17}{2} \times 5 x^{5-1}
= \frac{17}{2} \times 5 x^4
$$

Calculating the coefficient:

$$
\frac{17}{2} \times 5 = \frac{17 \times 5}{2} = \frac{85}{2}
$$

So,

$$
\frac{d}{dx} \left[\frac{17}{2} x^5 \right] = \frac{85}{2} x^4
$$

Term 2: $3 x^{1.8}$

- Coefficient: 3
- Applying the power rule:

$$
\frac{d}{dx} [3 x^{1.8}] = 3 \times 1.8 x^{1.8 - 1} = 3 \times 1.8 x^{0.8}
$$

Calculating the coefficient:

\[

$$
3 \times 1.8 = 5.4
$$

Thus,

$$
\frac{d}{dx} [3 x^{1.8}] = 5.4 x^{0.8}
$$

Step 4: Combine the Results

Adding the derivatives of each term, we arrive at the derivative of the entire function:

$$
F'(x) = \frac{85}{2} x^4 + 5.4 x^{0.8}
$$

For clarity, you can write it as:

$$
F'(x) = \boxed{\frac{85}{2} x^4 + 5.4 x^{0.8}}
$$

Additional Insights and Considerations

Simplification and Notation

- The derivative involves fractional exponents, which are common in calculus.
- You might prefer to express 0.8 as a fraction: $0.8 = \frac{4}{5}$. So, the derivative can also be written as:

$$
F'(x) = \frac{85}{2} x^4 + 5.4 x^{\frac{4}{5}}
$$

Numerical Approximation

If you need a numerical value at a specific point, substitute the x value into the derivative:

- For example, at $x=2$:

$$
F'(2) = \frac{85}{2} \times 2^4 + 5.4 \times 2^{0.8}
$$

Calculate step-by-step:

- $2^4 = 16$
- $\frac{85}{2} \times 16 = 42.5 \times 16 = 680$
- $2^{0.8} \approx e^{0.8 \ln 2} \approx e^{0.8 \times 0.6931} \approx e^{0.5545} \approx 1.741$
- $5.4 \times 1.741 \approx 9.394$

Thus,

$$
F'(2) \approx 680 + 9.394 \approx 689.394
$$

Common Pitfalls and Tips

- Remember the power rule applies to all terms with variable exponents, including fractional powers.
- Be cautious with negative or fractional exponents, especially when simplifying or plugging in specific values.
- Always double-check coefficients after applying the power rule.
- When dealing with complex functions, consider rewriting to simplify differentiation.

Applications of the Derivative

Knowing the derivative of this function allows you to:

- Identify critical points by setting $F'(x) = 0$ and solving for x.
- Determine intervals of increasing or decreasing behavior based on the sign of $F'(x)$.
- Analyze concavity and points of inflection by examining the second derivative.
- Model real-world phenomena where the rate of change varies with x, such as physics or economics.

Final Thoughts

The process of finding the derivative of $F(x) = \frac{17x^5}{2} + 3x^{1.8}$ exemplifies core calculus techniques involving the power rule and constant multiple rule. By systematically differentiating each term and combining the results, you obtain a comprehensive expression for the rate of change of the function.

Mastering these steps empowers you to tackle more complex derivatives and enhances your analytical toolkit in calculus. Whether you're analyzing the behavior of polynomial and fractional power functions or applying derivatives in practical contexts, this foundational skill opens the door to deeper mathematical understanding and problem-solving prowess.

Remember: Practice differentiating various functions regularly to build confidence, and always verify your derivatives for accuracy. Happy calculating!

Find The Derivative Of The Function F X 17x 5 2 3x 1 8 F X

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