If Tanx = 1/2 , Find SinxThe Values Of X Are In Between 0 And 2

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Understanding the relationship between the trigonometric functions is crucial in solving various mathematical problems, especially when dealing with angles and their corresponding ratios. In this comprehensive guide, we will explore how to find the sine of an angle x when $\tan x = \frac{1}{2}$, with the values of x lying between 0 and 2 radians. This problem involves understanding the fundamental trigonometric identities, the unit circle, and the inverse tangent function. By the end of this article, you'll be equipped with a clear methodology to determine $\sin x$ for the given condition, along with additional insights into related concepts.

Understanding the Basics of Trigonometric Functions

Before delving into the problem, it's essential to review some foundational concepts related to trigonometric functions.

What is the Tangent Function?

- The tangent of an angle x, denoted as $\tan x$, is the ratio of the sine to the cosine of that angle:

$$
\tan x = \frac{\sin x}{\cos x}
$$

- The tangent function is periodic with a period of π radians (180 degrees).

What is the Sine Function?

- The sine of an angle x, denoted as $\sin x$, is the ratio of the length of the side opposite the angle to the hypotenuse in a right-angled triangle.
- On the unit circle, $\sin x$ corresponds to the y-coordinate of the point on the circle at angle x.

Relationship Between $\sin x$, $\cos x$, and $\tan x$

- These functions are interconnected through the fundamental identity:

$$
\sin^2 x + \cos^2 x = 1
$$

- Given $\tan x = \frac{1}{2}$, the goal is to find $\sin x$ knowing this relationship.

Approach to Finding $\sin x$ When $\tan x = \frac{1}{2}$

To find $\sin x$ given $\tan x = \frac{1}{2}$, follow these steps:

Step 1: Use the definition of tangent to express $\sin x$ and $\cos x$

- Recall that:

$$
\tan x = \frac{\sin x}{\cos x}
$$

- Since $\tan x = \frac{1}{2}$, this implies:

$$
\frac{\sin x}{\cos x} = \frac{1}{2}
$$

- Rearranged as:

$$
\sin x = \frac{1}{2} \cos x
$$

Step 2: Use the Pythagorean identity to find $\sin x$ and $\cos x$

- From the identity:

$$
\sin^2 x + \cos^2 x = 1
$$

- Substitute $\sin x = \frac{1}{2} \cos x$:

$$
\left(\frac{1}{2} \cos x \right)^2 + \cos^2 x = 1
$$

- Simplify:

$$
\frac{1}{4} \cos^2 x + \cos^2 x = 1
$$

- Combine like terms:

$$
\left(\frac{1}{4} + 1 \right) \cos^2 x = 1
$$

$$
\frac{5}{4} \cos^2 x = 1
$$

- Solve for $\cos^2 x$:

$$
\cos^2 x = \frac{4}{5}
$$

- Take the square root:

$$
\cos x = \pm \sqrt{\frac{4}{5}} = \pm \frac{2}{\sqrt{5}}
$$

Step 3: Find $\sin x$ using the relation $\sin x = \frac{1}{2} \cos x$

- Plug in the value of $\cos x$:

$$
\sin x = \frac{1}{2} \times \pm \frac{2}{\sqrt{5}} = \pm \frac{1}{\sqrt{5}}
$$

- So, the possible values of $\sin x$ are:

$$
\boxed{\sin x = \pm \frac{1}{\sqrt{5}}}
$$

Determining the Correct Sign of $\sin x$ and $\cos x$

Since x is between 0 and 2 radians (which is approximately 0 to 114.59 degrees), we need to determine the sign of $\sin x$ and $\cos x$ in this interval.

Quadrant Analysis

- The interval from 0 to 2 radians covers:
- First Quadrant: 0 to $\frac{\pi}{2}$ (~0 to 1.57 radians)
- Second Quadrant: $\frac{\pi}{2}$ to π (~1.57 to 3.14 radians)
- Since 2 radians (~114.59 degrees) is less than π, the entire interval from 0 to 2 radians spans the first and part of the second quadrant.

Sign of $\sin x$ and $\cos x$ in these quadrants

- First Quadrant (0 to $\frac{\pi}{2}$):
- Both $\sin x$ and $\cos x$ are positive.
- Second Quadrant ($\frac{\pi}{2}$ to π):
- $\sin x$ is positive.
- $\cos x$ is negative.

Since x is between 0 and 2 radians (~0 to 114.59 degrees), the possible values for x are:
- In the first quadrant: $0 < x < 1.57$ radians, where both $\sin x$ and $\cos x$ are positive.
- In the second quadrant: $1.57 < x < 2$ radians, where $\sin x$ is positive and $\cos x$ is negative.

Given the signs, the specific values for $\sin x$ and $\cos x$ are:

Quadrant	$\sin x$	$\cos x$	$\tan x$
1st	$+ \frac{1}{\sqrt{5}}$	$+ \frac{2}{\sqrt{5}}$	$\frac{1/ \sqrt{5}}{2/ \sqrt{5}} = \frac{1}{2}$
2nd	$+ \frac{1}{\sqrt{5}}$	$- \frac{2}{\sqrt{5}}$	$- \frac{1}{2}$

Since $\tan x = \frac{1}{2}$ is positive, x is in the first quadrant, where both sine and cosine are positive. Therefore, the valid value for $\sin x$ is:

$$
\boxed{\sin x = \frac{1}{\sqrt{5}}}
$$

Similarly, $\cos x = \frac{2}{\sqrt{5}}$.

Calculating the Exact Value of $\sin x$

To provide a more precise answer, convert the value to a decimal or simplified radical form.

- Recall:

$$
\sin x = \frac{1}{\sqrt{5}} \approx 0.4472
$$

- Alternatively, rationalize the denominator:

$$
\sin x = \frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}
$$

- So, the exact value is:

$$
\boxed{\sin x = \frac{\sqrt{5}}{5}}
$$

Finding the Corresponding Angles x in Radians

To determine the actual angles, we use the inverse tangent function.

Step 1: Use the inverse tangent to find x

- Since $\tan x = \frac{1}{2}$,

$$
x = \arctan \left(\frac{1}{2} \right)
$$

Frequently Asked Questions

If tan(x) = 1/2 and x is between 0 and 2 radians, what is the value of sin(x)?

sin(x) = (1/2) cos(x). To find sin(x), first find cos(x) using tan(x) = sin(x)/cos(x) = 1/2. Using the Pythagorean identity, cos(x) = ±√(1 / (1 + tan^2(x))) = ±√(1 / (1 + (1/2)^2)) = ±√(1 / (1 + 1/4)) = ±√(1 / (5/4)) = ±√(4/5) = ±(2/√5). Since x is between 0 and 2 radians (approximately 0 to 114.59°), and tan(x) > 0, both sin(x) and cos(x) are positive in the first quadrant. Therefore, cos(x) = 2/√5. Then, sin(x) = tan(x) cos(x) = (1/2) (2/√5) = 1/√5 ≈ 0.447.

How do I find sin(x) given tan(x) = 1/2 in the interval 0 to 2 radians?

You can find sin(x) by expressing it as sin(x) = tan(x) cos(x). First, find cos(x) using the identity cos(x) = 1 / √(1 + tan^2(x)). With tan(x) = 1/2, cos(x) = 1 / √(1 + (1/2)^2) = 1 / √(1 + 1/4) = 1 / √(5/4) = 2 / √5. Then, sin(x) = tan(x) cos(x) = (1/2) (2/√5) = 1/√5 ≈ 0.447.

What is the value of sin(x) if tan(x) = 1/2 and x is between 0 and 2 radians?

The value of sin(x) is approximately 0.447, calculated as $\sin(x) = 1/\sqrt{5}$.

Can you determine sin(x) from tan(x) = 1/2 without knowing the quadrant?

Yes. Since tan(x) = 1/2 is positive and x is between 0 and 2 radians (roughly 0 to 114.59°), x likely lies in the first quadrant where both sin(x) and cos(x) are positive. Therefore, $\sin(x) \approx 0.447$.

What are the steps to find sin(x) if tan(x) = 1/2 in a restricted interval?

1. Use $\tan(x) = \sin(x)/\cos(x)$ to relate the functions.
2. Find cos(x) using the identity $\cos(x) = 1 / \sqrt{(1 + \tan^2(x))}$.
3. Calculate $\cos(x) = 2 / \sqrt{5}$.
4. Find $\sin(x) = \tan(x) \cos(x) = (1/2)(2/\sqrt{5}) = 1/\sqrt{5}$.
5. Approximate $\sin(x) \approx 0.447$.

Is the value of sin(x) the same in all quadrants for tan(x) = 1/2?

No. Since tangent is positive in both the first and third quadrants, the sine value differs depending on the quadrant. In this case, given the interval 0 to 2 radians, x is likely in the first quadrant where sin(x) is positive, approximately 0.447.

How does the interval 0 to 2 radians influence the value of sin(x) when tan(x) = 1/2?

In the interval 0 to 2 radians (roughly 0 to 114.59°), tan(x) = 1/2 indicates x is in the first quadrant, where both sin(x) and cos(x) are positive. This confirms that $\sin(x) \approx 0.447$.

If tan(x) = 1/2, what is the exact value of sin(x)?

The exact value of sin(x) is $1/\sqrt{5}$, which simplifies to $\sqrt{5}/5$.

Additional Resources

If Tan x = 1/2, Find Sin x: The Values of x Are Between 0 and 2π

Introduction

Trigonometry, a fundamental branch of mathematics, deals with the relationships between the angles and sides of triangles. It plays an integral role in various scientific and engineering domains, from physics to computer graphics. A common problem encountered in trigonometry involves determining unknown trigonometric functions given specific information about one function.

One such problem is: If tan x = 1/2, find sin x, and determine the values of x in the interval [0, 2π). Although seemingly straightforward, this problem involves understanding the relationship between tangent and sine functions, as well as the significance of the interval in which the solutions lie.

This detailed investigation aims to dissect this problem thoroughly, exploring the underlying concepts, step-by-step solutions, and broader implications of such trigonometric equations.

Fundamental Concepts and Preliminaries

Before delving into the problem's solution, it is essential to revisit some core trigonometric concepts and identities.

1. Definitions of Trigonometric Functions

- Sine (sin x): Ratio of the length of the side opposite angle x to the hypotenuse.
- Tangent (tan x): Ratio of the length of the side opposite x to the side adjacent to x, or equivalently, sin x / cos x.

2. Basic Identities

- Pythagorean Identity:
$$
\sin^2 x + \cos^2 x = 1
$$
- Relation between tan x and sin x, cos x:
$$
\tan x = \frac{\sin x}{\cos x}
$$

3. The Quadrants and Sign of Trigonometric Functions

In the interval [0, 2π):

Quadrant	x Range	sin x	cos x	tan x
I	0 to π/2	+	+	+
II	π/2 to π	+	-	-
III	π to 3π/2	-	-	+
IV	3π/2 to 2π	-	+	-

Since the tangent value is positive in Quadrants I and III, solutions for tan x = 1/2 will lie in these quadrants.

Step-by-Step Solution Approach

Step 1: Express tan x in terms of sin x and cos x

Given:

$$
\tan x = \frac{1}{2}
$$

Using the relation:

$$
\frac{\sin x}{\cos x} = \frac{1}{2}
$$

which implies:

$$
\sin x = \frac{1}{2} \cos x
$$

Step 2: Use Pythagorean identity to relate sin x and cos x

Substitute $\sin x = \frac{1}{2} \cos x$ into the Pythagorean identity:

$$
\left(\frac{1}{2} \cos x\right)^2 + \cos^2 x = 1
$$

Simplify:

$$
\frac{1}{4} \cos^2 x + \cos^2 x = 1
$$

$$
\left(\frac{1}{4} + 1\right) \cos^2 x = 1
$$

$$
\frac{5}{4} \cos^2 x = 1
$$

$$
\cos^2 x = \frac{4}{5}
$$

Taking square roots:

$$
\cos x = \pm \sqrt{\frac{4}{5}} = \pm \frac{2}{\sqrt{5}} = \pm \frac{2 \sqrt{5}}{5}
$$

Step 3: Find sin x corresponding to these cos x values

Recall:

$$
\sin x = \frac{1}{2} \cos x
$$

Thus:

$$
\sin x = \pm \frac{1}{2} \times \frac{2 \sqrt{5}}{5} = \pm \frac{\sqrt{5}}{5}
$$

Summary of possible solutions:

Cos x	Sin x	Quadrant
$+\frac{2 \sqrt{5}}{5}$	$+\frac{\sqrt{5}}{5}$	Quadrant I
$-\frac{2 \sqrt{5}}{5}$	$-\frac{\sqrt{5}}{5}$	Quadrant III

Since tan x = 1/2 is positive, solutions are in Quadrants I and III:

- Quadrant I: $\cos x > 0$, $\sin x > 0$
- Quadrant III: $\cos x < 0$, $\sin x < 0$

Hence, both solutions are valid in their respective quadrants.

Determining the Exact Values of x

To find the specific angles x in [0, 2π) where these conditions hold, we utilize inverse trigonometric functions.

Step 4: Find principal solutions

From the positive cosine and sine in Quadrant I:

$$
x = \arccos \left(\frac{2 \sqrt{5}}{5}\right)
$$

Similarly, for Quadrant III:

$$
x = \pi + \arccos \left(-\frac{2 \sqrt{5}}{5}\right)
$$

Note: Since $\cos x = -\frac{2 \sqrt{5}}{5}$, the inverse cosine yields an angle in [0, π], and the solutions in Quadrant III are obtained by adding π.

Step 5: Numerical approximations

Calculate $\arccos \left(\frac{2 \sqrt{5}}{5}\right)$:

- $\sqrt{5} \approx 2.236$
- $2 \sqrt{5} \approx 4.472$
- $\frac{2 \sqrt{5}}{5} \approx \frac{4.472}{5} \approx 0.8944$

Thus:

$$
x_1 \approx \arccos(0.8944) \approx 0.4636 \text{ radians}
$$

Similarly:

$$
x_2 \approx \pi + \arccos(-0.8944) \approx \pi + 2.6779 \approx 5.8195 \text{ radians}
$$

Final Solutions

Values of x in $[0, 2\pi)$:

- Solution 1 (Quadrant I):

$$
x \approx 0.464 \text{ radians}
$$

- Solution 2 (Quadrant III):

$$
x \approx 5.820 \text{ radians}
$$

Corresponding sin x values:

- For $x \approx 0.464$:

$$
\sin x \approx \frac{\sqrt{5}}{5} \approx 0.4472
$$

- For $x \approx 5.820$:

$$
\sin x \approx -0.4472
$$

Broader Context and Applications

The process of solving for sin x given tan x is not merely an academic exercise; it underpins numerous practical applications. For instance:

- Engineering and Physics: Understanding wave behaviors where phase angles relate to sine and tangent functions.
- Navigation: Calculating angles based on tangential measurements.
- Computer Graphics: Rotations and transformations often involve trigonometric calculations similar to these.

Moreover, the problem exemplifies the importance of understanding the relationships between different trigonometric functions and their signs in various quadrants, which is pivotal for solving real-world problems involving angles and directions.

Summary and Key Takeaways

- Given \(\tan x = 1/2\), the solutions for \(\sin x\) can be derived using fundamental identities.
- The solutions for x within [0, 2π) are approximately 0.464 radians (Quadrant I) and 5.820 radians (Quadrant III).
- The corresponding sine values are approximately 0.4472 and -0.4472, respectively.
- The problem illustrates the interconnectedness of trigonometric functions and the importance of quadrant analysis in solving equations.

Final Remarks

Understanding the relationship between tangent and sine functions, along with the application of identities, is crucial for tackling more complex trigonometric equations. This exercise not only reinforces core mathematical skills but also enhances problem-solving strategies that are widely applicable across scientific disciplines.

In conclusion, the exploration into "If tan x = 1/2, find sin x" demonstrates the elegance and utility of trigonometric identities, providing insight into the periodic and symmetric nature of these functions within the interval [0, 2π).

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