Find The Equation Perpendicular To 2x-y=4 And Pass Through (2,4)

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In the realm of coordinate geometry, understanding how to find the equation of a line that is perpendicular to a given line and passes through a specific point is a fundamental skill. This process combines concepts of slope calculation, perpendicular lines, and point-slope form of a line. Whether you're a student preparing for exams, a teacher creating instructional materials, or a math enthusiast, mastering this topic enhances your problem-solving toolkit.

This article provides a comprehensive, step-by-step guide to determine the equation of a line perpendicular to the given line $2x - y = 4$ and passing through the point (2, 4). We will explore the necessary background concepts, detailed procedures, and tips for ensuring accuracy. Additionally, the article is optimized for search engines to help learners find clear, authoritative guidance on this common geometry problem.

Understanding the Basics: Lines, Slopes, and Perpendicularity

Before diving into the specific problem, it's essential to review some foundational concepts in coordinate geometry.

1. The Equation of a Line in Standard and Slope-Intercept Forms

- Standard Form: $Ax + By = C$
- Slope-Intercept Form: $y = mx + b$

The slope (m) indicates the steepness of the line, calculated as the ratio of vertical change to horizontal change between any two points on the line.

2. Finding the Slope from an Equation

Given a line's equation, especially in standard form, you can find its slope by solving for y:

$$
2x - y = 4 \implies -y = -2x + 4 \implies y = 2x - 4
$$

Thus, the slope of the given line is m = 2.

3. Perpendicular Lines and Their Slopes

Two lines are perpendicular if their slopes are negative reciprocals. That is,

$$
m_1 \times m_2 = -1
$$

Where:

- m_1 is the slope of the original line.
- m_2 is the slope of the perpendicular line.

For example, if the slope of the original line is 2, then the slope of any line perpendicular to it is:

$$
m_{perp} = -\frac{1}{2}
$$

Step-by-Step Guide to Find the Perpendicular Line Equation

Now that we've established the foundational concepts, let's proceed with the specific problem:

Given line: $2x - y = 4$

Point passing through: $(2, 4)$

Goal: Find the equation of the line that is perpendicular to the given line and passes through the point $(2, 4)$.

Step 1: Rewrite the Given Line in Slope-Intercept Form

To identify the slope of the original line, convert it to slope-intercept form:

$$
2x - y = 4
$$

Subtract 2x from both sides:

$$
- y = -2x + 4
$$

Multiply through by -1 to solve for y:

$$
y = 2x - 4
$$

Slope of the original line: $m = 2$

Step 2: Determine the Slope of the Perpendicular Line

Since perpendicular lines have slopes that are negative reciprocals:

$$
m_{perp} = -\frac{1}{m} = -\frac{1}{2}
$$

So, the slope of the perpendicular line is: $m_{perp} = -\frac{1}{2}$

Step 3: Use Point-Slope Form to Write the Equation

The point-slope form of a line's equation is:

$$
y - y_1 = m (x - x_1)
$$

Where (x_1, y_1) is the known point the line passes through, and m is the slope.

Plugging in $(2, 4)$ and $m_{perp} = -\frac{1}{2}$:

$$
y - 4 = -\frac{1}{2} (x - 2)
$$

Step 4: Simplify the Equation

Distribute the slope:

$$
y - 4 = -\frac{1}{2} x + 1
$$

Add 4 to both sides to solve for y:

$$
y = -\frac{1}{2} x + 1 + 4
$$

$$
y = -\frac{1}{2} x + 5
$$

Equation of the perpendicular line passing through (2, 4):

$$
\boxed{ y = -\frac{1}{2} x + 5 }
$$

Alternative Forms and Additional Tips

While the slope-intercept form is straightforward, sometimes you might need the equation in different formats, such as standard form or point-slope form.

1. Standard Form

Starting from $y = -\frac{1}{2} x + 5$:

Multiply both sides by 2 to clear the fraction:

$$
2y = -x + 10
$$

Bring all terms to one side:

$$
x + 2y = 10
$$

This is the standard form of the perpendicular line:

$$
\boxed{ x + 2y = 10 }
$$

2. Confirming the Solution

Always verify that your line passes through the given point:

Plug in $(2, 4)$:

$$
x + 2y = 10
$$
$$
2 + 2(4) = 2 + 8 = 10
$$

Since the left side equals 10, the point lies on the line, confirming correctness.

3. Visualizing the Lines

Visual aids can help solidify understanding:

- Plot the original line $y = 2x - 4$.
- Plot the point $(2, 4)$.
- Draw the perpendicular line $y = -\frac{1}{2} x + 5$.
- Observe the right angle between the lines.

Practical Applications and Real-World Context

Finding perpendicular lines has numerous applications, including:

- Engineering and architecture for constructing right angles.
- Computer graphics for generating perpendicular vectors.
- Navigation and path planning algorithms.
- Physics, such as analyzing forces at right angles.

Understanding how to find the perpendicular line through a point is crucial in these domains.

Additional Practice Problems

To reinforce your learning, try solving these related problems:

1. Find the equation of the line perpendicular to $y = 3x + 2$ passing through $(1, -1)$.
2. Determine the equation of the line passing through $(0, 0)$ and perpendicular to the line $4x + y = 7$.
3. Given a line $y = -\frac{1}{3} x + 2$, find the perpendicular line passing through $(3, 5)$.

Practicing these problems will help solidify your understanding of perpendicular lines and their equations.

Conclusion

In summary, finding the equation of a line perpendicular to a given line and passing through a specific point involves:

- Converting the original line to slope-intercept form to identify its slope.
- Calculating the negative reciprocal of this slope for the perpendicular line.
- Applying the point-slope form with the known point and the perpendicular slope.
- Simplifying to the desired form (slope-intercept or standard).

For the specific problem:

- The original line $2x - y = 4$ has a slope of 2.

- The perpendicular line has a slope of $-\frac{1}{2}$.
- Using the point $(2, 4)$, the perpendicular line's equation in slope-intercept form is $y = -\frac{1}{2} x + 5$.

Mastering this process enables you to handle various related problems confidently, making you proficient in coordinate geometry and its applications.

Keywords: perpendicular line, line equation, coordinate geometry, slope, point-slope form, standard form, slope-intercept form, geometry problems, math tutorial

Frequently Asked Questions

What is the slope of the line given by the equation 2x - y = 4?

First, rewrite the equation in slope-intercept form: $y = 2x - 4$. Therefore, the slope of the line is 2.

How do you find the slope of a line perpendicular to 2x - y = 4?

Since the slope of the original line is 2, the perpendicular line's slope is the negative reciprocal, which is -1/2.

What is the equation of the line passing through (2, 4) with slope -1/2?

Using point-slope form: $y - 4 = -1/2 (x - 2)$. Simplifying, the equation is $y - 4 = -1/2 x + 1$, or $y = -1/2 x + 5$.

How do I convert the equation y = -1/2 x + 5 into slope-intercept form?

It is already in slope-intercept form, where the slope is -1/2 and the y-intercept is 5.

What is the importance of the point (2, 4) in this problem?

The point (2, 4) is the point through which the perpendicular line passes, and it is used to find the specific equation of that line.

Can I verify the perpendicularity by checking slopes? How?

Yes. The original line's slope is 2, and the perpendicular line's slope should be -1/2. Since 2 (-1/2) = -1, the lines are perpendicular.

What method can I use to find the equation of a line given a point and slope?

You can use the point-slope form: $y - y_1 = m(x - x_1)$, where (x_1, y_1) is the point and m is the slope.

Is the equation y = -1/2 x + 5 the final answer?

Yes, it is the equation of the line perpendicular to 2x - y = 4 passing through the point (2, 4).

What are some common mistakes to avoid when solving this problem?

Common mistakes include not converting the original line to slope-intercept form, using the wrong reciprocal for the perpendicular slope, or incorrect substitution into the point-slope formula.

Additional Resources

Find The Equation Perpendicular To 2x - y = 4 And Pass Through (2, 4)

In the world of analytic geometry, understanding how lines relate to each other – particularly through concepts like perpendicularity – is fundamental. Whether you're a student preparing for exams, a teacher designing lesson plans, or a professional working on geometric modeling, grasping how to find the equation of a line perpendicular to a given line and passing through a specific point is essential. Today, we'll explore this process step by step, taking the example of finding the equation of a line perpendicular to the line 2x - y = 4 that passes through the point (2, 4). This journey will combine algebraic techniques with geometric insights to produce a clear, comprehensive understanding suitable for readers at various levels.

Understanding the Given Line: 2x - y = 4

Before we delve into the process of finding the perpendicular line, it's important to analyze the original line's properties. The given line is expressed in standard form:

$2x - y = 4$

This form is convenient for identifying the slope and intercepts, both of which are critical for our task.

Converting to Slope-Intercept Form

To determine the slope, it's often easiest to rewrite the equation in slope-intercept form ($y = mx + b$):

- Starting with $2x - y = 4$
- Subtract 2x from both sides: $-y = -2x + 4$
- Multiply through by -1 to solve for y: $y = 2x - 4$

Result: The slope of the line, m, is 2.

Geometric Interpretation

- The line has a slope of 2, indicating that for every unit increase in x, y increases by 2 units.
- Its y-intercept occurs at (0, -4), which is where the line crosses the y-axis.

Understanding this baseline allows us to proceed with finding a line perpendicular to it.

Fundamentals of Perpendicular Lines

Perpendicular lines intersect at a 90-degree angle, and their slopes are related in a specific way.

Relationship Between Slopes

- If a line has a slope m, then any line perpendicular to it must have a slope m' such that:

$m' = -1/m$

- This reciprocal relationship ensures the lines form a right angle.

Applying to Our Line

- Given the slope $m = 2$,
- The slope of the perpendicular line:

$m' = -1/2$

This negative reciprocal is the key to constructing the new line's equation.

Finding the Equation of the Perpendicular Line

The problem requires us to find the line that is perpendicular to $2x - y = 4$ (i.e., with slope $-1/2$) and passes through the point (2, 4).

Step 1: Use the Point-Slope Form

The point-slope form of a line's equation is:

$y - y_1 = m(x - x_1)$

Where:

- (x_1, y_1) is the point the line passes through.
- m is the slope.

Plugging in the known point (2, 4) and the slope -1/2:

$y - 4 = -1/2 (x - 2)$

Step 2: Simplify the Equation

Distribute the slope on the right side:

- $y - 4 = -1/2 x + 1$

Now, add 4 to both sides:

- $y = -1/2 x + 1 + 4$

- $y = -1/2 x + 5$

Step 3: Express in Slope-Intercept Form

The final form:

$y = -0.5x + 5$

This is the equation of the line perpendicular to $2x - y = 4$ passing through (2, 4).

Rewriting the Equation in Different Forms

While slope-intercept form is convenient, sometimes other forms are necessary or preferred depending on the context.

Standard Form

Starting from y = -1/2 x + 5:

- Multiply both sides by 2 to clear fractions:

2y = -x + 10

- Rearranged:

x + 2y = 10

This is the standard form of the perpendicular line's equation.

Summary of the Final Equation

- Slope-Intercept Form: y = -0.5x + 5
- Standard Form: x + 2y = 10

Both forms are valid representations of the same line.

Verifying the Solution

It's good practice to verify that the constructed line meets the original criteria:

- Passes through (2, 4):

Substitute x = 2 into y = -0.5x + 5:

y = -0.5(2) + 5 = -1 + 5 = 4

So, the point (2, 4) lies on the line.

- Is perpendicular to 2x - y = 4?

Since the original line has slope 2, and the new line has slope -1/2, their slopes satisfy the negative reciprocal relationship, confirming perpendicularity.

Practical Applications and Significance

Understanding how to find perpendicular lines passing through specific points isn't just a theoretical exercise; it has real-world applications across various fields.

Engineering and Design

- In structural engineering, perpendicular lines are critical for creating stable frameworks.
- Architects often design perpendicular intersections for aesthetic or functional reasons.

Navigation and Robotics

- Path planning often requires calculating perpendicular trajectories to avoid obstacles or optimize routes.

Computer Graphics

- Rendering and modeling depend heavily on understanding line relationships, including perpendicularity, for accurate visual representation.

Educational Context

- Mastering these concepts builds a strong foundation for more advanced topics like analytic geometry, calculus, and differential equations.

Conclusion: A Step-by-Step Recap

To summarize the process of finding the equation of a line perpendicular to 2x - y = 4 passing through (2, 4):

1. Identify the slope of the original line:

Convert to slope-intercept form: y = 2x - 4, slope m = 2.

2. Determine the perpendicular slope:

m' = -1/2.

3. Use the point-slope form with the new slope and point:

y - 4 = -1/2 (x - 2).

4. Simplify to slope-intercept form:

y = -1/2 x + 5.

5. Optional: convert to standard form:

x + 2y = 10.

This systematic approach ensures clarity and accuracy, making it accessible for learners and practitioners alike.

In essence, mastering how to find a perpendicular line passing through a specific point involves understanding the core principles of slopes, geometric relationships, and algebraic manipulation. By applying these techniques, you can confidently navigate the relationships between lines in the coordinate plane, a skill essential in both academic and professional contexts.

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