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Understanding the geometric relationships between points on a coordinate plane is fundamental in coordinate geometry. One of the key concepts is determining whether two line segments are parallel, which involves analyzing their slopes. In this context, we are given four points: P (1,3), Q (8,-4), R (2,9), and S (7,y). Our goal is to find the value of y such that the line segment PQ is parallel to RS, denoted as PQ || RS.

This comprehensive guide will walk you through the process step-by-step, explaining the concepts of slope, how to calculate it, and how to set up and solve the equation to find the unknown y. Whether you're a student studying for an exam or someone interested in mastering coordinate geometry, this article provides clarity and detailed explanations.

Understanding the Problem Statement

Points Given

- P (1, 3)
- Q (8, -4)
- R (2, 9)
- S (7, y)

The problem involves the line segments PQ and RS. Since we are asked to find y such that PQ is parallel to RS, the first step is to recall what it means for two line segments to be parallel.

What Does Parallel Mean in Geometry?

Two line segments are parallel if they lie on lines that do not intersect and have the same slope. In coordinate geometry, this translates to:

> *Line segments PQ and RS are parallel if their slopes are equal.*

Therefore, the crux of the problem is to compute the slopes of PQ and RS and set them equal to each

other to find y.

Calculating the Slope of a Line Segment

Definition of Slope

The slope of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by:

$$m = (y_2 - y_1) / (x_2 - x_1)$$

This value represents the rate of change of y with respect to x, or how steep the line is.

Calculating Slope of PQ

Given points P (1,3) and Q (8,-4):

1. Identify the coordinates:
    - P: $(x_1, y_1) = (1, 3)$
    - Q: $(x_2, y_2) = (8, -4)$

2. Apply the slope formula:

   $$m_{PQ} = (-4 - 3) / (8 - 1) = (-7) / 7 = -1$$

Hence, the slope of segment PQ is -1.

Calculating the Slope of RS

Points R and S

- R: (2, 9)

- S: (7, y)

Applying the Slope Formula for RS

1. Identify the coordinates:
    - R: $(x_1, y_1) = (2, 9)$
    - S: $(x_2, y_2) = (7, y)$

2. Calculate the slope:

   $m_{RS} = (y - 9) / (7 - 2) = (y - 9) / 5$

Setting the Slopes Equal to Find y

Since PQ is parallel to RS, their slopes are equal:

$m_{PQ} = m_{RS}$

Plugging in the values:

$-1 = (y - 9) / 5$

Now, solve for y.

Solving the Equation

1. Multiply both sides by 5 to eliminate the denominator:

   $5 \cdot (-1) = y - 9$

2. Simplify:

   $-5 = y - 9$

3. Add 9 to both sides:

   $-5 + 9 = y$

4. Calculate:

   $y = 4$

Thus, the value of y is 4.

Verification of the Solution

It's always good practice to verify the solution to ensure correctness.

Check the Slope of RS with y=4

Calculate the slope:

$m_{RS} = (4 - 9) / (7 - 2) = (-5) / 5 = -1$

Since mPQ was also -1, the slopes are equal, confirming the segments are parallel.

Conclusion

The calculated y value of 4 ensures that the line segment RS is parallel to PQ.

Additional Insights and Related Concepts

Why is Slope Important?

- Slope determines the angle of inclination of a line.
- Equal slopes imply lines are parallel, which is fundamental in geometry and design.
- Understanding slopes helps in solving real-world problems involving angles, distances, and trajectories.

Other Ways to Approach the Problem

1. Using coordinate geometry formulas for distance and midpoints to analyze relationships between points.
2. Graphically plotting points to visually verify parallelism.
3. Applying vector methods to analyze directions of line segments.

Common Mistakes to Avoid

- Forgetting to use the correct order of points when calculating slopes.
- Dividing by zero when the x-coordinates are the same (vertical lines).
- Mixing up the points when setting up the equations.

Summary and Final Remarks

In this detailed analysis, we've determined that for the line segment PQ (with points P (1,3) and Q (8,-4)) to be parallel to RS (with points R (2,9) and S (7,y)), the value of y must be 4. This was achieved by calculating the slopes of both segments and setting them equal, then solving for y. Such problems reinforce the importance of understanding the concept of slope and its application in coordinate geometry.

Mastering these fundamental techniques equips you with the skills necessary to solve more complex geometric problems, analyze geometric figures, and understand the properties of lines and angles on the coordinate plane.

Practical Applications of Parallel Lines in Real Life

Understanding when and how lines are parallel has numerous practical applications beyond academic exercises:

- **Engineering and Architecture:** Ensuring structures like beams, walls, and bridges are aligned properly.
- **Navigation and Map Reading:** Determining routes and orientations that are parallel to certain landmarks or roads.
- **Art and Design:** Creating patterns and layouts with parallel lines for aesthetic appeal.
- **Robotics and Automation:** Programming movement paths that are parallel to each other for precision tasks.

Conclusion

This comprehensive guide has covered the process of finding the value of y such that the line segments PQ and RS are parallel, starting from understanding points, calculating slopes, setting their equality, and solving the resulting equation. Remember, the key steps involve:

1. Calculating the slope of the known segment PQ.
2. Expressing the slope of the unknown segment RS in terms of y.
3. Equating the slopes and solving for y.

The final answer, y=4, ensures the segments are parallel, illustrating a practical application of the fundamental concepts in coordinate geometry. Practice similar problems to strengthen your understanding and improve your problem-solving skills in geometry.

If you want further insights into coordinate geometry or related topics, exploring concepts like perpendicular lines, midpoint formulas, and distance calculations will expand your mathematical toolkit.

Frequently Asked Questions

Given points P(1,3), Q(8,-4), R(2,9), and S(7,y), how do we determine the value of y so that PQ is parallel to RS?

To find y, calculate the slopes of PQ and RS and set them equal: $(-4-3)/(8-1) = (y-9)/(7-2)$. Simplify: $(-7)/7 = (y-9)/5$. So, $-1 = (y-9)/5$. Multiply both sides by 5: $-5 = y-9$. Add 9 to both sides: $y = 4$.

What is the slope of line segment PQ given points P(1,3) and Q(8,-4)?

The slope of PQ is $(-4-3)/(8-1) = (-7)/7 = -1$.

How do you find the slope of line segment RS with points R(2,9) and S(7,y)?

The slope of RS is $(y-9)/(7-2) = (y-9)/5$.

What condition must y satisfy for lines PQ and RS to be parallel?

Their slopes must be equal: $(-7)/7 = (y-9)/5$. Simplifying gives $y-9 = -5$, so $y = 4$.

If y = 4, are segments PQ and RS parallel? Why?

Yes, because both have a slope of -1, making them parallel.

Can you verify that with y=4, lines PQ and RS are parallel? How?

Calculate slopes: PQ has slope -1; RS with y=4 has slope $(4-9)/5 = (-5)/5 = -1$. Since both slopes are equal, the lines are parallel.

What is the significance of setting the slopes equal in this problem?

Setting the slopes equal ensures that the two line segments are parallel, which is the condition given in the problem for line segments PQ and RS.

Additional Resources

Given P (1,3) Q(8,-4), R(2,9) And S(7,y). Find Y Such That Pq || Rs

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Introduction: Understanding the Problem

In the realm of coordinate geometry, understanding the relationships between lines—particularly their parallelism—is fundamental. The problem at hand involves four points: P(1,3), Q(8,-4), R(2,9), and S(7,y). The goal is to determine the value of y such that the line segment P Q (or P q, as sometimes notated) is parallel to the line segment R S (or R s). This task requires a clear understanding of how to compute slopes, interpret their significance, and apply the principles of parallel lines.

This article will explore the process step-by-step, providing clarity on the concepts involved, the calculations required, and the logical reasoning needed to find the value of y that satisfies the parallel condition.

Understanding Parallel Lines in Coordinate Geometry

The Concept of Slope

In coordinate geometry, the slope of a line is a measure of its steepness. For two points (x_1, y_1) and (x_2, y_2), the slope m is calculated as:

$$
m = \frac{y_2 - y_1}{x_2 - x_1}
$$

This ratio reflects the change in y over the change in x. If two lines have the same slope, they are parallel (assuming they are not coincident).

Applying Slope to Determine Parallelism

Given two line segments with endpoints (x_1, y_1) and (x_2, y_2), their slopes are:

- For line $P Q$: m_{PQ}
- For line $R S$: m_{RS}

To find the value of y such that $P Q \parallel R S$, we set the slopes equal:

$$
m_{PQ} = m_{RS}
$$

This equality forms the basis for solving for y.

Step-by-Step Calculation

1. Calculating the Slope of Line $P Q$

Given points:

- $P(1,3)$
- $Q(8,-4)$

The slope m_{PQ}:

$$
m_{PQ} = \frac{-4 - 3}{8 - 1} = \frac{-7}{7} = -1
$$

Thus, the slope of $P Q$ is -1.

2. Expressing the Slope of Line $R S$

Given points:

- $R(2,9)$
- $S(7,y)$

The slope m_{RS}:

$$
m_{RS} = \frac{y - 9}{7 - 2} = \frac{y - 9}{5}
$$

3. Setting the Slopes Equal for Parallelism

Since $P Q \parallel R S$:

$$
m_{PQ} = m_{RS}
$$

Substitute the known slope:

$$
-1 = \frac{y - 9}{5}
$$

4. Solving for y

Cross-multiplied:

$$
-1 \times 5 = y - 9
$$

$$
-5 = y - 9
$$

Add 9 to both sides:

$$
-5 + 9 = y
$$
$$
4 = y
$$

Therefore, the value of y that makes $P Q$ parallel to $R S$ is 4.

Validating the Solution

It's important to verify that the calculated y indeed produces lines with equal slopes:

- Slope of $P Q$: -1
- Slope of $R S$ with $y=4$:

$$
m_{RS} = \frac{4 - 9}{7 - 2} = \frac{-5}{5} = -1
$$

Since both slopes are -1, the lines are parallel, confirming the correctness of the solution.

Broader Implications and Applications

Real-World Contexts

Understanding the relationships between lines in coordinate space extends far beyond pure mathematics. For instance:

- Urban Planning: Ensuring roads or pathways are parallel for aesthetic or functional reasons.
- Engineering: Designing structures where components must run parallel to each other.
- Computer Graphics: Calculating and rendering parallel lines for visual consistency.

Educational Significance

Mastering such problems enhances spatial reasoning and prepares students for advanced topics like vector analysis, trigonometry, and analytical geometry, providing a foundation for careers in science, technology, engineering, and mathematics.

Additional Insights and Tips

- Always double-check the order of points when calculating slopes; reversing points affects the sign.
- Remember that vertical lines have undefined slopes, which require different approaches.
- When solving for y, carefully handle algebraic manipulations to avoid errors.
- Visualize the points and lines on a graph for intuitive understanding, especially when dealing with more complex problems.

Conclusion

Determining the value of y such that two line segments are parallel involves understanding the concept of slope and applying basic algebraic techniques. In this case, setting the slopes of PQ and RS equal led to the straightforward solution that $y = 4$. This process exemplifies the power of coordinate geometry in solving real-world and theoretical problems, reinforcing the importance of fundamental concepts like slope calculation and algebraic manipulation. Whether for academic purposes or practical applications, mastering these techniques unlocks a deeper understanding of spatial relationships in the coordinate plane.

Given P 1 3 Q 8 4 R 2 9 And S 7 Y Find Y Such That Pq Rs

Given P 1 3 Q 8 4 R 2 9 And S 7 Y Find Y Such That Pq Rs

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