Graph A Line With A Slope Of 3/4 That Contains The Point (2,-3)

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Understanding how to graph a line with a specific slope and a given point is a fundamental skill in algebra and coordinate geometry. In this article, we will explore the process of graphing the line that has a slope of 3/4 and passes through the point (2, -3). We will delve into the concepts of slope-intercept form, point-slope form, and how to translate these into a visual graph. Whether you're a student preparing for exams or someone interested in mastering coordinate geometry, this comprehensive guide will equip you with the knowledge to accurately graph such lines.

Understanding the Fundamentals of Line Graphing

What Is a Line Slope?

The slope of a line measures its steepness and direction. It is usually represented by the letter 'm' and is calculated as the ratio of the change in y to the change in x between two points on the line:

$$
\text{slope} (m) = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}
$$

A positive slope indicates the line rises as it moves from left to right, while a negative slope indicates it falls.

What Does the Slope 3/4 Signify?

A slope of 3/4 means that for every 4 units moved horizontally (to the right), the line moves up vertically by 3 units. This ratio defines the line's inclination and can be visualized on the coordinate plane as a relatively gentle upward slope.

Point (2, -3): Significance in Graphing

The point (2, -3) lies on the line, serving as a specific coordinate through which the line passes. It acts as an anchor point to help determine the line's position in the coordinate plane.

Formulating the Equation of the Line

To graph a line accurately, we need its equation. The two most common forms are the slope-intercept form and the point-slope form.

Using the Point-Slope Form

The point-slope form is especially useful when you know the slope and a point on the line:

$$
y - y_1 = m (x - x_1)
$$

Where:
- (x_1, y_1) is the known point,
- m is the slope.

Given:
- $m = \frac{3}{4}$,
- $(x_1, y_1) = (2, -3)$.

Plugging these into the point-slope form:

$$
y - (-3) = \frac{3}{4} (x - 2)
$$

Simplifies to:

$$
y + 3 = \frac{3}{4}(x - 2)
$$

Converting to Slope-Intercept Form

To make the equation more user-friendly for graphing, convert it to slope-intercept form $y = mx + b$:

$$
y + 3 = \frac{3}{4}x - \frac{3}{4} \times 2
$$

$$
y + 3 = \frac{3}{4}x - \frac{3}{2}
$$

Subtract 3 from both sides:

$$
y = \frac{3}{4}x - \frac{3}{2} - 3
$$

Express 3 as $\frac{6}{2}$ to combine with $-\frac{3}{2}$:

\[

y = \frac{3}{4}x - \frac{3}{2} - \frac{6}{2}
\]

\[
y = \frac{3}{4}x - \frac{9}{2}
\]

This is the slope-intercept form of the line:

\[
\boxed{y = \frac{3}{4}x - \frac{9}{2}}
\]

Summary:
- Equation in point-slope form: \(y + 3 = \frac{3}{4}(x - 2) \)
- Equation in slope-intercept form: \(y = \frac{3}{4}x - \frac{9}{2} \)

Step-by-Step Guide to Graphting the Line

Step 1: Plot the Known Point

Start by marking the point (2, -3) on the coordinate plane. This point is guaranteed to be on the line.

Step 2: Use the Slope to Find Additional Points

Since the slope is \(\frac{3}{4} \), from the point (2, -3):

- Move right 4 units (x increases by 4).
- Move up 3 units (y increases by 3).

The new point will be:

\[
(2 + 4, -3 + 3) = (6, 0)
\]

Plot this point as well.

Alternatively, to find a point to the left:

- Move left 4 units (x decreases by 4).
- Move down 3 units (y decreases by 3).

This leads to:

\[
(2 - 4, -3 - 3) = (-2, -6)

\]

Plot this point too.

Step 3: Draw the Line

Using the three points:
- (2, -3)
- (6, 0)
- (-2, -6)

Draw a straight line passing through these points, extending across the coordinate plane. Ensure the line is straight and accurately represents the slope.

Step 4: Verify Your Line

Check that the line passes through all the plotted points and that the slope between any two points matches 3/4.

Additional Tips for Accurate Graphing

- Use graph paper for precision.
- Label the points to avoid confusion.
- Extend the line beyond the plotted points for clarity.
- Use a ruler to draw the line for a clean, straight appearance.
- Confirm the slope by calculating the rise over run between different points.

Real-World Applications of Graphing Lines with Specific Slopes

Understanding how to graph lines with specific slopes and points has numerous applications:

- Physics: Modeling velocity and acceleration.
- Economics: Analyzing cost functions and supply-demand curves.
- Engineering: Designing systems with linear relationships.
- Data Analysis: Trend analysis and regression lines.

Practical Example: Budget Planning

Suppose a small business calculates revenue based on advertising spend. If the revenue increases by 3 dollars for every 4 dollars spent on advertising, the relationship can be represented by a line with slope 3/4. Plotting the line through a known point (like a fixed starting revenue) helps visualize future revenue projections.

Common Mistakes and How to Avoid Them

- Miscalculating the slope: Always verify the rise and run for multiple points.
- Incorrect plotting: Double-check coordinates before drawing the line.
- Ignoring the point: Ensure the line passes through the known point (2, -3).

Conclusion

Graphing a line with a slope of 3/4 that contains the point (2, -3) involves understanding the fundamental principles of coordinate geometry, formulating the correct equation, and accurately plotting the line on a graph. By following the step-by-step process outlined above, you can confidently create precise graphs for any linear equations with given slopes and points. Mastering these skills enhances your ability to interpret and analyze linear relationships in various academic and real-world contexts.

Meta Description: Learn how to graph a line with a slope of 3/4 passing through the point (2, -3). Step-by-step instructions, formulas, and tips for accurate coordinate plotting.

Frequently Asked Questions

What is the equation of the line with a slope of 3/4 passing through the point (2, -3)?

The equation is y = (3/4)x - 9/2.

How do you find the y-intercept of the line given its slope and a point it passes through?

Use the point-slope form: $y - y_1 = m(x - x_1)$. Plug in the slope m = 3/4 and the point (2, -3) to find the y-intercept.

What is the slope of the line that passes through (2, -3) with a slope of 3/4?

The slope is 3/4, as given in the problem statement.

Can you graph the line passing through (2, -3) with a slope of 3/4?

Yes, start at the point (2, -3), then from there, move up 3 units and right 4 units repeatedly to plot the line.

What is the significance of the point (2, -3) on the line with slope 3/4?

It is a specific point through which the line passes, helping to determine the exact equation of the line.

How does the slope of 3/4 affect the steepness of the line?

A slope of 3/4 indicates the line rises 3 units vertically for every 4 units horizontally, showing a moderate incline.

What is the general form of the line's equation given the point (2, -3) and slope 3/4?

The equation in point-slope form is $y + 3 = (3/4)(x - 2)$.

How can I verify that a point lies on the line with slope 3/4 passing through (2, -3)?

Substitute the point's x and y values into the line's equation and check if both sides are equal.

What are some real-world applications of understanding a line with a given slope and point?

Such lines model relationships like speed over time, cost vs. quantity, or other linear relationships in science, economics, and engineering.

Additional Resources

Graph A Line With A Slope Of 3/4 That Contains The Point (2, -3)

Introduction: Unraveling the Geometry of a Specific Linear Equation

In the realm of analytical geometry, the study of lines and their properties forms a fundamental cornerstone. Whether applied in physics, engineering, or pure mathematics, understanding the parameters that define a line—namely, its slope and specific points—enables a deeper comprehension of spatial relationships and algebraic expressions. One such intriguing problem involves analyzing the line characterized by a fixed slope of 3/4 that notably passes through the point (2, -3). This article delves into the detailed exploration of this line, uncovering its equation, properties, and implications within a broader mathematical context.

Foundations of Line Equations in Coordinate Geometry

Before exploring the specific line in question, it is vital to revisit the fundamental concepts that underpin the equation of a straight line in a Cartesian plane.

The Slope-Intercept Form

The most common representation of a line's equation is:

$$y = mx + b$$

where:
- m represents the slope of the line,
- b is the y-intercept, i.e., the point where the line crosses the y-axis.

This form is particularly useful when the slope and y-intercept are known or easily determined.

The Point-Slope Form

Alternatively, when a specific point (x_1, y_1) on the line and the slope m are known, the equation can be expressed as:

$$y - y_1 = m(x - x_1)$$

This form allows for straightforward derivation of the line's equation once a point and slope are specified.

Deriving the Equation of the Line with Slope 3/4 Passing Through (2, -3)

Given the parameters:
- Slope $m = \frac{3}{4}$,
- Point $(x_1, y_1) = (2, -3)$,

the point-slope form becomes:

$$y - (-3) = \frac{3}{4}(x - 2)$$

which simplifies to:

$$y + 3 = \frac{3}{4}(x - 2)$$

To express this in slope-intercept form, expand and simplify:

$$ y + 3 = \frac{3}{4}x - \frac{3}{2} $$

Subtract 3 from both sides:

$$ y = \frac{3}{4}x - \frac{3}{2} - 3 $$

Expressing 3 as $\frac{6}{2}$:

$$ y = \frac{3}{4}x - \frac{3}{2} - \frac{6}{2} $$

$$ y = \frac{3}{4}x - \frac{9}{2} $$

Final Equation of the Line:

$$ y = \frac{3}{4}x - \frac{9}{2} $$

This equation succinctly encapsulates the line's behavior, providing a foundation for further analysis.

Mathematical Properties and Characteristics

Having established the explicit equation, it is essential to examine the line's properties, such as intercepts, slope implications, and potential for intersection with other geometric entities.

Y-Intercept and X-Intercept

- Y-Intercept (b): The point where the line crosses the y-axis occurs when $x=0$:

$$ y = \frac{3}{4}(0) - \frac{9}{2} = -\frac{9}{2} = -4.5 $$

- X-Intercept: The point where the line crosses the x-axis when $y=0$:

Set $y=0$:

$$ 0 = \frac{3}{4}x - \frac{9}{2} $$

Add $\frac{9}{2}$ to both sides:

$$ \frac{3}{4}x = \frac{9}{2} $$

Multiply both sides by 4 to clear the denominator:

$$ 3x = 4 \times \frac{9}{2} = 4 \times 4.5 = 18 $$

Divide both sides by 3:

$$x = \frac{18}{3} = 6$$

Summary of intercepts:

- Y-intercept at $(0, -4.5)$,
- X-intercept at $(6, 0)$.

Slope Analysis

The slope of $\frac{3}{4}$ indicates that for every 4 units the line moves horizontally to the right, it ascends vertically by 3 units. This positive slope reflects an increasing linear relationship, ascending from left to right.

Graphical Visualization and Practical Implications

Visualizing the line on the Cartesian plane provides insight into its spatial orientation and potential applications.

Plotting Key Points

- Point $(2, -3)$, known to lie on the line.
- Y-intercept at $(0, -4.5)$.
- X-intercept at $(6, 0)$.

Plotting these points confirms the line's trajectory, emphasizing its gentle incline and intersection points.

Applications and Contexts

Lines with specified slopes and points are foundational in various fields:

- Physics: Modeling constant velocity or acceleration.
- Economics: Representing linear cost or revenue functions.
- Engineering: Analyzing stress-strain relationships.
- Computer Graphics: Rendering linear trajectories.

Understanding the precise equation enables precise predictions and modeling.

Variations and Extended Analyses

Beyond the basic analysis, several extensions and considerations enrich the understanding of this line.

Parameter Variations

- Altering the slope m affects the steepness and orientation.
- Changing the point (x_1, y_1) shifts the line without affecting its slope.

Perpendicular and Parallel Lines

- Parallel Lines: Have the same slope $\frac{3}{4}$, passing through different points.
- Perpendicular Lines: Have slopes that are negative reciprocals, i.e., $-\frac{4}{3}$.

For instance, the line perpendicular to our line passing through $(2, -3)$ would have the equation:

$$y - (-3) = -\frac{4}{3}(x - 2)$$

which simplifies similarly to the previous process.

Intersection with Other Lines or Curves

Analyzing intersections involves solving systems of equations, which can reveal points of concurrency, tangency, or other geometric relationships.

Educational and Pedagogical Significance

This problem exemplifies the process of deriving a line's equation from given parameters, reinforcing core concepts in algebra and geometry. It demonstrates:

- The utility of the point-slope form,
- Conversion between slope-intercept and point-slope forms,
- Calculations of intercepts,
- Graphical interpretation.

Such exercises serve as foundational building blocks for more advanced topics, including analytic geometry, calculus, and vector analysis.

Concluding Thoughts: The Broader Significance of Precise Line Analysis

Understanding the line with slope \(\frac{3}{4} \) through the point \((2, -3)\) is more than an academic exercise; it embodies the analytical approach necessary for solving real-world problems involving linear relationships. From engineering designs to economic modeling, the ability to translate parameters into explicit equations and analyze their properties is invaluable.

Through meticulous derivation and examination, this exploration underscores the importance of foundational geometric principles, paving the way for more complex analytical endeavors. As mathematics continues to evolve, the clarity and precision exemplified in this analysis remain essential tools for scientists, engineers, and educators alike.

In summary, the line characterized by a slope of 3/4 passing through (2, -3) is described by the equation:

\[y = \frac{3}{4}x - \frac{9}{2} \]

Its properties, intercepts, and graphical implications provide a comprehensive understanding of its behavior. Whether used as a teaching example or as a component within larger modeling frameworks, this line embodies the elegance and utility of algebraic geometry.

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