Graph The Solution To The Inequality In The Number Line:|u-5|_<2

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Understanding inequalities involving absolute values is a fundamental aspect of algebra that helps us analyze the range of possible solutions for various real-world problems. One common inequality that students encounter is |u - 5| < 2. This inequality is a representation of a range of values that the variable u can take, centered around a specific point—in this case, 5. Visualizing such inequalities on a number line provides an intuitive understanding of the solution set and enhances problem-solving skills. In this comprehensive guide, we will explore how to interpret and graph the inequality |u - 5| < 2, along with related concepts, methods, and examples.

Understanding the Absolute Value Inequality |u - 5| < 2

What Does the Absolute Value Represent?

Absolute value refers to the distance of a number from zero on the number line, regardless of its sign. For any real number x, |x| is always non-negative:

- |x| = x, if x ≥ 0
- |x| = -x, if x < 0

When dealing with inequalities such as |u - 5| < 2, the absolute value signifies the distance between u and 5 being less than 2. This means u is within 2 units of 5 on the number line.

Interpreting the Inequality |u - 5| < 2

The inequality |u - 5| < 2 can be read as:

- The distance between u and 5 is less than 2.

This can be translated into a compound inequality:

- -2 < u - 5 < 2

Adding 5 to all parts of the inequality gives:

- 3 < u < 7

Thus, the solution set includes all real numbers u that are greater than 3 and less than 7.

Solving the Inequality |u - 5| < 2

Step-by-Step Solution

1. Start with the inequality:

$|u - 5| < 2$

2. Express the absolute value inequality as a double inequality:

$-2 < u - 5 < 2$

3. Add 5 to all parts to isolate u:

$-2 + 5 < u - 5 + 5 < 2 + 5$

Simplifies to:

$3 < u < 7$

4. Conclusion:

The solution set is all real numbers u such that u is greater than 3 and less than 7.

Note: The inequality is strict (<), so the boundary points 3 and 7 are not included in the solution set.

Graphing the Solution on the Number Line

Visualizing the solution set involves plotting the relevant points on a number line and indicating the range of solutions.

Steps to Graph $|u - 5| < 2$

1. Draw a horizontal number line covering the interval of interest, for example, from 0 to 10.

2. Identify the boundary points:

- $u = 3$
- $u = 7$

3. Plot open circles at $u = 3$ and $u = 7$ because these points are not included in the solution (strict inequality).

4. Shade the region between 3 and 7 to indicate all values satisfying the inequality.

5. Label the graph to clarify the solution set: (3, 7).

This visual shows that all points between 3 and 7 satisfy $|u - 5| < 2$.

Understanding the Solution Set

The solution set is an interval:

- Open interval: (3, 7)

This means that any value of u strictly greater than 3 and less than 7 satisfies the inequality.

Key points to remember:

- The center of the inequality (here, 5) determines the middle point.
- The radius (here, 2) specifies how far u can be from the center.
- The solution set includes all points within that radius, but not including the boundary points if the inequality is strict.

Related Inequalities and Variations

Understanding the basic form $|u - a| < r$ helps in solving more complex inequalities involving absolute values.

1. $|u - a| < r$

- Represents all points u within r units of a.
- Solution: $(a - r, a + r)$

2. $|u - a| > r$

- Represents all points u more than r units away from a.
- Solution: $u < a - r$ or $u > a + r$
- Graphically, two rays extending left and right from the points $a - r$ and $a + r$.

3. $|u - a| \leq r$ or $\geq r$

- Include boundary points; use closed intervals $[a - r, a + r]$.

Examples of Graphing Different Inequalities

Example 1: Graph $|u - 4| > 3$

- Solution: $u < 1$ or $u > 7$
- Graph: open circles at 1 and 7, shading to the left of 1 and to the right of 7.

Example 2: Graph $|u - 6| \leq 2$

- Solution: u [?] [4, 8]
- Graph: closed circles at 4 and 8, shading between them.

Applications of Absolute Value Inequalities

Absolute value inequalities are not just academic exercises—they have practical applications:

- Error tolerance: In measurements, the acceptable deviation from a target value.
- Range of possible outcomes: For variables affected by uncertainties.
- Distance constraints: Ensuring a variable stays within a certain distance from a point.

Example: Suppose a factory produces items with weight u, and the acceptable weight range is within 2 grams of 5 grams. The inequality $|u - 5| < 2$ describes this condition, and graphing it helps visualize the permissible weights.

Summary and Key Takeaways

- The inequality $|u - 5| < 2$ describes all points u on the number line less than 2 units from 5.
- Solving involves translating the absolute value inequality into a double inequality and solving for u.
- The solution set is the open interval (3, 7).
- Graphing involves marking boundary points with open circles and shading the interval between them.
- Similar inequalities can be solved and graphed following the same principles, adjusting the boundary points accordingly.
- Visualizing inequalities on the number line helps in understanding the range of solutions and their applications.

Conclusion

Mastering the art of graphing and interpreting absolute value inequalities like $|u - 5| < 2$ is essential for developing a strong foundation in algebra. It enables students and professionals alike to visualize solutions, understand constraints, and apply these concepts across various fields such as engineering, statistics, and everyday problem-solving. Remember, the key steps include translating the inequality into a double inequality, solving for the variable, and accurately representing the solution on the number line. With practice, these skills become intuitive, empowering you to tackle more complex inequalities with confidence.

Frequently Asked Questions

What is the inequality $|u - 5| < 2$ representing on the number line?

It represents all the points u on the number line that are less than 2 units away from 5, meaning the points between 3 and 7.

How do you solve the inequality $|u - 5| < 2$?

You rewrite it as a compound inequality: $-2 < u - 5 < 2$, then add 5 to all parts to get $3 < u < 7$.

What is the solution set for $|u - 5| < 2$?

The solution set is all real numbers u such that $3 < u < 7$.

How is the solution to $|u - 5| < 2$ represented on the number line?

It is represented as an open interval between 3 and 7, often shown with open circles at 3 and 7 and shading between them.

What does the notation |u - 5| < 2 mean in real-world terms?

It means the value of u is within 2 units of 5, so u is somewhere between 3 and 7.

Can the inequality |u - 5| < 2 be written as two separate inequalities?

Yes, it can be written as u - 5 < 2 and u - 5 > -2, which simplifies to 3 < u < 7.

Additional Resources

Graph The Solution To The Inequality In The Number Line: |u - 5| < 2

Understanding inequalities involving absolute values is fundamental in algebra and provides critical insights into the relationships between numbers. Among these, the inequality |u - 5| < 2 stands out as a classic example of a problem that can be effectively visualized and understood through graphing on the number line. This article delves into the intricacies of this inequality, exploring its meaning, how to solve it algebraically, and how the graphing process offers a clear, visual solution. Whether you're a student seeking clarity or an educator aiming to enhance instructional methods, this comprehensive analysis aims to shed light on the significance of graphing absolute value inequalities.

Understanding Absolute Value and Its Significance

What Is Absolute Value?

Absolute value, denoted as |x|, measures the distance of a number x from zero on the number line, regardless of direction. For example:

- |3| = 3 (since 3 is 3 units from zero)
- |-3| = 3 (since -3 is also 3 units from zero)

This concept is crucial because it transforms the number into a non-negative value that explicitly expresses magnitude without regard to sign.

Why Are Absolute Value Inequalities Important?

Absolute value inequalities compare the magnitude of an expression to a number. They often take forms like |x| < a or |x| > a, which translate into ranges of values that satisfy certain distance constraints from a particular point (often zero). These inequalities are used in various fields, including physics, engineering, and economics, to model real-world scenarios such as tolerances, error margins, and spatial constraints.

Breaking Down the Inequality |u - 5| < 2

Interpreting the Inequality

The inequality $|u - 5| < 2$ can be read as:

- "The distance between u and 5 on the number line is less than 2."

This indicates all points u on the number line that are within 2 units of the point 5, but not including points exactly 2 units away (since the inequality is strict).

Algebraic Solution

To find the solution set algebraically, we leverage the definition of absolute value inequalities:

$|x| < a$ ⇔ $-a < x < a$, for $a > 0$.

Applying this to $|u - 5| < 2$:

1. Rewrite the inequality:

$-2 < u - 5 < 2$

2. Solve for u:

- Add 5 to all parts of the inequality:

$-2 + 5 < u - 5 + 5 < 2 + 5$

$3 < u < 7$

Thus, the solution set for the inequality is all real numbers u such that u is strictly between 3 and 7.

Key Point: The solution set is the open interval (3, 7).

Graphical Representation of the Inequality

Why Graphing Matters

While algebraic solutions are precise, graphing provides an intuitive understanding of the inequality. Visualizing the solution on the number line helps comprehend the range of solutions, especially for more complex inequalities.

Step-by-Step Graphing Process

1. Identify the Center Point: The inequality involves $|u - 5|$, which suggests a "center" at $u = 5$.

2. Determine the Radius: The boundary of the inequality is at a distance of 2 units from 5.

3. Plot the Critical Points:

- Mark points u = 3 and u = 7 on the number line (since 5 - 2 = 3 and 5 + 2 = 7).

4. Draw the Interval:

- Since the inequality is strict (< 2), the boundary points at 3 and 7 are not included, indicated by open circles.

- Shade the region between 3 and 7, representing all u satisfying the inequality.

Visual Summary:

- Open circle at u = 3
- Open circle at u = 7
- Shaded region between 3 and 7

This visual makes it clear that the solution set includes all points strictly between 3 and 7.

Advanced Concepts and Variations

Considering Different Inequality Forms

The approach used for |u - 5| < 2 can be adapted for other inequalities involving absolute values:

- |u - 5| ≤ 2: Includes boundary points 3 and 7; the solution set becomes [3, 7].

- |u - 5| > 2: Represents points more than 2 units away from 5; solution set includes u < 3 or u > 7, visualized as two rays extending outward from the boundary points.

- |u - 5| ≥ 2: Similar to the previous but includes boundary points, leading to solution set [3, 7] and the regions outside these points.

Understanding these variations enriches problem-solving skills, especially when interpreting different real-world constraints.

From Absolute Value to Distance on the Number Line

The absolute value inequality essentially models the concept of distance. For example:

- The inequality |u - 5| < 2 is equivalent to: "u is less than 2 units away from 5."

This distance perspective is useful in physics (e.g., tolerances), computer science (e.g., error margins), and other disciplines that rely on spatial reasoning.

Practical Applications of Graphing Absolute Value Inequalities

Real-World Scenarios

1. Tolerance in Manufacturing:

- Ensuring a part's dimension stays within a certain range, e.g., |measurement - target| < tolerance.

2. Error Margins in Data Analysis:

- Defining acceptable deviations from a mean value.

3. Location-Based Services:

- Finding all points within a certain distance of a landmark.

Graphing these inequalities provides a visual and intuitive understanding of the acceptable ranges, aiding in decision-making and communication.

Educational Benefits

Visual tools like graphs help students:

- Develop a better conceptual understanding of inequalities.
- Connect algebraic solutions with geometric interpretations.
- Enhance problem-solving skills through visual reasoning.

Conclusion: The Power of Graphing in Solving Inequalities

The inequality |u - 5| < 2 exemplifies how algebra and graphing work hand-in-hand to deepen understanding of mathematical concepts. Algebraically, it translates into a simple open interval (3, 7), but graphing elevates this understanding by providing a visual context. The process involves identifying the center point, determining the radius, marking boundary points, and shading the solution region—steps that reinforce the concept of distance and range.

Graphing absolute value inequalities is not merely an academic exercise; it is a powerful tool that bridges abstract mathematics with tangible visual intuition. Whether applied in engineering, physics, computer science, or everyday problem-solving, mastering this approach enhances analytical skills and promotes a more profound comprehension of the relationships between numbers.

Through consistent practice and exploration of various forms of absolute value inequalities, learners can develop a flexible, visual approach to algebra that will serve them well across numerous disciplines. Ultimately, the graphing of |u - 5| < 2 exemplifies the elegance of mathematical visualization in conveying complex concepts simply and effectively.

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