If R(t) = 2e2t, 2e2t, 2te2t , Find T(0), R''(0), And R'(t) R''(t).

If R(t) = 2e^{2t}, 2e^{2t}, 2te^{2t} , Find T(0), R''(0), And R'(t) R''(t).

Understanding how to analyze vector functions such as R(t) is fundamental in calculus, especially in fields like physics, engineering, and computer graphics. In this article, we will explore the steps to find the derivatives of the vector function R(t), evaluate these derivatives at specific points like t=0, and interpret what these derivatives represent in the context of motion or spatial analysis. We will break down the process systematically, explaining the concepts and calculations involved.

Analyzing the Vector Function R(t)

Given the vector function:

$$ R(t) = \langle 2e^{2t},\, 2e^{2t},\, 2t e^{2t} \rangle $$

our objectives include:

- Finding $T(0)$, which typically denotes the unit tangent vector at $t=0$.
- Computing $R''(0)$, the second derivative of R(t) evaluated at $t=0$.
- Determining $R'(t)$ and $R''(t)$, the first and second derivatives of R(t), respectively.
- Understanding how these derivatives relate to motion along a curve, such as velocity, acceleration, and curvature.

Step 1: Compute the First Derivative $R'(t)$

The derivatives are taken component-wise:

$$ R(t) = \langle R_1(t), R_2(t), R_3(t) \rangle = \langle 2e^{2t},\, 2e^{2t},\, 2t e^{2t} \rangle $$

Derivative of each component:

1. $R_1(t) = 2e^{2t}$

$$ R_1'(t) = \frac{d}{dt} (2e^{2t}) = 2 \cdot 2 e^{2t} = 4 e^{2t} $$

2. $R_2(t) = 2e^{2t}$ (same as $R_1(t)$)

$$ R_2'(t) = 4 e^{2t} $$

3. $R_3(t) = 2t e^{2t}$

Using the product rule:

$$ R_3'(t) = \frac{d}{dt}(2t e^{2t}) = 2 \cdot \frac{d}{dt}(t e^{2t}) $$

Applying product rule to $ t e^{2t} $:

$$ \frac{d}{dt}(t e^{2t}) = e^{2t} + t \cdot 2 e^{2t} = e^{2t} + 2t e^{2t} $$

Therefore:

$$ R_3'(t) = 2 (e^{2t} + 2t e^{2t}) = 2 e^{2t} + 4 t e^{2t} $$

Summary of $ R'(t) $:

$$ R'(t) = \langle 4 e^{2t},\, 4 e^{2t},\, 2 e^{2t} + 4 t e^{2t} \rangle $$

Step 2: Compute the Second Derivative $ R''(t) $

Differentiate each component of $ R'(t) $:

1. $ R_1'(t) = 4 e^{2t} $

$$ R_1''(t) = 4 \cdot 2 e^{2t} = 8 e^{2t} $$

2. $ R_2'(t) = 4 e^{2t} $

$$ R_2''(t) = 8 e^{2t} $$

3. $ R_3'(t) = 2 e^{2t} + 4 t e^{2t} $

Differentiate term-by-term:

- $ \frac{d}{dt} (2 e^{2t}) = 4 e^{2t} $
- $ \frac{d}{dt} (4 t e^{2t}) $

Apply product rule:

$$ \frac{d}{dt}(t e^{2t}) = e^{2t} + 2 t e^{2t} $$

Multiply by 4:

$$ 4 (e^{2t} + 2 t e^{2t}) = 4 e^{2t} + 8 t e^{2t} $$

Adding these:

$$ R_3''(t) = 4 e^{2t} + 4 e^{2t} + 8 t e^{2t} = (4 e^{2t} + 4 e^{2t}) + 8 t e^{2t} = 8 e^{2t} + 8 t e^{2t} $$

Summary of $ R''(t) $:

$$R''(t) = \langle 8 e^{2t},\, 8 e^{2t},\, 8 e^{2t} + 8 t e^{2t} \rangle$$

Step 3: Evaluate $R'(t)$ and $R''(t)$ at $t=0$

Evaluate $R'(0)$:

$$R'(0) = \langle 4 e^{0},\, 4 e^{0},\, 2 e^{0} + 4 \times 0 \times e^{0} \rangle = \langle 4,\, 4,\, 2 + 0 \rangle = \langle 4,\, 4,\, 2 \rangle$$

Evaluate $R''(0)$:

$$R''(0) = \langle 8 e^{0},\, 8 e^{0},\, 8 e^{0} + 8 \times 0 \times e^{0} \rangle = \langle 8,\, 8,\, 8 + 0 \rangle = \langle 8,\, 8,\, 8 \rangle$$

Step 4: Find $T(0)$, the Unit Tangent Vector at $t=0$

The unit tangent vector $T(t)$ is given by:

$$T(t) = \frac{R'(t)}{|R'(t)|}$$

where $|R'(t)|$ is the magnitude of $R'(t)$:

$$|R'(t)| = \sqrt{(R_1')^2 + (R_2')^2 + (R_3')^2}$$

At $t=0$:

$$R'(0) = \langle 4, 4, 2 \rangle$$

Calculate magnitude:

$$|R'(0)| = \sqrt{4^2 + 4^2 + 2^2} = \sqrt{16 + 16 + 4} = \sqrt{36} = 6$$

Therefore, the unit tangent vector at $t=0$:

$$T(0) = \frac{1}{6} \langle 4, 4, 2 \rangle = \left\langle \frac{4}{6}, \frac{4}{6}, \frac{2}{6} \right\rangle = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle$$

Interpreting the Derivatives in Context

The derivatives of the vector function $R(t)$ offer insights into the motion along a curve in space:

- First derivative $R'(t)$: Represents the velocity vector, indicating the direction and speed of motion at time t.
- Second derivative $R''(t)$: Represents the acceleration vector, describing how the velocity changes over time.
- Unit tangent vector $T(t)$: Indicates the direction of the curve at point t, normalized to length one.

Understanding these derivatives is crucial in applications such as physics (kinematics), robotics (path planning), and computer graphics (motion modeling).

Summary of Key Results

Quantity	Expression	Evaluation at $t=0$
$R'(t)$	$\langle 4 e^{2t},\, 4 e^{2t},\, 2 e^{2t} + 4 t e^{2t} \rangle$	$\langle 4,\, 4,\, 2 \rangle$
$R''(t)$	$\langle 8 e^{2t},\, 8 e^{2t},\, 8 e^{2t} + 8 t e^{2t} \rangle$	\(\langle 8,\, 8,\,

Frequently Asked Questions

Given the function R(t) = [2e^{2t}, 2e^{2t}, 2te^{2t}], how do we find R(0)?

To find R(0), substitute t = 0 into each component: R(0) = [2e^{0}, 2e^{0}, 20e^{0}] = [21, 21, 0] = [2, 2, 0].

What is R''(0) for the given vector function R(t)?

First, find R'(t), then differentiate again to get R''(t). At t=0, R''(0) can be computed after deriving R'(t).

How do we compute R'(t) for R(t) = [2e^{2t}, 2e^{2t}, 2te^{2t}]?

Differentiate each component: R'(t) = [d/dt(2e^{2t}), d/dt(2e^{2t}), d/dt(2t e^{2t})] = [4e^{2t}, 4e^{2t}, 2e^{2t} + 4t e^{2t}].

What is R''(t), the second derivative of R(t)?

Differentiate R'(t): R''(t) = [8e^{2t}, 8e^{2t}, d/dt(2e^{2t} + 4t e^{2t})] = [8e^{2t}, 8e^{2t}, 4e^{2t} + 4e^{2t} + 8t e^{2t}] = [8e^{2t}, 8e^{2t}, (8 + 8t) e^{2t}].

How do we evaluate R'(0) and R''(0)?

Substitute t=0 into R'(t): R'(0) = [4, 4, 2 + 0] = [4, 4, 2]. For R''(t), R''(0) = [8, 8, (8 + 0) 1] = [8, 8, 8].

What is the significance of these derivatives in understanding the behavior of R(t)?

R'(t) indicates the velocity or rate of change, while R''(t) indicates acceleration. Evaluating at t=0 gives initial velocity and acceleration, helping analyze the initial motion of the system.

Additional Resources

Understanding the Vector Function R(t) = (2e^{2t}, 2e^{2t}, 2t e^{2t}) and Its Derivatives

When exploring the behavior of vector functions such as R(t) = (2e^{2t}, 2e^{2t}, 2t e^{2t}), it's essential to analyze key properties like the initial value, derivatives, and the acceleration vector. These components help us understand the geometric and physical interpretations of the function, such as velocity, acceleration, and the path traced by the curve in space.

In this detailed guide, we will systematically compute:

- The value of R(0),
- The second derivative R''(0),
- The first derivative R'(t),
- The second derivative R''(t).

This comprehensive analysis provides insights not only into the behavior of the function at specific points but also into its overall dynamics.

Understanding the Vector Function R(t)

The function in question is:

$$R(t) = (2e^{2t}, \; 2e^{2t}, \; 2t e^{2t})$$

This is a vector-valued function in three-dimensional space, with each component dependent on the parameter t. The components suggest exponential growth in the first two components, and a product of a linear term with an exponential in the third.

Step 1: Computing R(0)

The initial value of the vector function, $R(0)$, provides the starting point of the curve in space.

Calculation:

- First component:

$$ R_1(0) = 2e^{2 \times 0} = 2e^{0} = 2 \times 1 = 2 $$

- Second component:

$$ R_2(0) = 2e^{0} = 2 \times 1 = 2 $$

- Third component:

$$ R_3(0) = 2 \times 0 \times e^{0} = 0 $$

Result:

$$
\boxed{
R(0) = (2,\, 2,\, 0)
}
$$

This point indicates where the curve begins in the three-dimensional space.

Step 2: Finding the First Derivative $R'(t)$

The first derivative, $R'(t)$, represents the velocity vector of the curve at any point t.

Computing derivatives component-wise:

- Component 1:

$$ R_1(t) = 2e^{2t} $$

Derivative:

$$ R_1'(t) = 2 \times \frac{d}{dt} e^{2t} = 2 \times 2 e^{2t} = 4 e^{2t} $$

- Component 2:

$$ R_2(t) = 2e^{2t} $$

Derivative:

$$ R_2'(t) = 4 e^{2t} $$

- Component 3:

$$ R_3(t) = 2t e^{2t} $$

Derivative (using the product rule):

$$ R_3'(t) = 2 \times e^{2t} + 2t \times 2 e^{2t} = 2 e^{2t} + 4 t e^{2t} $$

Vector form of R'(t):

$$
\boxed{
R'(t) = \left(4 e^{2t}, \; 4 e^{2t}, \; 2 e^{2t} + 4 t e^{2t} \right)
}
$$

Step 3: Computing R'(0)

To evaluate the velocity at t=0:

- First component:

$$ R_1'(0) = 4 e^{0} = 4 \times 1 = 4 $$

- Second component:

$$ R_2'(0) = 4 e^{0} = 4 $$

- Third component:

$$ R_3'(0) = 2 e^{0} + 4 \times 0 \times e^{0} = 2 + 0 = 2 $$

Result:

$$
\boxed{
R'(0) = (4,\, 4,\, 2)
}
$$

This vector tells us the initial velocity of the curve at t=0.

Step 4: Computing the Second Derivative R''(t)

The second derivative R''(t) corresponds to the acceleration vector, describing how the velocity changes over time.

Derivatives of each component:

- Component 1:

$$ R_1'(t) = 4 e^{2t} $$

Derivative:

$$ R_1''(t) = 4 \times 2 e^{2t} = 8 e^{2t} $$

- Component 2:

$$ R_2'(t) = 4 e^{2t} $$

Derivative:

$$ R_2''(t) = 8 e^{2t} $$

- Component 3:

$$ R_3'(t) = 2 e^{2t} + 4 t e^{2t} $$

Derivative (product rule):

$$
\begin{aligned}
R_3''(t) &= \frac{d}{dt} \left(2 e^{2t} + 4 t e^{2t} \right) \\
&= 2 \times 2 e^{2t} + 4 \left(e^{2t} + 2 t e^{2t} \right) \\
&= 4 e^{2t} + 4 e^{2t} + 8 t e^{2t} \\
&= (4 e^{2t} + 4 e^{2t}) + 8 t e^{2t} \\
&= 8 e^{2t} + 8 t e^{2t}
\end{aligned}
$$

Vector form of R''(t):

$$
\boxed{
R''(t) = \left(8 e^{2t}, \; 8 e^{2t}, \; 8 e^{2t} + 8 t e^{2t} \right)
}
$$

Step 5: Computing R''(0)

Evaluating at t=0:

- First component:

$$ R_1''(0) = 8 e^{0} = 8 \times 1 = 8 $$

- Second component:

$$ R_2''(0) = 8 $$

- Third component:

$$ R_3''(0) = 8 e^{0} + 8 \times 0 \times e^{0} = 8 + 0 = 8 $$

Result:

$$
\boxed{
R''(0) = (8,\, 8,\, 8)
}
$$

This vector indicates the initial acceleration of the curve at $t=0$.

Summary of Key Results

Quantity	Result	Explanation
$R(0)$	$(2, 2, 0)$	Starting point of the curve.
$R'(0)$	$(4, 4, 2)$	Initial velocity vector.
$R''(0)$	$(8, 8, 8)$	Initial acceleration vector.

Additional Insights: Derivatives $R'(t)$ $R''(t)$

Understanding the behavior of $R'(t)$ and $R''(t)$ across different t-values can reveal the nature of the curve—whether it's accelerating, decelerating, or changing direction.

Analyzing $R'(t)$:

- The components involve exponential functions, which grow rapidly as t increases.
- The third component includes a linear term multiplied by an exponential, indicating a combination of linear and exponential growth.

Analyzing $R''(t)$:

- All components involve exponential growth, scaled by constants.
- The third component combines exponential and linear terms, highlighting complex acceleration behavior.

Geometric Interpretation:

- The acceleration vector points in the same general direction as the velocity at $t=0$, given their components are proportional.
- The exponential growth suggests the curve is expanding rapidly in space as t increases.

Practical Applications of this Analysis

- Physics: Determining the velocity and acceleration of particles moving along a trajectory described by R(t).
- Engineering: Analyzing the motion of robotic arms or components where the position depends on exponential functions.
- Mathematics: Studying the curvature, torsion, and overall shape of parametric curves.

Final Remarks

This detailed analysis of the vector function R(t) = (2e^{2t}, 2e^{2t}, 2t e^{2t}) illustrates the importance of component-wise differentiation and evaluation at specific points. Computing derivatives at t=0 provides a snapshot of the initial position, velocity, and acceleration, which are fundamental in understanding the dynamic behavior of the system modeled by R(t).

By mastering these derivative calculations and interpretations, you can extend similar techniques to more complex vector functions, enhancing your skills in vector calculus and its applications in science and engineering.

Remember: Always verify your derivatives carefully and consider the geometric implications of your calculations to gain deeper insights into the behavior of vector-valued functions.

If R T 2e2t 2e2t 2te2t Find T 0 R 0 And R T R T

If R T 2e2t 2e2t 2te2t Find T 0 R 0 And R T R T

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