Given That F(x)=x^2-2 And G(x)=5x+3, Find (f+g)(4), If It Exists

Given That F(x)=x^2-2 And G(x)=5x+3, Find (f+g)(4), If It Exists

When working with functions in mathematics, particularly in algebra and calculus, understanding how to manipulate and combine functions is essential. One common operation is the addition of two functions, denoted as (f+g)(x), which essentially means adding the outputs of the functions f(x) and g(x) for a given input x. In this article, we will explore how to find (f+g)(4) given the specific functions F(x) = x^2 - 2 and G(x) = 5x + 3. We will also discuss the conditions under which this operation exists, how to evaluate the sum, and explore related concepts for a comprehensive understanding.

Understanding Functions and Their Addition

What Is a Function?

A function in mathematics is a relation that assigns exactly one output to each input from a given set. For example, the functions provided:
- F(x) = x^2 - 2
- G(x) = 5x + 3

are both functions that take a real number as input and produce a real number as output.

Adding Functions: The (f+g)(x) Operation

The sum of two functions f(x) and g(x) is defined as:
- (f+g)(x) = f(x) + g(x)

This means that for any input x, the output of (f+g)(x) is simply the sum of f(x) and g(x).

Existence of (f+g)(x)

The operation (f+g)(x) exists at a specific point x if both f(x) and g(x) are defined at that point. Since both functions in our case are polynomial functions, which are defined for all real numbers, the sum (f+g)(x) exists for every real x.

Step-by-Step Solution to Find (f+g)(4)

Step 1: Identify the functions

Given:
- f(x) = x^2 - 2

- $g(x) = 5x + 3$

Step 2: Evaluate each function at $x = 4$

Calculate $f(4)$:
- $f(4) = (4)^2 - 2 = 16 - 2 = 14$

Calculate $g(4)$:
- $g(4) = 5(4) + 3 = 20 + 3 = 23$

Step 3: Add the results to find $(f+g)(4)$

- $(f+g)(4) = f(4) + g(4) = 14 + 23 = 37$

Conclusion

Therefore, the value of $(f+g)(4)$ is 37. Since both functions are defined at $x=4$, the sum exists, and the calculation is straightforward.

Further Exploration: Generalization and Related Concepts

Evaluating $(f+g)(x)$ for Any x

The process demonstrated can be extended to any real number x:
- $(f+g)(x) = (x)^2 - 2 + 5x + 3$
- Simplify: $(f+g)(x) = x^2 + 5x + 1$

This expression allows us to evaluate $(f+g)(x)$ efficiently for any x.

Graphical Interpretation

Visualizing the functions:
- The graph of $f(x) = x^2 - 2$ is a parabola opening upwards, shifted downward by 2 units.
- The graph of $g(x) = 5x + 3$ is a straight line with slope 5 and y-intercept 3.
- The sum $(f+g)(x)$ corresponds to the combined effect of these two functions, resulting in a parabola shifted and stretched, which can be useful in understanding their combined behavior.

Applications of Function Addition

Adding functions arises in various fields:
- Physics: Combining displacement functions
- Engineering: Summing signals or responses
- Economics: Aggregating functions representing different economic factors

Understanding how to add functions and evaluate their sums at specific points is fundamental across these disciplines.

Common Mistakes and Troubleshooting

- Assuming the sum exists without verifying domain compatibility: For polynomial functions, the domain is all real numbers, so the sum always exists. For other types of functions (like square roots or logarithms), check the domain restrictions.
- Incorrect substitution: Always substitute the specific x-value into each function carefully.
- Forgetting to simplify: Combining like terms after substitution can make calculations more straightforward.

Summary

- The sum of functions f(x) = x^2 - 2 and g(x) = 5x + 3 is (f+g)(x) = x^2 + 5x + 1.
- To find (f+g)(4), evaluate each function at x=4 and add the results.
- The calculation yields (f+g)(4) = 37.
- Since the functions are polynomials, their sum exists for all real x, including x=4.
- Understanding these operations enhances problem-solving skills in algebra and prepares you for more advanced topics in calculus and analysis.

Additional Practice Problems

To strengthen your understanding, try solving these:

1. Given F(x) = 3x - 4 and G(x) = x^2 + 1, find (F+G)(2).

2. For functions H(x) = √x and K(x) = 2x + 5, determine if (H+K)(x) exists at x = 9 and evaluate it.

3. Given F(x) = 2x^3 - x and G(x) = -x^3 + 4x, find (f+g)(x) and simplify.

Final Thoughts

Mastering the addition of functions is a stepping stone toward more complex operations such as composition, subtraction, and division of functions. It also lays the groundwork for understanding function transformations, derivatives, and integrals in calculus. Whether you're a student learning algebra or a professional applying mathematical models, these skills are fundamental in analyzing and interpreting real-world data and phenomena.

Remember, always verify the domain restrictions of your functions and carefully perform substitutions to ensure accurate results. With practice, evaluating sums of functions at specific points becomes a quick and reliable process, enabling you to approach more advanced mathematical problems with confidence.

Frequently Asked Questions

What is the value of (f+g)(4) given f(x)=x^2 - 2 and g(x)=5x + 3?

First, find f(4): 4^2 - 2 = 16 - 2 = 14. Then, find g(4): 5(4) + 3 = 20 + 3 = 23. Now, (f+g)(4) = f(4) + g(4) = 14 + 23 = 37.

How do you compute (f+g)(x) for given functions f(x)=x^2 - 2 and g(x)=5x + 3?

To compute (f+g)(x), add the functions: (f+g)(x) = f(x) + g(x). For the given functions, it becomes (x^2 - 2) + (5x + 3) = x^2 + 5x + 1.

Is (f+g)(4) defined for the functions provided, and what is its value?

Yes, it is defined since both functions are polynomial functions. The value at x=4 is 37, as calculated by substituting 4 into both functions and summing the results.

Can (f+g)(x) be expressed as a single polynomial function for the given functions?

Yes, by adding f(x) and g(x), (f+g)(x) = x^2 + 5x + 1, which is a quadratic polynomial.

What is the significance of finding (f+g)(4) in function analysis?

Finding (f+g)(4) helps evaluate the combined effect of functions f and g at a specific point, useful in understanding their combined behavior at that point.

Are there any conditions under which (f+g)(x) might not exist for the given functions?

Since both f(x) and g(x) are polynomial functions, their sum exists for all real x. Therefore, (f+g)(x) exists for all x, including x=4.

Additional Resources

Given That F(x)=x^2-2 And G(x)=5x+3, Find (f+g)(4), If It Exists is a classic problem in algebra that tests understanding of function operations and the ability to evaluate composite functions at specific points. This problem not only emphasizes the fundamental concept of function addition but also encourages a systematic approach to evaluating composite functions at given values. As students or learners encounter such questions, they develop critical skills in function manipulation, algebraic simplification, and problem-solving strategies, which are essential in higher mathematics and various applied sciences. In this article, we will explore the problem thoroughly, breaking down the process into manageable steps, discussing the underlying concepts, and providing insights into related topics.

Understanding the Functions F(x) and G(x)

Before diving into the calculations, it's important to understand the nature of the given functions:

- $F(x) = x^2 - 2$ is a quadratic function. Its graph is a parabola opening upward with a vertex at the point (0, -2). The function is symmetric about the y-axis and exhibits quadratic growth as x moves away from zero.

- $G(x) = 5x + 3$ is a linear function. Its graph is a straight line with a slope of 5 and a y-intercept at (0, 3). The function exhibits a steady, linear increase as x increases.

These two functions are simple yet representative examples of polynomial functions of degree 2 and 1, respectively, making them ideal for illustrating the process of combining functions and evaluating at specific points.

Defining (f+g)(x)

The notation (f+g)(x) refers to the sum of the two functions evaluated at x. Formally:

$$ (f+g)(x) = f(x) + g(x) $$

This means that to find (f+g)(x), you evaluate each function at x and then add the results.

Key points:

- Function addition combines the outputs of two functions for the same input.
- The sum of functions is itself a function, often of higher degree if the original functions are polynomials.

In our case, since:

- $f(x) = x^2 - 2$
- $g(x) = 5x + 3$

we have:

$$ (f+g)(x) = (x^2 - 2) + (5x + 3) $$

which simplifies to:

$$ (f+g)(x) = x^2 + 5x + 1 $$

Calculating (f+g)(4)

The core task is to evaluate the combined function at a specific point, $x = 4$.

Step-by-step process:

1. Evaluate $f(4)$:

$$
f(4) = (4)^2 - 2 = 16 - 2 = 14
$$

2. Evaluate $g(4)$:

$$
g(4) = 5 \times 4 + 3 = 20 + 3 = 23
$$

3. Add the results:

$$
(f+g)(4) = f(4) + g(4) = 14 + 23 = 37
$$

Answer:

$$
\boxed{(f+g)(4) = 37}
$$

This straightforward calculation demonstrates how understanding the individual functions and their sum allows for quick and accurate evaluation at any point.

Alternative Approach: Direct Simplification of (f+g)(x)

Instead of evaluating each function separately at $x=4$, an alternative involves first simplifying the sum function:

$$
(f+g)(x) = x^2 + 5x + 1
$$

Then, simply substitute $x=4$:

$$
(f+g)(4) = (4)^2 + 5 \times 4 + 1 = 16 + 20 + 1 = 37
$$

Advantages of this approach:

- Reduces repetitive calculations.
- Provides a general formula for (f+g)(x), useful if multiple evaluations are needed.
- Clarifies the combined effect of the functions.

Discussion: When Does (f+g)(x) Exist?

In the context of real-valued functions, the sum (f+g)(x) exists if both f(x) and g(x) exist at the given x. Since both functions are polynomial functions, they are defined for all real numbers:

- Polynomial functions are continuous and defined everywhere.
- Thus, (f+g)(x) exists for all real x, including x=4.

In more complex scenarios involving functions with restrictions (like rational functions with potential division by zero, or functions involving roots with negative radicands), verifying the domain is crucial before evaluating.

In our case:

- Domain of f(x): all real numbers.
- Domain of g(x): all real numbers.
- Therefore, (f+g)(4) definitely exists.

Related Topics and Extensions

Understanding the process of adding functions and evaluating at specific points opens doors to a wide range of mathematical concepts:

- Function Composition: Instead of addition, functions can be composed, e.g., $(f\circ g)(x) = f(g(x))$. This involves plugging one function into another.
- Difference and Product of Functions: Similar procedures apply for subtraction and multiplication.
- Graphical Interpretation: The sum of functions corresponds to the vertical sum of their graphs at each x-value.
- Higher-Degree Polynomial Operations: When dealing with polynomials of higher degrees, addition and evaluation follow similar steps but can involve more complex algebra.

Pros and Cons of Function Addition

Pros:

- Simplifies complex functions into manageable expressions.
- Facilitates the understanding of combined effects of multiple functions.

- Useful in modeling scenarios where quantities are summed, such as total cost or combined forces.

Cons:

- Can lead to higher-degree polynomials, which may be more complex to analyze.
- When functions are not defined everywhere, domain restrictions must be carefully considered.
- In some cases, the algebra can become cumbersome, especially with non-polynomial functions.

Features and Applications

- Mathematical Modeling: Combining functions to represent real-world phenomena like total revenue (price function + quantity function).
- Calculus: Deriving or integrating the sum of functions to find rates of change or areas.
- Engineering: Summing signals or forces modeled by different functions.
- Computer Science: Function operations in algorithms and data transformations.

Conclusion

The problem of finding (f+g)(4) given specific functions exemplifies fundamental concepts in algebra and function operations. By understanding the individual functions, their domains, and how to combine them algebraically, students can efficiently evaluate complex expressions at specific points. The process highlights the importance of simplifying expressions before substitution, ensuring clarity and reducing computational errors. Moreover, recognizing that the sum function exists for all real numbers in this case emphasizes the significance of domain considerations in more advanced problems. Overall, mastering such problems lays a solid foundation for more advanced topics in mathematics, science, and engineering, where functions serve as essential tools for modeling and analysis.

In summary:

- The functions $f(x) = x^2 - 2$ and $g(x) = 5x + 3$ are straightforward to work with.
- Their sum simplifies to $x^2 + 5x + 1$.
- Evaluating at $x=4$ yields $\boxed{37}$.
- The process demonstrates core algebraic skills and highlights the importance of understanding function operations.

By practicing such problems, learners develop confidence in manipulating functions, which is crucial for success in higher mathematics and numerous applied fields.

Given That F X X 2 2 And G X 5x 3 Find F G 4 If It Exists

Given That F X X 2 2 And G X 5x 3 Find F G 4 If It Exists

Related Articles

- Please Help With All And Brainliesting Please Help Really Urgent
- ABAB Or ABBC Is An Example Of A Rhyming _____.questionsimilepattern
- What Is An Example Of A Reserved Power Of The State Governments?

Back to Home