Graph The Line With Slope -1/3 Passing Through The Point (2, 1).

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Understanding how to graph a line given its slope and a point it passes through is a fundamental skill in algebra and coordinate geometry. In this article, we will explore the process of graphing a line with a slope of -1/3 that passes through the point (2, 1). By the end, you'll be equipped with the knowledge and steps necessary to accurately plot this line and grasp the underlying concepts involved.

Understanding the Basics: Slope-Intercept Form and Coordinates

What Is Slope?

Slope is a measure of the steepness of a line, represented mathematically as the ratio of the vertical change (rise) to the horizontal change (run) between two points on the line. It is often denoted by the letter 'm'.

- Positive slope: Indicates the line rises from left to right.
- Negative slope: Indicates the line falls from left to right.
- Zero slope: Represents a horizontal line.
- Undefined slope: Represents a vertical line.

In our case, the slope is -1/3, which indicates a line that declines gently as it moves from left to right.

The Coordinates of a Point

The point (2, 1) specifies a location on the Cartesian plane, with 2 being the x-coordinate and 1 being the y-coordinate. This point will serve as a reference point for graphing the line.

Formulating the Equation of the Line

Using Point-Slope Form

The point-slope form of a linear equation is particularly useful when you know a point on the line and its slope:

$$ y - y_1 = m(x - x_1) $$

Where:
- (x_1, y_1) is a known point on the line.
- m is the slope.

Applying our values:

$$ y - 1 = -\frac{1}{3}(x - 2) $$

Converting to Slope-Intercept Form

To make graphing easier, convert the above equation to slope-intercept form $y = mx + b$:

$$
\begin{aligned}
y - 1 &= -\frac{1}{3}(x - 2) \\
y - 1 &= -\frac{1}{3}x + \frac{2}{3} \\
y &= -\frac{1}{3}x + \frac{2}{3} + 1 \\
y &= -\frac{1}{3}x + \frac{2}{3} + \frac{3}{3} \\
y &= -\frac{1}{3}x + \frac{5}{3}
\end{aligned}
$$

This is the slope-intercept form of the line passing through (2, 1) with a slope of -1/3.

Steps to Graph the Line

1. Plot the Known Point

Begin by marking the point (2, 1) on the Cartesian plane. This point is your starting reference.

2. Use the Slope to Find Another Point

Since the slope is -1/3, for every 3 units you move horizontally, the line will move 1 unit vertically in the opposite direction.

- Moving Right (positive direction):

- From (2, 1), move 3 units to the right:

- $x = 2 + 3 = 5$

- $y = 1 + (-1) \times 1 = 1 - 1 = 0$

- So, the point (5, 0) lies on the line.

- Moving Left (negative direction):

- From (2, 1), move 3 units to the left:

- $x = 2 - 3 = -1$

- $y = 1 + 1 = 2$

- The point (-1, 2) is also on the line.

Plot these points: (5, 0) and (-1, 2).

3. Draw the Line

Using a straightedge, connect the three points: (2, 1), (5, 0), and (-1, 2). Extend the line across the graph, and ensure it passes through all plotted points.

4. Verify the Line

Check that the line maintains the slope of -1/3 by confirming the rise over run between different points on the line.

Additional Techniques and Tips for Accurate Graphing

Using the Equation to Find More Points

Beyond the two points identified, you can choose any x-value, substitute it into the slope-intercept form $y = -\frac{1}{3}x + \frac{5}{3}$, and determine the corresponding y-coordinate.

Example:
- For $x = 0$:

$$
y = -\frac{1}{3} \times 0 + \frac{5}{3} = \frac{5}{3} \approx 1.67
$$

- For $x = 3$:

$$
y = -\frac{1}{3} \times 3 + \frac{5}{3} = -1 + \frac{5}{3} = -1 + 1.67 \approx 0.67
$$

Plot these points (0, 1.67) and (3, 0.67) for a more detailed graph.

Understanding the Slope and Intercepts

The slope of -1/3 indicates the line descends one unit vertically for every three units it moves horizontally to the right. The y-intercept, $b = 5/3$, is where the line crosses the y-axis (at approximately 1.67).

Graphing Tools and Resources

- Graph Paper: Ensures precision and helps visualize the line accurately.
- Graphing Calculators or Software: Tools like Desmos, GeoGebra, and online graph plotters simplify the process.
- Manual Calculation: Always double-check points by substituting x-values into the line equation.

Real-World Applications of Graphing Lines with Negative Slope

Graphing lines with negative slopes has practical implications across various fields:

- Economics: Representing decreasing demand or declining revenue.
- Physics: Describing negative acceleration or deceleration motion.
- Biology: Modeling decreasing populations over time.
- Engineering: Analyzing stress and strain relationships where the response diminishes.

Understanding how to graph such lines helps in interpreting data trends, creating accurate models, and solving real-world problems.

Summary and Key Takeaways

- The slope of -1/3 indicates a line that declines gently as it moves from left to right.
- Using the point (2, 1) and the slope, the line's equation is $y = -\frac{1}{3}x + \frac{5}{3}$.
- Plotting involves marking the known point, using the slope to find additional points, and drawing a straight line through them.

- Additional points can be found by substituting various x-values into the line's equation.
- Understanding the slope and intercepts enhances comprehension of the line's behavior and its real-world applications.

By mastering these steps, you can confidently graph lines with any slope passing through a given point, enriching your algebraic and geometric skills.

Conclusion

Graphing a line with a slope of -1/3 passing through the point (2, 1) is a straightforward yet essential skill in algebra. It involves understanding the slope-intercept form, plotting key points based on the slope, and accurately drawing the line. This process not only solidifies your grasp of coordinate geometry but also prepares you for more complex graphing tasks and data analysis in various scientific and practical contexts. Practice regularly, utilize graphing tools, and analyze the behavior of lines to become proficient in representing linear relationships visually.

Frequently Asked Questions

How do you find the equation of a line with slope -1/3 passing through the point (2, 1)?

Use the point-slope form: $y - y_1 = m(x - x_1)$. Plug in m = -1/3, $x_1 = 2$, $y_1 = 1$: y - 1 = -1/3(x - 2). Simplify to get the equation in slope-intercept form.

What is the slope-intercept form of the line passing through (2, 1) with slope -1/3?

Starting from y - 1 = -1/3(x - 2), distribute and simplify: y - 1 = -1/3 x + 2/3. Then, y = -1/3 x + 2/3 + 1 = -1/3 x + 5/3.

How can I graph the line with slope -1/3 passing through (2, 1)?

Plot the point (2, 1). From there, use the slope -1/3 to find another point: move 3 units right and 1 unit down to reach (5, 0). Draw a straight line through these points.

What is the significance of the slope -1/3 in the line's graph?

A slope of -1/3 indicates the line decreases 1 unit vertically for every 3 units it moves horizontally to the right, showing a gentle downward tilt.

Can the line with slope -1/3 passing through (2, 1) be written in standard form?

Yes. Starting from y - 1 = -1/3(x - 2), multiply both sides by 3 to clear the fraction: 3(y - 1) = - (x - 2). Simplify to get 3y - 3 = -x + 2, then rearranged as x + 3y = 5.

What are some real-world examples where a line with a slope of -1/3 might be applicable?

Such a slope could model situations like a decreasing rate of resource consumption over time, or a scenario where temperature drops at a rate of 1 degree for every 3 hours.

Additional Resources

Graphing the Line with Slope -1/3 Passing Through the Point (2, 1): An In-Depth Exploration

Understanding how to graph a line given its slope and a point it passes through is a fundamental skill in coordinate geometry. In this comprehensive guide, we will delve into the detailed process of graphing the line with a slope of -1/3 that passes through the point (2, 1). This exploration aims to demystify each step, clarify underlying concepts, and provide practical insights to enhance your grasp of line graphing techniques.

Introduction to the Problem

Graphing a straight line in the coordinate plane requires knowledge of two key pieces of information:

- The slope (m): Indicates the steepness and the direction of the line.
- A point on the line (x_1, y_1): A specific coordinate through which the line passes.

In our case:

- Slope (m): -1/3
- Point: (2, 1)

The goal is to accurately plot this line on a coordinate plane, understanding the process from the algebraic formulation to the visual representation.

Understanding the Slope -1/3

The Concept of Slope

The slope of a line measures its inclination or steepness. It is calculated as the ratio of the change in y (rise) to the change in x (run):

$$
m = \frac{\Delta y}{\Delta x}
$$

- A positive slope indicates the line rises as it moves from left to right.
- A negative slope indicates the line falls as it moves from left to right.
- A slope of zero corresponds to a horizontal line.
- An undefined slope (division by zero) corresponds to a vertical line.

Implication of a Slope of -1/3

- The negative sign indicates the line slopes downward from left to right.
- The magnitude 1/3 indicates that for every 3 units moved horizontally to the right, the line drops by 1 unit vertically.

This relatively gentle downward inclination makes the line less steep than, say, a slope of -1 or -2 but clearly sloping downward.

Formulating the Equation of the Line

To graph the line accurately, it's essential to derive its algebraic form. The most straightforward approach is to use the point-slope form:

$$
y - y_1 = m(x - x_1)
$$

where (x_1, y_1) is the known point, and m is the slope.

Applying the Point-Slope Formula

Given:

- Point (2, 1)
- Slope $m = -1/3$

Plugging these into the formula:

$$
y - 1 = -\frac{1}{3}(x - 2)
$$

Converting to Slope-Intercept Form

Expanding and simplifying:

$$
y - 1 = -\frac{1}{3}x + \frac{2}{3}
$$

Adding 1 to both sides:

$$
y = -\frac{1}{3}x + \frac{2}{3} + 1
$$

Express 1 as $\frac{3}{3}$:

$$
y = -\frac{1}{3}x + \frac{2}{3} + \frac{3}{3} = -\frac{1}{3}x + \frac{5}{3}
$$

Final equation in slope-intercept form:

$$
\boxed{y = -\frac{1}{3}x + \frac{5}{3}}
$$

This form makes it easy to identify the slope and y-intercept directly.

Plotting the Line Step-by-Step

1. Plot the Known Point (2, 1)

Begin by locating the point:

- From the origin (0,0), move 2 units to the right.
- From there, move 1 unit up.
- Mark this point clearly; it will serve as a reference for the rest of the graph.

2. Use the Slope to Find a Second Point

Since the slope is -1/3, it provides a pattern for movement:

- For every 3 units moved to the right (positive x-direction), move 1 unit down (negative y-direction).
- Alternatively, for every 3 units to the left, move 1 unit up.

Applying this:

- Starting from (2, 1):

- Move 3 units to the right to x = 5:

- y decreases by 1: y = 1 - 1 = 0

- The second point: (5, 0)

- Or, moving to the left:

- From x = 2 to x = -1 (move left 3 units):

- y increases by 1: y = 1 + 1 = 2

- The second point: (-1, 2)

Plot these points to establish the line's direction.

3. Draw the Line

- Use a ruler to draw a straight line passing through the two points: (2, 1) and (5, 0), or (2, 1) and (-1, 2).
- Extend the line across the graph to cover the coordinate plane as needed.
- Ensure the line is smooth and straight, indicating the continuous nature of a linear function.

Alternative Approaches to Graphing

While the above steps are straightforward, various methods can be employed depending on context and preference.

Method 1: Plotting Using the Equation (y = -1/3 x + 5/3)

- Choose values for x, compute corresponding y-values, and plot points:

- x = 0:

$$
y = -\frac{1}{3} \times 0 + \frac{5}{3} = \frac{5}{3} \approx 1.67
$$

- x = 3:

$$
y = -\frac{1}{3} \times 3 + \frac{5}{3} = -1 + \frac{5}{3} \approx 1.67
$$

- x = 6:

$$
y = -\frac{1}{3} \times 6 + \frac{5}{3} = -2 + \frac{5}{3} \approx -0.33
$$

- Plot these points: (0, 1.67), (3, 1.67), (6, -0.33).

- Connect the points with a straight line.

Method 2: Using Slope-Intercept and Y-Intercept

- Identify the y-intercept (b) from the equation:

$$
y = -\frac{1}{3}x + \frac{5}{3}
$$

- The y-intercept is at $\left(0, \frac{5}{3} \right) \approx (0, 1.67)$. Plot this point.

- Use the slope (-1/3) to find another point:

- From (0, 1.67), move right 3 units, down 1 unit:

$$
(0 + 3, 1.67 - 1) = (3, 0.67)
$$

- Plot (3, 0.67) and draw the line through these points.

Key Features and Characteristics of the Line

Understanding the line's properties enhances comprehension and provides context for its behavior in the coordinate plane.

Slope and Direction

- As established, the slope is -1/3.
- The line slopes downward from left to right, consistent with the negative slope.
- The line’s inclination is gentle, indicating a slow decline.

Y-Intercept (b)

- The y-intercept occurs at $\frac{5}{3} \approx 1.67$.
- This is the point where the line crosses the y-axis (x=0).

X-Intercept

- To find the x-intercept, set y=0 in the slope-intercept form:

$$
0 = -\frac{1}{3}x + \frac{5}{3}
$$

- Solving for x:

$$
\frac{1}{3}x = \frac{5}{3}
$$
$$
x = 5
$$

- The x-intercept is at (5, 0).

Summary of Key Points

Feature	Coordinates or Value
Slope (m)	-1/3
Y-intercept (b)	(0, 5/3) or approximately (0, 1.67)
X-intercept	(5, 0)
Known Point	(2, 1)

Practical Applications and Insights

Understanding how to graph this line is not just an academic exercise but has real-world

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