Find The Two Solutions To The Following Equation:x(x + 10) = 0

Find The Two Solutions To The Following Equation:x(x + 10) = 0 is a fundamental problem in algebra that introduces students and learners to the concept of solving quadratic equations, especially those that are expressed in factored form. Understanding how to find solutions to such equations is essential for mastering algebraic techniques and building a strong foundation for more advanced mathematics. In this article, we will explore the process of solving the equation $x(x + 10) = 0$ step by step, discuss the underlying principles, and provide detailed explanations to help you grasp the concepts thoroughly.

Understanding the Equation x(x + 10) = 0

Before diving into the solution process, it's important to understand the structure of the equation itself.

Factored Form of Equations

The equation $x(x + 10) = 0$ is expressed in factored form, where the expression is written as a product of factors set equal to zero. This form is particularly useful because it allows us to apply the Zero Product Property, a key principle in algebra.

The Zero Product Property

The Zero Product Property states that:

- If the product of two factors equals zero, then at least one of the factors must be zero.

Mathematically, if:

$a \times b = 0$

then:

$a = 0$ or $b = 0$

This property simplifies solving equations where factors are multiplied together.

Step-by-Step Solution Process

Applying the Zero Product Property to the equation $x(x + 10) = 0$ involves identifying the factors and setting each equal to zero separately.

Step 1: Recognize the Factors

The given equation is already factored:

- First factor: x
- Second factor: $(x + 10)$

Step 2: Set Each Factor Equal to Zero

Using the Zero Product Property, solve for x in each case:

1. $x = 0$
2. $x + 10 = 0$

Step 3: Solve each simple equation

- For $x = 0$, the solution is directly $x = 0$.
- For $x + 10 = 0$, subtract 10 from both sides to get $x = -10$.

Solutions to the Equation

By solving each factor, we find the two solutions:

- $x = 0$
- $x = -10$

These are the solutions to the equation $x(x + 10) = 0$.

Verifying the Solutions

Verification ensures that the solutions satisfy the original equation.

Check for x = 0:

Substitute $x = 0$ into the original equation:

$0 \times (0 + 10) = 0 \times 10 = 0$

Since the result is 0, $x = 0$ is a valid solution.

Check for x = -10:

Substitute $x = -10$ into the original equation:

$-10 \times (-10 + 10) = -10 \times 0 = 0$

Again, the result is 0, confirming that $x = -10$ is a solution.

Graphical Interpretation of the Solutions

Understanding the solutions geometrically offers additional insight.

Graph of y = x(x + 10)

- The equation $y = x(x + 10)$ represents a quadratic function.
- It can be expanded to $y = x^2 + 10x$.
- The solutions correspond to the x-intercepts where the graph crosses the x-axis.

Plotting the Graph

- The parabola opens upward (since the coefficient of x^2 is positive).
- The x-intercepts are at $x = 0$ and $x = -10$, which are the solutions.

Additional Methods to Solve Similar Equations

While factoring is straightforward for equations like $x(x + 10) = 0$, other methods can be used for different types of quadratic equations.

Quadratic Formula

For equations not easily factored, the quadratic formula:

$x = [-b \pm \sqrt{b^2 - 4ac}] / (2a)$

can be used, where the quadratic is in the form $ax^2 + bx + c = 0$.

Completing the Square

This method involves rewriting the quadratic in a perfect square form to solve for x.

Real-World Applications of Solving Such Equations

Understanding solutions to equations like $x(x + 10) = 0$ has practical applications in various fields:

- Physics: Calculating points where an object reaches zero velocity or displacement.
- Engineering: Designing systems where certain parameters must reach zero.
- Economics: Finding break-even points in profit equations.

Summary and Key Takeaways

- The equation $x(x + 10) = 0$ is a quadratic in factored form.
- Applying the Zero Product Property allows us to set each factor equal to zero.
- The solutions are $x = 0$ and $x = -10$.
- Verification confirms these solutions satisfy the original equation.
- Graphical interpretation shows the solutions as points where the parabola crosses the x-axis.
- Similar equations can be solved using various algebraic methods, including the quadratic formula and completing the square.

Conclusion

Solving the equation $x(x + 10) = 0$ is a fundamental skill in algebra that illustrates how factoring simplifies problem-solving. Recognizing the structure of factored equations and applying the Zero Product Property efficiently yields solutions that are easy to verify and interpret. Mastery of these techniques not only helps in solving similar equations but also builds a strong foundation for tackling more complex mathematical problems. Whether you're a student, educator, or enthusiast, understanding these principles enhances your overall mathematical proficiency and prepares you for advanced topics in algebra, calculus, and beyond.

Frequently Asked Questions

What are the solutions to the equation $x(x + 10) = 0$?

The solutions are $x = 0$ and $x = -10$.

How do I solve the equation $x(x + 10) = 0$?

Set each factor equal to zero: $x = 0$ and $x + 10 = 0$, which gives $x = -10$.

Why does setting each factor to zero work in solving x(x + 10) = 0?

Because the product of two factors equals zero only when at least one of the factors is zero, according to the Zero Product Property.

Are there any other solutions to the equation x(x + 10) = 0 besides x = 0 and x = -10?

No, these are the only solutions since the equation is factored into linear factors.

What type of equation is x(x + 10) = 0?

It is a quadratic equation in factored form, specifically a product of linear factors set equal to zero.

Can I verify the solutions to x(x + 10) = 0?

Yes, substitute x = 0 and x = -10 back into the original equation to check that both satisfy the equation.

What is the graphical interpretation of the solutions to x(x + 10) = 0?

The solutions correspond to the points where the graph of y = x(x + 10) crosses the x-axis at x = 0 and x = -10.

How do the solutions x = 0 and x = -10 relate to the factors of the equation?

They directly correspond to the roots of each linear factor: x = 0 from the first factor and x = -10 from the second.

What is the significance of finding solutions to equations like x(x + 10) = 0?

Finding solutions helps identify the values of x that satisfy the equation, which is fundamental in algebra and problem-solving contexts.

Additional Resources

Find The Two Solutions To The Following Equation: x(x + 10) = 0

When exploring algebraic equations, one of the fundamental goals is to find the solutions—values of the variable that satisfy the equation. The equation x(x + 10) = 0 is a classic example of a factored quadratic expression and provides an excellent opportunity to delve into the principles of solving equations, especially through factoring. This comprehensive review aims to guide you through understanding the equation, the methods to solve it, and the significance of its solutions in various contexts.

Understanding the Equation: $x(x + 10) = 0$

Before jumping into solving the equation, it’s crucial to interpret its structure and what it represents.

Factored Form of the Equation

The equation is written as a product of two factors:

- x
- $(x + 10)$

This form is important because it directly links to the Zero Product Property, which states:

> If the product of two factors is zero, then at least one of the factors must be zero.

Mathematically, if `A B = 0`, then either `A = 0` or `B = 0`.

Applying this principle simplifies solving the equation significantly.

Graphical Interpretation

Graphically, the equation corresponds to the x-intercepts of the function:

- $f(x) = x(x + 10)$

The solutions are the points where the graph crosses the x-axis.

- The graph is a parabola opening upward (since the coefficient of x^2 is positive).
- The roots (solutions) are where $f(x) = 0$.

Understanding this helps visualize why the solutions are located where the factors equal zero.

Methodology to Find the Solutions

The process of solving the equation involves applying algebraic principles, primarily the Zero

Product Property. Let’s explore the steps in detail.

Step 1: Set Each Factor Equal to Zero

Given the factored form:

$$ x(x + 10) = 0 $$

Apply the Zero Product Property:

- Factor 1: $x = 0$
- Factor 2: $x + 10 = 0$

This step transforms a complex-looking equation into simple linear equations.

Step 2: Solve Each Linear Equation

Now, solve each of these equations separately:

- For $x = 0$, the solution is straightforward:

$$ x = 0 $$

- For $x + 10 = 0$:

$$ x = -10 $$

These are the two solutions to the original equation.

Step 3: Verify the Solutions

Verification involves substituting the solutions back into the original equation:

- For $x = 0$:

$$ 0 \times (0 + 10) = 0 \times 10 = 0 $$

The equation holds true.

- For $x = -10$:

$$ (-10) \times (-10 + 10) = (-10) \times 0 = 0 $$

The equation holds true again.

Verification confirms the correctness of the solutions.

Deep Dive into the Solutions

Having identified the solutions, it’s essential to understand their implications and how they fit into the broader context of algebra and mathematics.

The Two Solutions: x = 0 and x = -10

These solutions are roots or zeros of the function:

- $x = 0$: The point where the graph intersects the x-axis at the origin.
- $x = -10$: The point where the parabola intersects the x-axis at $x = -10$.

Both points satisfy the original equation, representing the x-values where the product of the factors is zero.

Significance of the Solutions

- Mathematically, solutions provide critical points for understanding the behavior of the function.
- Graphically, they mark the intercepts, giving insight into the shape and position of the parabola.
- Practically, these solutions could represent physical points of equilibrium, zero net change, or boundary conditions depending on the context of the problem.

Properties of the Solutions

- Both solutions are real numbers, which indicates the equation has real roots.
- The solutions are distinct; the parabola crosses the x-axis at two different points.

Alternative Methods for Solving the Equation

While the factoring method is straightforward, other techniques can be employed, especially for more complex equations.

Method 1: Using the Zero Product Property Directly

As demonstrated, this is the most direct method for factored equations.

Method 2: Expanding and Using Quadratic Formula

Suppose the equation was not initially factored; it could be expanded:

$$ x(x + 10) = 0 $$
$$ x^2 + 10x = 0 $$

Rewrite as a standard quadratic:

$$ x^2 + 10x = 0 $$

Apply the quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

where $ a = 1 $, $ b = 10 $, $ c = 0 $.

Calculations:

- Discriminant:

$$ \Delta = b^2 - 4ac = 10^2 - 4(1)(0) = 100 $$

- Roots:

$$ x = \frac{-10 \pm \sqrt{100}}{2} = \frac{-10 \pm 10}{2} $$

- When using $ + $:

$$ x = \frac{-10 + 10}{2} = \frac{0}{2} = 0 $$

- When using $ - $:

$$ x = \frac{-10 - 10}{2} = \frac{-20}{2} = -10 $$

This confirms the solutions obtained earlier.

Method 3: Graphical Solution

Plotting the function $ f(x) = x(x + 10) $ reveals:

- The parabola crosses the x-axis at $ x = 0 $ and $ x = -10 $.

- This visual approach emphasizes understanding the solutions' locations and relationships.

Real-World Applications and Contexts

Understanding the solutions to equations like $x(x + 10) = 0$ extends beyond pure mathematics into various practical fields.

Physics and Engineering

- Zero points in motion or force equations can represent equilibrium states.
- For example, in mechanical systems, solutions may indicate points where net force or displacement is zero.

Economics and Business

- Equations similar to this can model profit or loss functions; solutions indicate break-even points.

Biology and Ecology

- Population models often involve quadratic equations where solutions denote stable or unstable states.

Mathematics Education and Problem Solving

- This example serves as a fundamental teaching tool for understanding zero product property, factoring, and quadratic equations.

Common Mistakes and Misconceptions

When solving equations like $x(x + 10) = 0$, students often encounter pitfalls.

1. Overlooking the Zero Product Property

- Failing to set each factor equal to zero can lead to incomplete solutions.

2. Misinterpreting the Solutions

- Thinking that solutions must be positive or ignoring negative roots.

3. Not Verifying Solutions

- Substituting solutions back into the original equation is an essential step to confirm correctness, especially in more complex equations.

4. Confusing the Solutions with the Roots of the Equation

- Remember, solutions are the roots; in this case, $x = 0$ and $x = -10$.

Summary and Final Thoughts

To conclude, solving the equation $x(x + 10) = 0$ demonstrates a fundamental algebraic principle: the zero product property. By recognizing the factored form, setting each factor equal to zero, and solving the resulting linear equations, we find the solutions:

- $x = 0$
- $x = -10$

These solutions are not just numbers; they represent key points where the function intersects the x-axis, with implications in various scientific and mathematical contexts. The process underscores the importance of understanding factored forms, linear equations, and verification techniques in algebra.

Mastering this type of problem builds a solid foundation for tackling more complex quadratic equations, polynomial functions, and real-world applications. Additionally, visualizing solutions through graphing enhances comprehension and provides a holistic view of the equation's behavior.

Whether you're a student learning algebra or a professional applying mathematical modeling, the ability to find and interpret solutions like these is a vital skill. With practice, recognizing factored forms and applying the zero product property will become second nature, empowering you to solve a wide range of equations efficiently and accurately.

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