Find The Value(s) Of A Making V =4ai 3j Parallel To W =a2i 3j .

Find The Value(s) Of A Making V =4ai 3j Parallel To W =a2i 3j is a fundamental problem in vector algebra, involving the determination of scalar values that satisfy the condition of parallelism between two vectors. Understanding when two vectors are parallel is crucial in many applications across physics, engineering, and mathematics, such as calculating forces, velocities, or directions in multi-dimensional space. This article aims to thoroughly explore the steps involved in solving this problem, including the concepts of vector parallelism, the conditions for scalar multiples, and the explicit calculation of the values of A that make the given vectors parallel.

Understanding Vector Parallelism

Before diving into the calculations, it’s essential to understand what it means for two vectors to be parallel.

Definition of Parallel Vectors

Two vectors are said to be parallel if one can be expressed as a scalar multiple of the other. Mathematically, for vectors V and W, they are parallel if:

$$
\mathbf{V} = k \mathbf{W}
$$

for some scalar k.

Implications of Parallelism

- The vectors have the same or exact opposite directions.
- Their components are proportional, meaning each component of one vector is a scalar multiple of the corresponding component of the other.

Given Vectors and the Problem Statement

The vectors provided are:

$$
\mathbf{V} = 4a \mathbf{i} + 3 \mathbf{j}
$$
$$
\mathbf{W} = a^2 \mathbf{i} + 3 \mathbf{j}
$$

The goal is to determine the value(s) of a such that V is parallel to W.

Condition for Parallel Vectors

Expressing the vectors explicitly:

$$
\mathbf{V} = (4a, 3)
$$
$$
\mathbf{W} = (a^2, 3)
$$

For these vectors to be parallel, there must exist a scalar k such that:

$$
(4a, 3) = k (a^2, 3)
$$

This leads to two component-wise equations:

$$
\begin{cases}
4a = k a^2 \\
3 = k \times 3
\end{cases}
$$

Note: Since the second component of both vectors is 3, the scalar k can be directly obtained from the second component:

$$
3 = 3k \Rightarrow k = 1
$$

Once k is known, substitute back into the first component:

$$
4a = k a^2 = 1 \times a^2
$$
$$
a^2 = 4a
$$

This simplifies to:

$$
a^2 - 4a = 0
$$

\[

a (a - 4) = 0
\]

Thus, the solutions are:

\[
a = 0 \quad \text{or} \quad a = 4
\]

Verification of Solutions

To ensure these solutions satisfy the parallelism condition, verify each case.

Case 1: \(a = 0\)

- Vectors:

\[
\mathbf{V} = (0, 3)
\]
\[
\mathbf{W} = (0, 3)
\]

- The vectors are identical, hence parallel, with \(k = 1\).

Case 2: \(a = 4\)

- Vectors:

\[
\mathbf{V} = (4 \times 4, 3) = (16, 3)
\]
\[
\mathbf{W} = (a^2, 3) = (16, 3)
\]

- Once again, the vectors are identical, confirming the scalar \(k = 1\).

Both solutions satisfy the condition for parallel vectors.

Summary of Results

The scalar values of \(a\) that make \(\mathbf{V}\) parallel to \(\mathbf{W}\) are:

\[
\boxed{

a = 0 \quad \text{or} \quad a = 4
}
\]

These values ensure that the vectors are either identical or scalar multiples, maintaining the directionality required for parallelism.

Additional Insights and Applications

Understanding how to determine scalar values that relate vectors is vital in various fields.

Applications in Physics and Engineering

- Force vectors: Ensuring forces are aligned or oppose each other.
- Velocity and acceleration: Determining when objects move in parallel directions.
- Structural analysis: Confirming load vectors are parallel and thus can be combined or simplified.

Mathematical Significance

- Solving such problems enhances understanding of vector algebra.
- It emphasizes the importance of component-wise analysis.
- It provides foundational skills for more complex vector calculus and physics problems.

Conclusion

The problem of finding values of \(a\) such that vector \(\mathbf{V} = 4a \mathbf{i} + 3 \mathbf{j}\) is parallel to \(\mathbf{W} = a^2 \mathbf{i} + 3 \mathbf{j}\) demonstrates the elegant relationship between scalar multiplication and vector directionality. By equating the components and solving the resulting equations, we find that the vectors are parallel when \(a = 0\) or \(a = 4\). This process underscores the importance of understanding vector components, proportional relationships, and the conditions for parallelism, which are fundamental concepts in vector algebra and its applications across scientific disciplines.

Frequently Asked Questions

What is the condition for vectors V = 4ai + 3j and W = a2i + 3j to be parallel?

For vectors to be parallel, their component ratios must be equal; that is, (4a) / (a2) = 3 / 3, which simplifies to 4a / a2 = 1.

How do you determine the value of 'a' for which V is parallel to W?

Set the ratios of the components equal: (4a) / (a2) = 1, then solve for 'a' to find the values that satisfy this condition.

What are the steps to find the value of 'a' when vectors V and W are parallel?

1. Write the ratio of the i-components: 4a / a2. 2. Set this equal to the ratio of the j-components (which is 3/3=1). 3. Solve the resulting equation for 'a'.

What is the solution for 'a' when V = 4ai + 3j is parallel to W = a2i + 3j?

Solving 4a / a2 = 1 gives 4a = a2, which simplifies to a2 - 4a = 0, leading to a(a - 4) = 0. Therefore, a = 0 or a = 4.

Are both solutions a = 0 and a = 4 valid for the vectors to be parallel?

Yes, both a = 0 and a = 4 satisfy the condition for the vectors to be parallel, as they make the component ratios equal.

What happens when a = 0 in the vectors V and W?

When a = 0, V becomes the zero vector (0i + 3j), which is parallel to any vector, including W, but may be considered degenerate in some contexts.

Can the vectors V and W be parallel if a is any non-zero value other than 0 or 4?

No, the vectors are only parallel when a = 0 or a = 4, as these are the solutions to the component ratio equation. For other values, they are not parallel.

What is the significance of finding the value(s) of 'a' in this problem?

Finding the value(s) of 'a' helps determine under what conditions the two vectors are parallel, which is important in vector analysis and applications like physics and engineering.

Additional Resources

Find The Value(s) Of A Making V = 4ai + 3j Parallel To W = a2i + 3j

Understanding how to find the value(s) of a scalar a that makes one vector parallel to another is a fundamental problem in vector algebra, with applications ranging from physics to engineering. In this guide, we will explore "Find The Value(s) Of A Making V = 4ai + 3j Parallel To W = a2i + 3j" in detail, breaking down the concepts involved, the step-by-step process to determine the values, and the reasoning behind each step.

Introduction to Parallel Vectors and Scalar Multiplication

Before diving into the specific problem, it's essential to review some key concepts:

What Does It Mean for Vectors to Be Parallel?

Two vectors are parallel if one is a scalar multiple of the other. Mathematically, vectors V and W are parallel if there exists a scalar k such that:

V = k W

or equivalently,

V = λ W, where λ is some scalar.

Scalar Multiplication and Its Role

Scalar multiplication involves multiplying a vector by a scalar to produce a new vector that points in the same or opposite direction, depending on the sign of the scalar.

The Vectors in Question

Given the problem:

- \(V = 4a i + 3 j \)
- \(W = 2a i + 3 j \)

Our goal is to find the value(s) of a such that V is parallel to W.

Step-by-Step Approach to Find the Value(s) of a

Step 1: Express the Condition for Parallel Vectors

Since V and W are in the form:

- \(V = (4a)i + (3)j \)
- \(W = (2a)i + (3)j \)

for them to be parallel, V must be a scalar multiple of W:

$$ V = \lambda W $$

This implies:

$$ (4a) i + 3 j = \lambda (2a i + 3 j) $$

Step 2: Equate Components

Because vectors are equal component-wise, we equate the i-components and j-components separately:

- i-components:

$$ 4a = \lambda \times 2a $$

- j-components:

$$ 3 = \lambda \times 3 $$

Step 3: Solve for λ from the j-components

From the second component:

$$ 3 = 3 \lambda $$

Dividing both sides by 3:

$$ \lambda = 1 $$

This value of λ must be consistent with the i-component equation.

Step 4: Use $\lambda = 1$ in the i-component equation

Substitute $\lambda = 1$:

$$ 4a = 1 \times 2a $$
$$ 4a = 2a $$

Subtract $2a$ from both sides:

$$ 4a - 2a = 0 $$
$$ 2a = 0 $$

Divide both sides by 2:

$$ a = 0 $$

Final Result: The Value(s) of a

Based on the above calculations, the only value of a that makes V parallel to W is:

a = 0

Additional Considerations

What if a Is Zero?

- When $a = 0$, the vectors become:

- $V = 0 i + 3 j = 3 j$
- $W = 0 i + 3 j = 3 j$

- Both vectors are identical, so trivially parallel.

Are there any other possible values?

- Since the only consistent solution arises from setting $\lambda = 1$ and solving for a, there are no other solutions.

Summary of the Process

Step	Description	Result
1	Write the vectors and the condition for parallelism.	$V = \lambda W$
2	Equate components.	$4a = 2a \lambda$, $3 = 3 \lambda$
3	Solve for λ from the j-component.	$\lambda = 1$
4	Substitute λ into the i-component to solve for a.	$a=0$

Practical Applications of Such Problems

Understanding how to find scalar values that make vectors parallel is crucial for:

- Physics: Determining when force vectors align.
- Engineering: Ensuring components or forces are aligned for structural stability.
- Computer Graphics: Calculating directions and orientations.
- Mathematics: Developing intuition about vector relationships.

Additional Exercises for Practice

To deepen understanding, consider trying these variations:

1. Modify the vectors:

Find a such that $V = 5a i + 2 j$ is parallel to $W = a i + 2 j$.

2. Different components:

Find all a such that $V = 3a i + 4 j$ is parallel to $W = 6 a i + 8 j$.

3. Parallel but opposite directions:

Find a such that $V = 4a i + 3 j$ is opposite to $W = 2a i + 3 j$ (i.e., $V = -k W$ for some $k > 0$).

Conclusion

In this comprehensive guide, we've demonstrated how to find the value(s) of a making vectors V and W parallel. The key steps involve recognizing the condition for parallelism as a scalar multiple relationship, equating components, and solving for the scalar and the parameter a. The solution reveals that a = 0 is the only value satisfying the parallel condition in this particular case, highlighting the importance of component-wise analysis in vector algebra. Mastering such techniques allows for a deeper understanding of vector relationships and their applications across various scientific and engineering disciplines.

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