Graph A Line That Contains A Point (−6,1) And Has A Slope Of 5

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Understanding how to graph a line given a point and a slope is fundamental in coordinate geometry. In this article, we will explore the process of graphing such a line step-by-step, focusing on the specific example of a line passing through the point (−6, 1) with a slope of 5. We will discuss the concepts of slope, point-slope form, slope-intercept form, and practical methods for plotting the line accurately. Whether you're a student learning the basics or someone seeking to deepen your understanding, this guide aims to provide clarity and detailed explanations.

Fundamental Concepts in Graphing a Line

Understanding Slope

The slope of a line, often represented as m, measures the steepness and direction of the line. It is calculated as the ratio of the vertical change to the horizontal change between two points on the line:

$$
m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

In our case, the slope $m = 5$ indicates that for every 1 unit increase in x, the value of y increases by 5 units. The positive slope suggests the line rises as it moves from left to right.

Point-Slope Form of a Line

Given a point (x_1, y_1) and a slope m, the equation of the line can be written in point-slope form:

$$
y - y_1 = m(x - x_1)
$$

Using our specific point and slope:

$$
y - 1 = 5(x + 6)
$$

This form is particularly useful for graphing because it directly incorporates the known point and slope.

Slope-Intercept Form of a Line

The slope-intercept form, $y = mx + b$, expresses the line as a function of x, where b is the y-intercept—the point where the line crosses the y-axis. To convert the point-slope form to slope-intercept form:

$$
y - 1 = 5(x + 6)
$$

$$
y - 1 = 5x + 30
$$

$$
y = 5x + 31
$$

This form makes it straightforward to identify the y-intercept, which in this case is at $(0, 31)$.

Step-by-Step Guide to Graphting the Line

1. Write the Equation in Slope-Intercept Form

As shown above, the line passing through $(-6, 1)$ with a slope of 5 can be expressed as:

$$
y = 5x + 31
$$

This form simplifies the process of plotting the line because it provides direct information about the y-intercept.

2. Plot the Y-Intercept

Begin by plotting the y-intercept point $(0, 31)$ on the coordinate plane. Since this point has a very high y-value, ensure your graph has sufficient vertical range. This point is where the line crosses the y-axis.

3. Use the Slope to Find a Second Point

From the y-intercept $(0, 31)$, use the slope $m = 5$ to find another point. Recall that a slope of 5 means:

- Rise: 5 units upward
- Run: 1 unit to the right

Starting at $(0, 31)$:

- Move 1 unit to the right: $x = 1$
- Move 5 units up: $y = 36$

Plot this second point at $(1, 36)$.

Alternatively, moving in the negative x-direction:

- Move 1 unit left: $x = -1$
- Move 5 units down: $y = 26$

Plot at $(-1, 26)$.

Using these two points, you can verify the line's accuracy.

4. Draw the Line

Using a ruler, draw a straight line passing through the two points $(0, 31)$ and $(1, 36)$. Extend the line across the graph, ensuring it's straight and continuous.

5. Confirm the Line Passes Through the Given Point

Since the point $(-6, 1)$ was given, verify it lies on the line. Plugging $x = -6$ into the equation:

$$
y = 5(-6) + 31 = -30 + 31 = 1
$$

Indeed, the point $(-6, 1)$ satisfies the equation, confirming that your graph is correct.

Alternative Methods for Graphing the Line

Using the Point-Slope Form

Instead of converting to slope-intercept form, you can directly plot using the point-slope form:

$$
y - 1 = 5(x + 6)
$$

Choose additional values for x:

- For $x = 0$:

$$
y - 1 = 5(0 + 6) = 30 \Rightarrow y = 31
$$

- For $x = -7$:

$$
y - 1 = 5(-7 + 6) = 5(-1) = -5 \Rightarrow y = -4
$$

Plot these points $(0, 31)$ and $(-7, -4)$, then draw the line through them.

Advantages: This method is flexible when you have the point and slope but prefer not to convert equations.

Graphing Using a Table of Values

Create a table with chosen x-values:

x	$y = 5x + 31$
-8	$5(-8) + 31 = -40 + 31 = -9$
-6	$5(-6) + 31 = -30 + 31 = 1$ (given point)
0	31 (intercept)
1	$5(1) + 31 = 5 + 31 = 36$

Plotting these points helps visualize the line accurately.

Practical Tips for Accurate Graphing

- Use graph paper for precision.
- Label the points clearly to avoid confusion.
- Ensure the scale on axes is consistent.
- Draw the line with a ruler to maintain straightness.
- Verify key points, especially the given point, to confirm accuracy.

Applications and Importance of Graphing Lines

Understanding how to graph a line from a point and slope has numerous applications:

- Solving real-world problems: such as predicting trends, calculating costs, or modeling physical phenomena.
- Analyzing relationships: in fields like economics, physics, and biology.
- Graphing functions: which is foundational for calculus and advanced mathematics.

- Data visualization: representing data points and their relationships.

Mastering this skill enhances problem-solving abilities and deepens comprehension of algebraic concepts.

Conclusion

Graphing a line that contains a specific point and has a given slope involves understanding the fundamental forms of a line’s equation and being able to manipulate them effectively. Starting from the point-slope form, converting to slope-intercept form simplifies plotting, especially when the y-intercept is easily identifiable. Using the slope to find additional points ensures accuracy and clarity in the graph. The process described above, applied to the point $(-6, 1)$ with a slope of 5, demonstrates a systematic approach to visualizing linear relationships. With practice, these techniques become intuitive tools in the broader context of algebra and coordinate geometry.

Remember: Always verify your points and equations, and use precise tools for drawing. These steps not only help in accurately graphing the line but also deepen your understanding of the underlying mathematical principles.

Frequently Asked Questions

How do you write the equation of a line that passes through the point (-6, 1) with a slope of 5?

Using point-slope form: $y - 1 = 5(x + 6)$. Simplifying, the equation is $y = 5x + 31$.

What is the slope-intercept form of the line passing through (-6, 1) with a slope of 5?

The slope-intercept form is $y = 5x + 31$.

How can I graph the line that passes through (-6, 1) with a slope of 5?

Start at the point (-6, 1), then from there, move up 5 units and right 1 unit repeatedly to plot additional points, then draw the line through these points.

What is the significance of the point (-6, 1) and slope 5 in the line's equation?

The point (-6, 1) is the line's known point through which it passes, and the slope of 5 indicates the line's steepness, meaning it rises 5 units for every 1 unit it runs horizontally.

Can I find the equation of the line without using the point-slope form?

Yes. Since the line passes through (-6, 1) and has slope 5, you can substitute into the slope-intercept form y = mx + b and solve for b: 1 = 5(-6) + b, so b = 31. The equation is y = 5x + 31.

What are some real-world examples of a line with slope 5 passing through a specific point?

An example could be a car starting at a position (-6, 1) on a coordinate grid and moving so that its position increases by 5 units vertically for every 1 unit horizontally traveled, representing a rapid upward trend or speed.

How does knowing the point and slope help in graphing the line accurately?

Knowing the point (-6, 1) and slope 5 allows you to plot the initial point precisely and then use the slope to determine subsequent points, ensuring an accurate and precise graph of the line.

Additional Resources

Graph A Line That Contains A Point (-6,1) And Has A Slope Of 5

Understanding how to graph a straight line given specific information about its position and inclination is a fundamental skill in algebra and coordinate geometry. Today, we delve into the process of graphing a particular line that passes through the point (-6, 1) and has a slope of 5. This exercise not only enhances comprehension of linear equations but also provides insight into the practical applications of these concepts in various fields such as engineering, computer graphics, and data analysis. By breaking down each step with clarity and precision, we aim to make this topic accessible and engaging for learners at different levels.

The Foundations: Understanding the Slope-Point Form of a Line

Before diving into the specific line described, it's essential to establish a clear understanding of the fundamental principles involved in graphing a line with given conditions.

What is the Slope of a Line?

In simple terms, the slope of a line measures its steepness. It is calculated as the ratio of the vertical change (rise) to the horizontal change (run) between any two points on the line. Mathematically, it’s expressed as:

$$ m = \frac{\Delta y}{\Delta x} $$

where:
- Δy is the change in the y-coordinate,
- Δx is the change in the x-coordinate.

A positive slope indicates the line ascends from left to right, while a negative slope indicates it descends.

The Point-Slope Form Equation

The point-slope form of a linear equation is a powerful tool when a specific point and slope are known:

$$ y - y_1 = m(x - x_1) $$

where:
- (x_1, y_1) is a known point on the line,
- m is the slope.

This form allows us to derive the line’s equation directly once the point and slope are known, which then can be graphed with ease.

Applying the Point-Slope Form to the Given Line

Given:
- Point: $(-6, 1)$
- Slope: 5

Using the point-slope form:

$$ y - 1 = 5(x + 6) $$

This is the starting point for our graphing process.

Step-by-Step Derivation of the Equation

Let's expand and rearrange the equation to slope-intercept form, which is often more straightforward for graphing:

1. Expand the right side:

$$ y - 1 = 5x + 30 $$

2. Add 1 to both sides to solve for y:

$$ y = 5x + 31 $$

Now, the equation of the line in slope-intercept form is:

$$ y = 5x + 31 $$

This equation indicates that the line has a slope of 5 and crosses the y-axis at $y = 31$.

Visualizing the Line: Graphing Step-by-Step

To accurately sketch this line on a coordinate plane, follow these systematic steps.

1. Plot the Known Point

Start by marking the point $(-6, 1)$ on the graph:

- Locate -6 on the x-axis.
- From there, move vertically to y = 1.
- Place a point at this coordinate.

This point is a guaranteed point on the line, serving as a reference.

2. Use the Slope to Find Additional Points

Since the slope $m = 5$ (which is $\frac{5}{1}$), it indicates that:

- For every 1 unit increase in x, y increases by 5 units.
- Alternatively, for every 1 unit decrease in x, y decreases by 5 units.

Applying this:

- From the point $(-6, 1)$, move 1 unit to the right (x = -5):

- y-coordinate increases by 5: $1 + 5 = 6$.

- Plot the point $(-5, 6)$.

- Similarly, move 1 unit to the left (x = -7):

- y decreases by 5: $1 - 5 = -4$.

- Plot the point $(-7, -4)$.

These points help establish the steepness and direction of the line.

3. Draw the Line

Using a ruler, draw a straight line through the points $(-6, 1)$, $(-5, 6)$, and $(-7, -4)$. Extend the line across the graph, ensuring it passes through all these points smoothly.

4. Confirming the Graph

To verify accuracy:

- Pick another x-value (say, $x = 0$):

$$y = 5(0) + 31 = 31$$

- Mark the point $(0, 31)$. This point should lie on the line.

- Check if the line passes through this point.

This consistency check ensures the line is correctly graphed.

Real-World Applications and Significance

Understanding how to graph lines with specific points and slopes has practical relevance far beyond classroom exercises.

Engineering and Architecture

Designers often work with slopes to determine the incline of ramps, roofs, or roads. Precise graphing ensures safety and compliance with standards.

Computer Graphics

Rendering lines with specific slopes and points helps in creating accurate visual representations and modeling.

Data Analysis

Linear models describe relationships between variables. Plotting these lines visually aids in interpreting data trends.

Extending the Concept: From Graph to Equation

While graphing is a visual skill, translating the visual back into an algebraic equation is equally important. The process involves:

- Identifying a point on the line.
- Calculating the slope based on known or inferred points.
- Applying the point-slope form or slope-intercept form to derive the equation.

This process bridges geometric intuition and algebraic formalism, reinforcing a comprehensive understanding of linear functions.

Challenges and Tips for Students

Graphing a line with a given point and slope can seem straightforward, but certain challenges may arise:

- Choosing the correct points: Always verify points are on the line.
- Maintaining accuracy: Use a ruler and graph paper to ensure straightness and precision.
- Understanding the slope: Visualize the steepness; remember that a large positive slope means a steep ascent.

Tips:

- Practice plotting multiple points using the slope to develop intuition.
- Convert between different forms of linear equations to enhance flexibility.
- Use technology tools like graphing calculators or software for verification.

Conclusion: Mastering the Art of Graphing Lines

Graphing a line that contains a point like (-6, 1) and has a slope of 5 exemplifies the beautiful interplay between algebra and geometry. Through understanding the fundamental principles, applying methodical steps, and verifying results, learners can develop confidence in their ability to visualize and interpret linear relationships. Such skills are invaluable not only in academic pursuits but also in real-world problem-solving scenarios where precision and clarity are paramount. As you continue exploring the world of linear equations, remember that each graph tells a story—a story of how variables relate, how steepness reflects change, and how mathematics models the world around us with elegance and exactness.

Graph A Line That Contains A Point 6 1 And Has A Slope Of 5

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