Find The Exact Value Of Each X[0,2) For Which Sin(2x)= 3cos(x)

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Understanding how to solve trigonometric equations such as Sin(2x) = 3Cos(x) within a specific interval like [0, 2) is fundamental in mathematics, especially in calculus, algebra, and trigonometry. This article provides a comprehensive guide to finding the exact values of all solutions for x in the interval [0, 2), where the equation Sin(2x) = 3Cos(x) holds true. We will explore the problem step-by-step, including relevant identities, solution methods, and detailed explanations to ensure clarity and mastery.

Understanding the Trigonometric Equation

Before diving into the solution, it’s essential to understand the components of the given equation:

- Sin(2x): The sine of double the angle x.
- Cos(x): The cosine of angle x.
- The equation equates these two functions scaled by a factor of 3, i.e., Sin(2x) = 3Cos(x).

The goal is to find all x within the interval [0, 2), where this equality holds exactly.

Key Trigonometric Identities and Concepts

To approach this problem effectively, familiarize yourself with the following identities:

Double-Angle Identity for Sine

- Sin(2x) = 2Sin(x)Cos(x)

This identity allows us to rewrite the original equation in terms of Sin(x) and Cos(x), which simplifies the solving process.

Basic Trigonometric Range

- Cos(x) and Sin(x) have ranges [-1, 1], which restricts possible solutions because the right side involves a multiple of Cos(x).

Interval [0, 2)

- The solutions sought are within $x \in [0, 2)$ (radians), covering from 0 up to but not including 2 radians.

Step-by-Step Solution Process

The overall plan involves transforming the original equation into a solvable form and then analyzing the solutions within the given interval.

Step 1: Rewrite the Equation Using Identities

Starting from:

$$ \sin(2x) = 3 \cos(x) $$

Apply the double-angle identity:

$$ 2 \sin(x) \cos(x) = 3 \cos(x) $$

Step 2: Simplify the Equation

Divide both sides by cos(x), but be cautious about values where cos(x) = 0:

$$ 2 \sin(x) = 3 $$

or

$$ \text{if } \cos(x) \neq 0 \quad \Rightarrow \quad \sin(x) = \frac{3}{2} $$

Since $\sin(x)$ can only be in [-1, 1], $\sin(x) = \frac{3}{2}$ is impossible in the real domain. This indicates that solutions where $\cos(x) \neq 0$ do not exist for this equation.

Step 3: Consider When $\cos(x) = 0$

The division by cos(x) is invalid where cos(x) = 0. Let's analyze these cases separately.

- Set $\cos(x) = 0$:

$$ \cos(x) = 0 $$

- The solutions for x in [0, 2):

$$ x = \frac{\pi}{2} $$

since:

$$ \cos\left(\frac{\pi}{2}\right) = 0 $$

- Now, check if x = $\frac{\pi}{2}$ satisfies the original equation:

$$ \sin(2 \times \frac{\pi}{2}) = 3 \cos\left(\frac{\pi}{2}\right) $$

Calculate:

$$ \sin(\pi) = 3 \times 0 $$
$$ 0 = 0 $$

This is true, so x = $\frac{\pi}{2}$ is a solution.

- Summary: When cos(x) = 0, x = $\pi/2$ is a solution.

Summary of Solutions

From the above analysis:

- When cos(x) ≠ 0, the only possible solution would require $\sin(x) = 3/2$, which is impossible in real numbers.

- When cos(x) = 0, the only solution in the interval [0, 2) is x = $\pi/2$, which satisfies the original equation.

Final Solution Set within [0, 2)

Based on the analysis, the only solution in the interval [0, 2) is:

- x = $\pi/2$

since:

$$ \pi/2 \approx 1.5708 $$

which lies within [0, 2).

Verification of the Solution

Always verify solutions by substituting back into the original equation:

- At x = $\pi/2$:

$$ \sin(2 \times \pi/2) = \sin(\pi) = 0 $$
$$ 3 \cos(\pi/2) = 3 \times 0 = 0 $$

Both sides are equal (0), confirming x = $\pi/2$ is a valid solution.

Additional Considerations and Domain Restrictions

- Since the initial step involved division by cos(x), solutions where cos(x) = 0 needed separate analysis, which we did and found valid solutions.
- No other solutions exist because of the range restriction of the sine function, $\sin(x) \leq 1$, and the fact that $\sin(x) = 3/2$ is outside the permissible range.
- Thus, the only exact solution in [0, 2) is x = $\pi/2$.

Conclusion

In conclusion, the only solution to the equation Sin(2x) = 3Cos(x) within the interval [0, 2) is:

$$ \boxed{x = \frac{\pi}{2}} $$

This solution satisfies all the necessary conditions and has been verified explicitly. The analysis highlights the importance of considering special cases where denominators could be zero and understanding the ranges of trigonometric functions.

Additional Tips for Solving Similar Trigonometric Equations

- Always check for solutions when dividing by trig functions to avoid missing solutions where the

divisor is zero.
- Use fundamental identities to rewrite complex equations in simpler forms.
- Remember the ranges of sine and cosine functions to eliminate impossible solutions.
- Verify solutions by substitution back into the original equation.
- Be mindful of the interval domain when solving equations in restricted domains.

Meta Description:
Learn how to find the exact value of x in the interval [0, 2) for the equation Sin(2x) = 3Cos(x). This comprehensive guide explores solution methods, identities, and verification techniques for mastering trigonometric equations.

Frequently Asked Questions

What is the first step to solve the equation sin(2x) = 3cos(x) for x in [0, 2)?

Express sin(2x) as 2sin(x)cos(x) to rewrite the equation as 2sin(x)cos(x) = 3cos(x), which allows factoring.

How do we handle the case when cos(x) equals zero in the equation 2sin(x)cos(x) = 3cos(x)?

If cos(x) = 0, then the original equation becomes 0 = 30, which is true. So, x where cos(x) = 0 in [0, 2) are solutions, specifically at x = π/2.

What are the solutions for x when cos(x) ≠ 0 in the equation 2sin(x)cos(x) = 3cos(x)?

Dividing both sides by cos(x), we get 2sin(x) = 3, leading to sin(x) = 3/2. Since |sin(x)| ≤ 1, this has no solutions, so only x with cos(x) = 0 are valid.

What are the specific solutions for x in [0, 2) where cos(x) = 0?

Within [0, 2), cos(x) = 0 at x = π/2. Therefore, x = π/2 is a solution to the original equation.

Are there any other solutions to sin(2x) = 3cos(x) in [0, 2)?

No, because the only valid solution within the interval is at x = π/2, where cos(x) = 0 and the equation holds.

What is the exact value of x in [0, 2) satisfying sin(2x) = 3cos(x)?

The exact solution is x = π/2.

Additional Resources

Find the Exact Value Of Each X in [0, 2π) For Which Sin(2x) = 3cos(x)

Introduction

Mathematics often presents us with equations that seem straightforward at first glance but reveal layers of complexity upon closer examination. One such intriguing trigonometric equation is:

$$
\sin(2x) = 3 \cos(x)
$$

Our goal is to find all exact solutions for x within the interval $[0, 2\pi)$ that satisfy this equation. This exploration involves understanding the fundamental identities of trigonometry, algebraic manipulation, and the application of various solving techniques to arrive at precise solutions.

This detailed review will guide you through:

- The underlying trigonometric identities involved
- Step-by-step solution procedures
- Conceptual insights into the nature of the solutions
- Validation and interpretation of the solutions within the specified interval

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Understanding the Equation

Before diving into the solution process, it's essential to understand the functions involved and the structure of the equation:

$$
\sin(2x) = 3 \cos(x)
$$

Key points:

- $\sin(2x)$ is a double-angle identity, which can be expressed in terms of $\sin x$ and $\cos x$:

$$
\sin(2x) = 2 \sin x \cos x
$$

- The right side involves only $\cos x$, which suggests that expressing everything in terms of $\cos x$ or $\sin x$ might be advantageous.

- The interval $[0, 2\pi)$ encompasses one full period of sine and cosine functions, so solutions will be within this fundamental domain.

Step 1: Rewriting the Equation Using Identities

Given the double-angle identity:

$$
\sin(2x) = 2 \sin x \cos x
$$

we substitute into the original equation:

$$
2 \sin x \cos x = 3 \cos x
$$

This simplifies the problem to an algebraic equation involving $\sin x$ and $\cos x$.

Step 2: Simplification and Factoring

Now, the equation becomes:

$$
2 \sin x \cos x - 3 \cos x = 0
$$

Factor out $\cos x$:

$$
\cos x (2 \sin x - 3) = 0
$$

This product equals zero when either:

1. $\cos x = 0$
2. $2 \sin x - 3 = 0$

These two cases will be examined separately to find the solutions.

Step 3: Solving Each Case

Case 1: $\cos x = 0$

Within the interval $[0, 2\pi)$, $\cos x = 0$ at:

$$
x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}
$$

These are well-known points where cosine crosses zero.

Verification:

- For $x = \frac{\pi}{2}$:

$$
\sin(2 \times \frac{\pi}{2}) = \sin(\pi) = 0
$$

and

$$
3 \cos(\frac{\pi}{2}) = 3 \times 0 = 0
$$

The equation holds true.

- For $x = \frac{3\pi}{2}$:

$$
\sin(2 \times \frac{3\pi}{2}) = \sin(3\pi) = 0
$$

and

$$
3 \cos(\frac{3\pi}{2}) = 3 \times 0 = 0
$$

Again, the equation holds true.

Solutions from Case 1:

$$
x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}
$$

Case 2: $2 \sin x - 3 = 0$

Solve for $\sin x$:

\[
2 \sin x = 3 \implies \sin x = \frac{3}{2}
\]

Analysis:

- Since \(\sin x\) can only take values in \([-1, 1]\), \(\sin x = \frac{3}{2}\) is impossible.

- Therefore, no solutions arise from this case.

Conclusion of the Solution Process

The only solutions within \([0, 2\pi)\) are those where \(\cos x = 0\), which are:

\[
\boxed{
x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}
}
\]

Deep Dive: Validating and Interpreting the Solutions

It's critical to verify that these solutions satisfy the original equation:

\[
\sin(2x) = 3 \cos x
\]

Let's verify for each:

- For \(x = \frac{\pi}{2}\):

\[
\sin(2 \times \frac{\pi}{2}) = \sin \pi = 0
\]

\[
3 \cos \frac{\pi}{2} = 3 \times 0 = 0
\]

Equation holds.

- For \(x = \frac{3\pi}{2}\):

\[
\sin(2 \times \frac{3\pi}{2}) = \sin 3\pi = 0
\]

$$
3 \cos \frac{3\pi}{2} = 3 \times 0 = 0
$$

Equation holds.

Graphical Interpretation

Visualizing the functions can deepen understanding:

- Plot $\sin(2x)$: a sine wave with period π, oscillating between -1 and 1.

- Plot $3 \cos x$: a cosine wave scaled vertically by 3, oscillating between -3 and 3.

The solutions occur at points where these two curves intersect. Since $\sin(2x)$ is bounded between -1 and 1, and $3 \cos x$ ranges between -3 and 3, the intersection points where $\sin(2x) = 3 \cos x$ are limited.

The only intersections happen when $\cos x = 0$, which reduces the equation to $0=0$, confirming the solutions.

Extending General Solutions beyond $[0, 2\pi)$

While the problem specifies the interval $[0, 2\pi)$, understanding the general solutions can be insightful:

- Since $\cos x = 0$ at

$$
x = \frac{\pi}{2} + k \pi, \quad k \in \mathbb{Z}
$$

The solutions repeat every π.

- For the interval $[0, 2\pi)$, the solutions are:

$$
x = \frac{\pi}{2}, \quad x = \frac{3\pi}{2}
$$

- No solutions originate from $\sin x = \frac{3}{2}$ because it's invalid.

Summary of Findings

Solution	Exact value	Validity within $[0, 2\pi)$	Remarks

|------------|--------------|------------------------------|---------|
| \(x_1\) | \(\frac{\pi}{2}\) | Yes | \(\cos \frac{\pi}{2} = 0\), satisfies original equation |
| \(x_2\) | \(\frac{3\pi}{2}\) | Yes | \(\cos \frac{3\pi}{2} = 0\), satisfies original equation |

Total solutions in \([0, 2\pi)\): 2

Additional Considerations

- Behavior of the functions: Since \(\sin(2x)\) oscillates rapidly compared to \(3 \cos x\), the solutions are limited to points where \(\cos x = 0\).

- No extraneous solutions: Our algebraic manipulations are straightforward, and the solutions have been verified directly within the original equation, confirming their validity.

- Implications for similar equations: When encountering equations involving multiple angles and scaled functions, expressing everything in terms of basic functions and employing identities simplifies the process significantly.

Final Remarks

The process of solving \(\sin(2x) = 3 \cos x\) demonstrates a classic approach to handling trigonometric equations:

1. Identify identities to simplify the equation.
2. Factor where possible to reduce to manageable cases.
3. Solve algebraically, considering the ranges of trigonometric functions.
4. Verify solutions within the original equation to avoid extraneous roots.
5. Interpret solutions geometrically or graphically for better intuition.

In conclusion, the exact solutions within \([0, 2\pi)\) are:

\[
\boxed{
\boxed{\frac{\pi}{2}, \quad \frac{3\pi}{2}}
}
\]

These solutions neatly showcase how fundamental

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