Find The Equation Of The Line Joining The Points (2,-8) And (5,6)

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Understanding how to find the equation of a line that passes through two given points is a fundamental skill in coordinate geometry. Whether you're a student preparing for exams or someone interested in mathematical applications, mastering this process is essential. In this comprehensive guide, we'll walk through the step-by-step method to find the equation of the line passing through the points (2, -8) and (5, 6). We'll cover key concepts, formulas, and practical examples to ensure clarity and confidence in solving such problems.

Introduction to the Equation of a Line

Before diving into the specific points, it's important to understand what the equation of a line represents and the various forms it can take.

What Is an Equation of a Line?

An equation of a line is a mathematical expression that describes all the points lying on that line in the coordinate plane. It relates the x-coordinate and y-coordinate of any point on the line.

Common Forms of Line Equations

The most frequently used forms are:

- **Slope-Intercept Form:** $y = mx + c$
- **Point-Slope Form:** $y - y_1 = m(x - x_1)$
- **Two-Point Form:** $(y - y_1) = [(y_2 - y_1) / (x_2 - x_1)] (x - x_1)$
- **Standard Form:** $Ax + By + C = 0$

In this guide, our primary focus will be on finding the equation using the slope and point-slope forms, which are most straightforward when two points are given.

Step-by-Step Process to Find the Equation of the Line

Let's elaborate on the step-by-step process using the given points: (2, -8) and (5, 6).

Step 1: Identify the Coordinates of the Points

Given points:

- Point 1: $(x_1, y_1) = (2, -8)$
- Point 2: $(x_2, y_2) = (5, 6)$

Step 2: Calculate the Slope (m) of the Line

The slope indicates how steep the line is and is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Substituting the values:

$$
m = \frac{6 - (-8)}{5 - 2} = \frac{6 + 8}{3} = \frac{14}{3}
$$

Thus, the slope of the line passing through (2, -8) and (5, 6) is $\frac{14}{3}$.

Step 3: Choose a Point to Use in the Equation

You can choose either point. For simplicity, we'll use (2, -8).

Step 4: Write the Equation in Point-Slope Form

Using the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Substituting the known values:

\[

y - (-8) = \frac{14}{3}(x - 2)
\]
Simplify:
\[
y + 8 = \frac{14}{3}(x - 2)
\]

Step 5: Convert to Slope-Intercept Form (Optional)

To express the equation in y = mx + c form:
\[
y + 8 = \frac{14}{3}x - \frac{14}{3} \times 2
\]
Calculate:
\[
y + 8 = \frac{14}{3}x - \frac{28}{3}
\]
Subtract 8 from both sides:
\[
y = \frac{14}{3}x - \frac{28}{3} - 8
\]

Express 8 as a fraction with denominator 3:
\[
8 = \frac{24}{3}
\]
So:
\[
y = \frac{14}{3}x - \frac{28}{3} - \frac{24}{3} = \frac{14}{3}x - \frac{52}{3}
\]

Final equation in slope-intercept form:
\[
\boxed{
y = \frac{14}{3}x - \frac{52}{3}
}
\]

Understanding the Derived Equation

The equation \(y = \frac{14}{3}x - \frac{52}{3} \) fully describes the line passing through the points (2, -8) and (5, 6).

Key Components of the Equation:

- **Slope (m):** $\frac{14}{3}$ indicates the line rises 14 units for every 3 units it moves horizontally.
- **Y-intercept (c):** $-\frac{52}{3}$ is where the line crosses the y-axis.

Additional Methods and Considerations

While the above method is straightforward, it's beneficial to understand alternative approaches and related concepts.

Using the Two-Point Formula Directly

The two-point form of the line equation is:

$$
\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}
$$

which is essentially the slope formula rearranged to an equation form. From here, you can manipulate algebraically to derive the slope-intercept form.

Standard Form of the Line Equation

Converting the slope-intercept form to standard form involves rearrangement:

$$
y = \frac{14}{3}x - \frac{52}{3}
$$

Multiply both sides by 3 to clear denominators:

$$
3y = 14x - 52
$$

Rearranged as:

$$
14x - 3y = 52
$$

This is the standard form.

Practical Applications of Finding Line Equations

Understanding how to find the equation of a line through two points is essential in various fields:

- **Graphing and Visualization:** Plotting lines accurately based on two points.
- **Physics:** Calculating trajectories or motion paths.
- **Economics:** Analyzing cost functions or supply-demand curves.
- **Engineering:** Designing structures and systems that depend on linear relationships.

Common Mistakes to Avoid

While performing these calculations, be mindful of:

- Dividing by zero if the points have the same x-coordinate (vertical line case).
- Incorrectly calculating the slope, especially sign errors.
- Mixing units or misapplying formulas during algebraic manipulations.

Summary and Key Takeaways

To find the equation of the line passing through (2, -8) and (5, 6):

1. Calculate the slope: \(m = \frac{14}{3} \)
2. Use either point in the point-slope form: \(y - y_1 = m(x - x_1) \)
3. Simplify to slope-intercept or standard form as needed.

The resulting line equation:
\[
\boxed{
y = \frac{14}{3}x - \frac{52}{3}

}
\]
fully characterizes the line through the given points.

Conclusion

Mastering how to derive the equation of a line through two points is a vital skill in mathematics. It combines understanding of the slope, algebraic manipulation, and geometric interpretation. Whether for academic purposes, practical applications, or enhancing problem-solving skills, this process forms a foundational component of coordinate geometry. Remember to carefully perform each step, verify calculations, and understand the geometric meaning behind the algebraic expressions.

Approach each problem systematically, and soon you'll be comfortable finding line equations for any pair of points!

Frequently Asked Questions

How do I find the slope of the line passing through the points (2, -8) and (5, 6)?

The slope m is calculated as (y2 - y1) / (x2 - x1). So, m = (6 - (-8)) / (5 - 2) = 14 / 3.

What is the step-by-step process to find the equation of the line through (2, -8) and (5, 6)?

First, find the slope m = 14/3. Then, use point-slope form: y - y1 = m(x - x1). Plug in one point, for example (2, -8), to get y + 8 = (14/3)(x - 2). Simplify to find the equation.

What is the slope of the line passing through (2, -8) and (5, 6)?

The slope is 14/3.

How do I write the equation of the line in slope-intercept form using the points (2, -8) and (5, 6)?

After calculating the slope m = 14/3 and using point (2, -8), the equation in point-slope form is y + 8 = (14/3)(x - 2). Convert it to slope-intercept form: y = (14/3)x - (28/3) - 8, which simplifies to y = (14/3)x - (28/3) - (24/3) = (14/3)x - (52/3).

Can I verify the equation of the line by plugging in the given points?

Yes. Substitute x=2 and y=-8 into the equation y = (14/3)x - (52/3). For x=2, y should be -8; for x=5, y should be 6, confirming the correctness.

What is the final equation of the line passing through (2, -8) and (5, 6)?

The final equation in slope-intercept form is y = (14/3)x - (52/3).

Is there an alternative form to express the line equation joining these points?

Yes, you can also write the equation in point-slope form: y + 8 = (14/3)(x - 2), or as a standard form: 14x - 3y = 52.

How can I convert the line equation to standard form from slope-intercept form?

Start with y = (14/3)x - (52/3). Multiply both sides by 3 to clear denominators: 3y = 14x - 52. Rearrange to standard form: 14x - 3y = 52.

Additional Resources

Line Equation

Understanding how to find the equation of a line that passes through two given points is a fundamental skill in coordinate geometry, often encountered in various mathematical and real-world applications. Whether you're a student preparing for exams, a teacher designing lesson plans, or an engineer working on spatial analysis, mastering this process is invaluable. In this article, we will explore the step-by-step method to find the equation of the line passing through the points (2, -8) and (5, 6), dissecting each component for clarity and depth.

Introduction to the Problem

Imagine you're tasked with drawing a straight line that precisely connects two specific locations on a coordinate plane: point A at (2, -8) and point B at (5, 6). To do this accurately, you need to determine the algebraic equation that describes this line. The equation not only signifies the path connecting these points but also allows you to predict points on the line, understand its slope, and analyze its behavior in relation to other geometrical figures.

The core challenge is to derive an explicit formula—either in slope-intercept form, point-slope form, or standard form—that uniquely characterizes the line passing through these coordinates.

Understanding the Foundations

Before delving into calculations, it's essential to review some fundamental concepts:

Coordinate Points

- Points in a plane are represented as ordered pairs: (x, y).
- In our case:
- Point A: (2, -8)
- Point B: (5, 6)

Slope of a Line

- The slope (m) measures the steepness of the line, defined as the ratio of the change in y-values to the change in x-values between two points:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

- A positive slope indicates the line rises from left to right, whereas a negative slope indicates it falls.

Line Equations Forms

- Slope-Intercept Form: $y = mx + c$
- Point-Slope Form: $y - y_1 = m(x - x_1)$
- Standard Form: $Ax + By + C = 0$

The goal is to find the most suitable form that accurately describes the line passing through the given points.

Step-by-Step Derivation

Let's now methodically derive the equation of the line connecting (2, -8) and (5, 6).

Step 1: Calculate the Slope (m)

The slope formula:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

Plugging in the points:

$$
m = \frac{6 - (-8)}{5 - 2} = \frac{6 + 8}{3} = \frac{14}{3}
$$

Result: The slope of the line is $\frac{14}{3}$.

This value indicates a relatively steep incline, with y increasing approximately 4.67 units for each 1-unit increase in x.

Step 2: Choose a Point for the Equation

While either point can be used, selecting the point with easier coordinates can simplify calculations. Here, both are straightforward, but using point (2, -8) is convenient.

Step 3: Use the Point-Slope Form

The point-slope form is particularly handy for deriving the line's equation:

$$
y - y_1 = m(x - x_1)
$$

Substituting the known point (2, -8):

$$
y - (-8) = \frac{14}{3}(x - 2)
$$

Simplify:

$$
y + 8 = \frac{14}{3}(x - 2)
$$

Step 4: Simplify to Slope-Intercept Form

Distribute the slope:

$$
y + 8 = \frac{14}{3}x - \frac{14}{3} \times 2
$$

Calculate:

$$
\frac{14}{3} \times 2 = \frac{28}{3}
$$

Thus,

$$
y + 8 = \frac{14}{3}x - \frac{28}{3}
$$

Subtract 8 from both sides to isolate y:

$$
y = \frac{14}{3}x - \frac{28}{3} - 8
$$

Express 8 as a fraction with denominator 3:

$$
8 = \frac{24}{3}
$$

Now, combine:

$$
y = \frac{14}{3}x - \frac{28}{3} - \frac{24}{3} = \frac{14}{3}x - \frac{52}{3}
$$

Final Equation (Slope-Intercept Form):

$$
\boxed{
y = \frac{14}{3}x - \frac{52}{3}
}
$$

This equation wholly describes the line passing through the points, with slope $\frac{14}{3}$ and y-intercept $-\frac{52}{3}$.

Alternative Forms and Additional Insights

While slope-intercept form is often the most intuitive, other forms can be useful depending on the context.

Point-Slope Form

Using the initial point (2, -8):

$$
y + 8 = \frac{14}{3}(x - 2)
$$

This form is particularly helpful for quickly plugging in new points or visualizing the line's behavior relative to a specific point.

Standard Form

To convert to standard form, clear denominators:

$$
y = \frac{14}{3}x - \frac{52}{3}
$$

Multiply through by 3:

$$
3y = 14x - 52
$$

Rearranged:

$$
14x - 3y = 52
$$

Standard Form:

$$
\boxed{
14x - 3y = 52
}
$$

This form is often preferred in algebraic manipulations and for systems of equations.

Verifying the Equation

Verification ensures the correctness of our derived formula.

- Check with point (5, 6):

Plug x = 5 into the slope-intercept form:

$$
y = \frac{14}{3} \times 5 - \frac{52}{3} = \frac{70}{3} - \frac{52}{3} = \frac{18}{3} = 6
$$

Matches the y-coordinate, confirming the line passes through (5, 6).

- Check with point (2, -8):

Plug x = 2:

$$
y = \frac{14}{3} \times 2 - \frac{52}{3} = \frac{28}{3} - \frac{52}{3} = -\frac{24}{3} = -8
$$

Again, matches the y-coordinate, confirming accuracy.

Practical Applications and Significance

Understanding how to find a line's equation connecting two points extends beyond pure mathematics:

- Engineering: Designing roads or pipelines that connect two points with specific slopes.
- Navigation: Plotting straight-line paths between locations.
- Physics: Describing motion trajectories or linear relationships.
- Data Analysis: Fitting a line through two data points for predictive modeling.

Mastering this calculation enhances problem-solving skills, spatial reasoning, and algebraic fluency.

Summary & Key Takeaways

- The slope of the line between (2, -8) and (5, 6) is $\frac{14}{3}$.
- The line's equation in slope-intercept form is:

$$

y = \frac{14}{3}x - \frac{52}{3}
\]

- Alternative forms, such as point-slope and standard form, provide flexibility for different applications.
- Verification confirms the accuracy of the derived equation.

Through meticulous calculation and verification, we've established a comprehensive understanding of how to find and express the equation of a line passing through any two given points. This foundational technique is a cornerstone of coordinate geometry and essential for advanced mathematical pursuits.

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