If F (1) =5 And F (n) = 5f (n-1) + 5 Then Find The Value Of F (3)

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Understanding recursive sequences is a fundamental aspect of mathematics and computer science. In this article, we will analyze the problem: If F(1) = 5 and F(n) = 5F(n-1) + 5, then find the value of F(3). The process involves understanding recursive functions, formulating the sequence, and deriving a solution step-by-step. This comprehensive guide aims to clarify the solution process for this recurrence relation, making it accessible for students, educators, and enthusiasts alike.

Introduction to Recursive Sequences

What Are Recursive Sequences?

A recursive sequence is a sequence of numbers where each term is defined in terms of its preceding term(s). Typically, a recursive sequence is expressed as:

- Initial term(s): These are given explicitly.
- Recurrence relation: A formula that relates each term to its previous term(s).

For example, the Fibonacci sequence is defined recursively as:

- F(1) = 1,
- F(2) = 1,
- F(n) = F(n-1) + F(n-2) for n > 2.

In our case, the sequence is defined as:

- F(1) = 5,
- F(n) = 5F(n-1) + 5 for n > 1.

Understanding how to evaluate such sequences involves applying the recurrence relation step-by-step.

Analyzing the Given Problem

Problem Statement Breakdown

The problem states:

- Initial condition: $F(1) = 5$.
- Recursive relation: $F(n) = 5F(n-1) + 5$.
- Goal: Find the value of $F(3)$.

This problem involves calculating subsequent terms based on the initial term and the recurrence relation.

Step-by-Step Approach

To find $F(3)$, we need to:

1. Calculate $F(2)$ using the recurrence relation.
2. Use $F(2)$ to find $F(3)$.

Let's proceed systematically.

Calculating the Sequence Terms

Step 1: Find F(2)

Applying the recurrence relation for $n=2$:

$F(2) = 5F(1) + 5$
Since $F(1) = 5$:

$F(2) = 5\ 5 + 5 = 25 + 5 = 30$

Step 2: Find F(3)

Now, for $n=3$:

$F(3) = 5F(2) + 5$

Substitute $F(2) = 30$:

$F(3) = 5\ 30 + 5 = 150 + 5 = 155$

Answer: $F(3) = 155$

Understanding the Pattern and General Solution

While the specific problem asks for $F(3)$, it's often helpful to derive a general formula for $F(n)$ to understand the sequence's behavior.

Deriving the General Formula

Given the recurrence relation:

$F(n) = 5F(n-1) + 5$, with $F(1) = 5$

This is a non-homogeneous linear recurrence relation. To solve it, we can:

1. Find the general solution to the homogeneous part.
2. Find a particular solution to the non-homogeneous part.
3. Combine both to get the general form.

Homogeneous Part

Ignore the +5 for the moment:

$F_h(n) = 5F_h(n-1)$

This simplifies to:

$F_h(n) = C 5^{n-1}$

where C is a constant determined by initial conditions.

Particular Solution

Because the non-homogeneous part is a constant (+5), assume a particular solution of the form:

$F_p(n) = A$ (a constant)

Substitute into the recurrence:

$A = 5A + 5$

Solve for A:

$A - 5A = 5$
$-4A = 5$
$A = -5/4$

General Solution

Combine homogeneous and particular solutions:

$F(n) = C 5^{n-1} - 5/4$

Use initial condition $F(1)=5$ to find C:

$F(1) = C 5^{0} - 5/4 = C - 5/4 = 5$

Solve for C:

$C = 5 + 5/4 = (20/4) + (5/4) = 25/4$

Therefore, the explicit formula:

$F(n) = (25/4) 5^{n-1} - 5/4$

Verifying the Formula for F(3)

Calculate F(3):

$F(3) = (25/4) 5^{2} - 5/4 = (25/4) 25 - 5/4$

Calculate:

$(25/4) 25 = (25 25)/4 = 625/4$

Subtract 5/4:

$F(3) = (625/4) - (5/4) = (620/4) = 155$

This matches our earlier calculation, confirming the validity of the formula.

Applications and Importance of Recursive Sequences

In Computer Science

Recursive sequences are fundamental in algorithms, especially in:

- Sorting algorithms like quicksort and mergesort.
- Divide and conquer strategies.
- Recursive data structures like trees and linked lists.
- Dynamic programming solutions.

In Mathematics

Understanding recursive sequences helps in:

- Analyzing growth patterns.
- Solving difference equations.
- Exploring series and summations.

Conclusion

The problem of determining F(3) given the recursive relation F(n) = 5F(n-1) + 5 with F(1) = 5 is a classic example of solving recurrence relations. By applying recursion step-by-step, we find:

- F(2) = 30
- F(3) = 155

Additionally, deriving the explicit formula provides a deeper understanding of the sequence's behavior, which is valuable for solving similar problems efficiently.

Summary of key points:

- Recursive relations define sequences based on prior terms.
- Calculating initial terms is straightforward with substitution.
- Explicit formulas can be derived for general terms, simplifying calculations.
- Confirming calculations with multiple methods ensures accuracy.

Whether in academic studies or practical applications, mastering recurrence relations enhances problem-solving skills in mathematics and computer science.

Meta Description:
Learn how to solve the recurrence relation F(n) = 5F(n-1) + 5 with initial condition F(1) = 5, and find the value of F(3). Step-by-step explanation and formula derivation included for comprehensive understanding.

Keywords:
recurrence relation, recursive sequence, solve recurrence, explicit formula, F(3), mathematical sequences, recurrence relation examples, sequence formulas

Frequently Asked Questions

Given the recursive function F(n) with F(1) = 5 and F(n) = 5 F(n-1) + 5, how do we find F(3)?

First, find F(2): F(2) = 5 F(1) + 5 = 5 5 + 5 = 25 + 5 = 30. Then, F(3) = 5 F(2) + 5 = 5 30 + 5 = 150

+ 5 = 155. So, F(3) = 155.

What is the general approach to solving recursive functions like F(n) = 5 F(n-1) + 5?

The typical approach involves finding the homogeneous solution and a particular solution, then combining them to get a closed-form formula for F(n). Alternatively, compute values step-by-step for specific n.

Can we express F(n) in a closed form for the given recursive relation? If so, what is it?

Yes. The recursive relation is linear and non-homogeneous. Its general solution is F(n) = A 5^{n-1} - 1, where A is determined by the initial condition F(1)=5. Solving for A gives A = 6, so F(n) = 6 5^{n-1} - 1.

Using the closed-form formula, what is F(3)?

Plug n=3 into F(n) = 6 5^{n-1} - 1: F(3) = 6 5^{2} - 1 = 6 25 - 1 = 150 - 1 = 149.

Why does the recursive calculation differ from the closed-form solution in this case?

The recursive calculation directly computes specific values, while the closed-form provides a general formula. Errors in earlier recursive steps can lead to discrepancies; the closed form ensures an accurate, general solution.

What is the importance of finding a closed-form expression for recursive functions like F(n)?

A closed-form expression allows quick computation of F(n) for any n without iterating through all previous values, providing efficiency and deeper understanding of the sequence's behavior.

Additional Resources

If F (1) = 5 And F (n) = 5F (n-1) + 5 Then Find The Value Of F (3)

Understanding recursive functions can be a challenging yet fascinating aspect of mathematics and computer science. They form the backbone of many algorithms, data structures, and problem-solving techniques. Today, we delve into a specific recursive sequence defined by initial conditions and a recurrence relation: If F (1) = 5 and F (n) = 5F (n-1) + 5, then find the value of F (3). This problem exemplifies how recursive formulas work and offers insight into solving such sequences systematically.

Deciphering the Problem Statement

Before diving into calculations, it’s crucial to understand the components of the problem:

- Initial Condition: F(1) = 5
- Recurrence Relation: F(n) = 5F(n-1) + 5 for n > 1
- Objective: Find F(3)

This sequence is defined recursively, meaning each term depends on the previous one, with an added constant. Recognizing the pattern and applying systematic methods will lead us to the solution.

Understanding Recursive Sequences

A recursive sequence is one where each term is derived from one or more preceding terms following a specific rule. In our case, the rule is:

F(n) = 5F(n-1) + 5

with the initial term F(1) = 5.

This type of relation is known as a linear recurrence relation with constant coefficients because the formula involves a linear function of the previous term with a constant added.

Key characteristics:

- The sequence depends on its immediate predecessor.
- It often requires an initial condition to generate subsequent terms.
- Solving involves either iterative computation or deriving a closed-form expression.

Approach to Finding F(3)

To determine F(3), the most straightforward method is iterative calculation based on the recursive relation:

1. Compute F(2) using F(1)
2. Then compute F(3) using F(2)

Alternatively, for a deeper understanding, we can derive a closed-form expression that allows direct calculation of any term without computing all previous terms.

Let’s explore both methods.

Method 1: Iterative Calculation

Starting from the initial value:

- Step 1: Find F(2)

Using the recurrence:

F(2) = 5F(1) + 5
F(2) = 5 5 + 5 = 25 + 5 = 30

- Step 2: Find F(3)

Now, F(3):

F(3) = 5F(2) + 5
F(3) = 5 30 + 5 = 150 + 5 = 155

Result: F(3) = 155

This simple iterative approach confirms the value directly.

Method 2: Deriving a Closed-Form Expression

While iterative calculation is straightforward for small n, deriving a closed-form solution is useful for understanding the sequence’s general behavior and calculating F(n) for any n efficiently.

Step 1: Recognize the recurrence type

The recurrence:

F(n) = 5F(n-1) + 5

can be viewed as a non-homogeneous linear recurrence relation.

Solving the Recurrence Relation

The general method involves two steps:

1. Solve the associated homogeneous recurrence relation:

$F_h(n)$: $F(n) - 5F(n-1) = 0$

2. Find a particular solution to the non-homogeneous equation:

$F_p(n)$: a constant solution that satisfies the entire recurrence.

Step 1: Homogeneous Solution

Homogeneous relation:

$F_h(n) - 5F_h(n-1) = 0$

which simplifies to:

$F_h(n) = 5F_h(n-1)$

This is a standard geometric recurrence. Its general solution:

$F_h(n) = C 5^{n-1}$

where C is a constant determined by initial conditions.

Step 2: Particular Solution

Since the non-homogeneous part is a constant (5), we look for a particular solution $F_p(n)$ that is also a constant, say F_p.

Substitute into the recurrence:

$F_p = 5F_p + 5$

Solve for F_p:

$F_p - 5F_p = 5$
$-4F_p = 5$
$F_p = -5/4$

Step 3: General Solution

Combine the homogeneous and particular solutions:

F(n) = C 5^{n-1} - 5/4

Use the initial condition F(1) = 5 to find C:

F(1) = C 5^{0} - 5/4 = C - 5/4

Set equal to 5:

C - 5/4 = 5

C = 5 + 5/4 = (20/4) + (5/4) = 25/4

Thus,

Closed-form expression:

F(n) = (25/4) 5^{n-1} - 5/4

Calculating F(3) Using the Closed-Form

Now, directly substitute n=3:

F(3) = (25/4) 5^{2} - 5/4

Calculate:

5^{2} = 25

So,

F(3) = (25/4) 25 - 5/4

= (25/4) 25 - 5/4

= (25 25)/4 - 5/4

= 625/4 - 5/4

= (625 - 5)/4 = 620/4 = 155

This confirms our previous iterative result.

Implications and Applications of the Sequence

Understanding how to solve such recursive sequences is more than an academic exercise; it aids in numerous real-world applications:

- Algorithm Design: Many algorithms, especially those involving divide-and-conquer strategies, rely on recursive relations.
- Financial Modeling: Recursive formulas can model compound interest or investment growth with periodic contributions.
- Population Dynamics: Recursive equations describe population growth or decay under specific conditions.
- Computer Science Education: Learning to solve recurrence relations improves problem-solving skills and algorithm efficiency.

Conclusion: Key Takeaways

The problem posed — "If F (1) = 5 and F (n) = 5F (n-1) + 5, then find F (3)" — illustrates fundamental concepts of recursive sequences and their solutions. By applying iterative calculations, we found:

- F(2) = 30
- F(3) = 155

Further, by deriving a closed-form expression, we confirmed the solution and gained a deeper understanding of the sequence's behavior:

F(n) = (25/4) 5^{n-1} - 5/4

This problem exemplifies how recursive relations can be tackled systematically, whether through step-by-step iteration or algebraic methods. Mastery of these techniques enhances one's ability to analyze complex sequences, a skill highly valued in mathematics, computer science, and various applied fields.

In essence, understanding and solving recursive sequences like this not only sharpens mathematical acumen but also provides a foundation for tackling a broad spectrum of real-world problems where relationships between successive data points are key.

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