Find The Point On The Line Y = 2x 3 That Is Closest To The Origin.

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Determining the point on a line closest to the origin (0,0) is a common problem in coordinate geometry. This task involves understanding the geometric relationship between a given line and a point, typically the origin, and applying optimization techniques to find the minimal distance. In this article, we will explore step-by-step how to find the point on the line y = 2x + 3 that is closest to the origin, delve into the mathematical principles behind the solution, and discuss practical applications of such calculations in fields like engineering, physics, and computer graphics.

Understanding the Problem: Closest Point on a Line to the Origin

Before diving into the solution, it's essential to understand what the problem entails and why it matters. The goal is to find a point (x, y) on the line y = 2x + 3 such that the distance from (0,0) to (x, y) is minimized.

What Does It Mean for a Point to Be Closest to the Origin?

- The distance between a point (x, y) and the origin (0, 0) is given by the Euclidean distance formula:
\[D = \sqrt{(x - 0)^2 + (y - 0)^2} = \sqrt{x^2 + y^2} \]
- Since the square root function is monotonically increasing, minimizing D is equivalent to minimizing D^2, which simplifies calculations:
\[D^2 = x^2 + y^2 \]

Reformulating the Problem

- Given the line y = 2x + 3, every point on the line can be represented as (x, 2x + 3).
- The problem reduces to finding the value of x that minimizes:
\[D^2(x) = x^2 + (2x + 3)^2 \]

Mathematical Approach to Find the Closest Point

To find the point on the line y = 2x + 3 closest to the origin, we use calculus to minimize the distance squared function.

Step 1: Write the Distance Squared Function

$$
D^2(x) = x^2 + (2x + 3)^2
$$

Expanding the second term:

$$
D^2(x) = x^2 + (4x^2 + 12x + 9)
$$

$$
D^2(x) = x^2 + 4x^2 + 12x + 9
$$

$$
D^2(x) = 5x^2 + 12x + 9
$$

Step 2: Find the Critical Points by Differentiation

Differentiate $D^2(x)$ with respect to x:

$$
\frac{d}{dx} D^2(x) = 10x + 12
$$

Set the derivative equal to zero to find the minimum:

$$
10x + 12 = 0
$$

$$
10x = -12
$$

$$
x = -\frac{12}{10} = -\frac{6}{5} = -1.2
$$

Step 3: Find the Corresponding y-coordinate

Recall the line equation:

$$
y = 2x + 3
$$

Substitute $x = -1.2$:

$$
y = 2(-1.2) + 3 = -2.4 + 3 = 0.6
$$

Step 4: Verify the Point and Calculate the Distance

- The closest point to the origin on the line y = 2x + 3 is:
$$
\boxed{\left(-\frac{6}{5}, \frac{3}{5}\right) = (-1.2, 0.6)}
$$
- The minimal distance can be computed as:
$$
D = \sqrt{(-1.2)^2 + (0.6)^2} = \sqrt{1.44 + 0.36} = \sqrt{1.8} \approx 1.3416
$$

Geometric Interpretation of the Solution

The shortest distance from a point to a line is along the perpendicular from the point to the line.
- The point closest to the origin on y = 2x + 3 is the foot of the perpendicular from (0,0) to the line.
- Our calculus-based solution aligns with this geometric principle, confirming the correctness of the approach.

Perpendicular Distance Formula

Alternatively, the shortest distance from the origin to the line y = 2x + 3 can be calculated directly using the point-line distance formula:
$$
d = \frac{|A \cdot x_0 + B \cdot y_0 + C|}{\sqrt{A^2 + B^2}}
$$
where the line is in the form Ax + By + C = 0.

Convert y = 2x + 3 to standard form:
$$
2x - y + 3 = 0
$$
- A = 2
- B = -1
- C = 3

Calculate the distance:
$$
d = \frac{|2 \cdot 0 + (-1) \cdot 0 + 3|}{\sqrt{2^2 + (-1)^2}} = \frac{|3|}{\sqrt{4 + 1}} = \frac{3}{\sqrt{5}} \approx 1.3416
$$
which matches our earlier calculation.

Applications of Finding the Closest Point on a Line

Understanding how to find the closest point on a line to a given point has numerous real-world applications:

1. Engineering and Robotics

- Path planning and obstacle avoidance often require calculating the shortest distance from a robot or component to a path or boundary.

2. Computer Graphics and Visualization

- Rendering scenes and collision detection involve proximity calculations between objects and lines or planes.

3. Geospatial Analysis

- Determining the closest point on a geographical feature (like a road or river) to a location for navigation or planning.

4. Physics and Mechanics

- Calculating minimum energy configurations where particles or objects are constrained along certain paths.

5. Data Fitting and Regression Analysis

- Identifying the best-fit line that minimizes the distance to a set of data points.

Summary of Key Points

- The problem involves minimizing the Euclidean distance from the origin to a line.
- By expressing the line parametrically and formulating a quadratic function, calculus techniques can be applied to find the minimum.
- The critical point for the quadratic function leads to the closest point on the line.
- The geometric principle of perpendicular distance confirms the solution.
- The approach is versatile and widely applicable across numerous scientific and engineering disciplines.

Conclusion

Finding the point on a line closest to the origin is a fundamental problem in coordinate geometry with practical implications. By translating the geometric problem into an algebraic optimization problem, we leverage calculus to efficiently identify the closest point. In the case of y = 2x + 3, the closest point to the origin is (-1.2, 0.6), with a minimum distance of approximately 1.3416 units. Mastering this technique enhances problem-solving skills in various scientific fields, enabling precise calculations in complex systems involving lines and points.

Further Resources

- "Coordinate Geometry" by David E. Joyce
- Khan Academy's tutorials on the distance from a point to a line
- Online tools for graphing and algebraic calculations, such as Desmos and Wolfram Alpha

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FAQs

1. **How do I find the closest point on a different line?**
   Apply the same principles: express the line in algebraic form, formulate the distance squared function, differentiate to find the minimum, and verify the point and distance.

2. **Can this method be used for lines in three dimensions?**
   Yes, but the calculations involve three coordinates, and the minimization process includes vector projections and the perpendicular distance in 3D space.

3. **What if the line is given in parametric form?**
   You can substitute parametric equations directly into the distance formula and minimize accordingly.

By understanding and applying these principles, you can confidently solve problems involving the shortest distance from a point to a line across various contexts, making this knowledge a valuable addition to your mathematical toolkit.

Frequently Asked Questions

What is the goal when finding the point on the line y = 2x + 3 closest to the origin?

The goal is to find the point on the line $y = 2x + 3$ that has the shortest distance from the origin (0,0).

How can the shortest distance from the origin to a line be mathematically determined?

By minimizing the distance formula between the origin and a point on the line, or equivalently, finding the point on the line where the perpendicular from the origin intersects the line.

What is the slope of the line y = 2x + 3?

The slope of the line is 2.

How do you find the point on the line closest to the origin using calculus?

Set up the distance function from the origin to a point (x, y) on the line, then differentiate and find the x-value that minimizes this distance, considering $y = 2x + 3$.

What is the formula for the perpendicular distance from a point to a line?

The perpendicular distance from a point (x_0, y_0) to the line $Ax + By + C = 0$ is $|A x_0 + B y_0 + C| / \sqrt{A^2 + B^2}$.

What is the closest point on the line y = 2x + 3 to the origin?

The closest point is found at $(x, y) = (-12/5, 6/5)$.

How is the point on the line y = 2x + 3 closest to the origin derived without calculus?

By finding the perpendicular line from the origin to the given line and solving the system of equations, we can determine the intersection point which is closest.

Additional Resources

Find The Point On The Line Y = 2x + 3 That Is Closest To The Origin

Understanding how to find the point on a given line closest to the origin is a fundamental problem in coordinate geometry, often encountered in various fields such as mathematics, engineering, and computer science. The task involves identifying the point on the specified line that minimizes the distance to the origin (0, 0). In this article, we will explore the problem of finding the point on the line Y = 2x + 3 that is closest to the origin, detailing the mathematical approach, geometric intuition, and practical applications.

Understanding the Problem

Before diving into the solution, it is essential to clearly understand what the problem entails.

The problem statement:
Given the line defined by the equation Y = 2x + 3, find the point (x, y) on this line that has the shortest Euclidean distance to the origin.

Key points to consider:

- The line is expressed in slope-intercept form: y = 2x + 3.
- The origin (0, 0) is our point of reference.
- The goal is to minimize the distance function between any point on the line and the origin.

Why is this problem important?
Finding the closest point on a line to a specific point is a classic optimization problem. It has applications in collision detection, location optimization, and even in physics for finding the shortest path or minimal energy configurations.

The Mathematical Approach

To solve this problem, we typically employ methods from calculus or analytic geometry. The core idea is to formulate the distance function, then find its minimum with respect to the variable x.

Step 1: Express the Point on the Line

Any point on the line Y = 2x + 3 can be represented as:
$$ P(x) = (x, 2x + 3) $$

Step 2: Write the Distance Function

The Euclidean distance D from the origin to an arbitrary point $P(x)$ is:
$$ D(x) = \sqrt{(x - 0)^2 + (2x + 3 - 0)^2} $$

Simplify:
$$ D(x) = \sqrt{x^2 + (2x + 3)^2} $$

Since the square root function is monotonically increasing, minimizing $D(x)$ is equivalent to minimizing its square:
$$ D^2(x) = x^2 + (2x + 3)^2 $$

Expand the second term:
$$ D^2(x) = x^2 + (4x^2 + 12x + 9) = 5x^2 + 12x + 9 $$

Step 3: Minimize the Distance Squared Function

Find the critical points by taking the derivative with respect to x:
$$ \frac{d}{dx} D^2(x) = 10x + 12 $$

Set derivative to zero to find the minimum:
$$ 10x + 12 = 0 \Rightarrow x = -\frac{12}{10} = -\frac{6}{5} $$

Step 4: Find the Corresponding y-Coordinate

Substitute $x = -\frac{6}{5}$ into the line equation:
$$ y = 2 \left(-\frac{6}{5} \right) + 3 = -\frac{12}{5} + 3 = -\frac{12}{5} + \frac{15}{5} = \frac{3}{5} $$

Resultant point:
$$ P \left(-\frac{6}{5}, \frac{3}{5} \right) $$

This is the point on the line closest to the origin.

Verifying the Solution

To confirm that this point indeed minimizes the distance, we can check the second derivative or analyze the nature of the critical point.

The second derivative of $D^2(x)$:

$$\frac{d^2}{dx^2} D^2(x) = 10 > 0$$

Since it is positive, the critical point corresponds to a minimum.

Geometric Interpretation

Understanding the problem geometrically can provide additional insights.

Shortest distance from a point to a line can be found by dropping a perpendicular from the point to the line. The point where this perpendicular intersects the line is the closest point.

Equation of the line:

$$y = 2x + 3$$

Line's slope:

$$m = 2$$

Perpendicular slope:

$$m_{\perp} = -\frac{1}{m} = -\frac{1}{2}$$

Equation of the perpendicular from the origin:

$$y = -\frac{1}{2} x$$

Find the intersection point between this perpendicular and the original line:

Set equal:

$$2x + 3 = -\frac{1}{2} x$$

Multiply through by 2 to clear the fraction:

$$4x + 6 = -x$$

Solve for x:

$$4x + x = -6 \Rightarrow 5x = -6 \Rightarrow x = -\frac{6}{5}$$

Find y:

$$y = -\frac{1}{2} \times -\frac{6}{5} = \frac{3}{5}$$

The intersection point is the same as our earlier solution, confirming that the shortest distance is indeed along this perpendicular.

Calculating the Shortest Distance

Now that we have the closest point, the minimal distance from the origin to the line is:

$$ D_{min} = \sqrt{\left(-\frac{6}{5}\right)^2 + \left(\frac{3}{5}\right)^2} $$

Calculate:

$$ D_{min} = \sqrt{\frac{36}{25} + \frac{9}{25}} = \sqrt{\frac{45}{25}} = \sqrt{\frac{9}{5}} = \frac{3}{\sqrt{5}} $$

Optional rationalization:

$$ D_{min} = \frac{3 \sqrt{5}}{5} $$

Features and Applications

Understanding how to find the closest point on a line to a given point has numerous practical applications:

- Collision Detection: In computer graphics and robotics, determining the minimum distance between objects helps prevent overlaps.
- Optimization Problems: Many real-world problems involve minimizing distances, such as locating facilities relative to a set of points.
- Navigation and Path Planning: Finding shortest paths or minimal distances from a point to a route.
- Physics: Calculating minimal energy configurations where the path or position is constrained.

Pros and Cons of the Analytical Method

Pros:

- Precise and exact solution.
- Straightforward to implement with basic calculus.
- Generalizable to other lines and points.

Cons:

- Requires familiarity with derivatives and algebraic manipulation.
- May become complex with non-linear constraints or higher dimensions.

- Not always intuitive; geometric visualization can help.

Alternative Methods

While the calculus-based approach is most direct, alternative methods include:

- Geometric Approach: Using perpendiculars and geometric properties for a visual solution.
- Vector Methods: Employing dot products to find the projection of the point onto the line, which directly yields the closest point.
- For example, if $\vec{A}$ is the point (origin) and $\vec{B}$ is a point on the line, then the closest point is the projection of $\vec{A}$ onto the line vector.

Conclusion

Finding the point on the line $Y = 2x + 3$ closest to the origin involves formulating the problem mathematically, deriving the minimum of the squared distance function, and verifying the result either through calculus or geometric reasoning. The critical point at $x = -\frac{6}{5}$ with corresponding $y = \frac{3}{5}$ provides the exact location, and the minimal distance is $\frac{3 \sqrt{5}}{5}$. Such problems reinforce fundamental principles of analytic geometry and are widely applicable in scientific and engineering contexts. Mastery of these techniques enhances problem-solving skills and deepens understanding of spatial relationships in coordinate systems.

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