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Understanding how to solve quadratic equations is a fundamental skill in algebra that opens the door to solving complex mathematical problems. One intriguing example is the equation $(x - 3)^2 = 5$, which invites us to explore the values of x that satisfy this condition. In this article, we'll delve into the methods for solving such equations, interpret what these solutions mean, and discuss related concepts to enhance your understanding of quadratic equations.

Deciphering the Equation $(x - 3)^2 = 5$

Before identifying the solutions, it’s essential to comprehend what the equation represents and how to approach solving it.

Understanding the Structure of the Equation

The equation $(x - 3)^2 = 5$ is a quadratic in disguise. It features a squared binomial on the left side and a constant on the right. The goal is to find all real numbers x that make the equation true.

This type of equation exemplifies a basic quadratic form, where a squared expression equals a constant. Solving it involves applying inverse operations and understanding the properties of squares.

Rewriting the Equation for Clarity

To solve $(x - 3)^2 = 5$, consider the following steps:

1. Recognize that the square of a number equals 5.
2. Take the square root of both sides to find the possible values of $x - 3$.
3. Solve for x by isolating it.

Solving the Equation $(x - 3)^2 = 5$

Let's walk through the process step-by-step.

Step 1: Take the Square Root of Both Sides

Since $(x - 3)^2 = 5$, applying the square root to both sides yields:

$$ \sqrt{(x - 3)^2} = \pm \sqrt{5} $$

which simplifies to:

$$ |x - 3| = \sqrt{5} $$

This step utilizes the property that the square root of a square yields the absolute value of the original expression.

Step 2: Solve for x

Because the absolute value of $x - 3$ equals $\sqrt{5}$, $x - 3$ can be either $\sqrt{5}$ or $-\sqrt{5}$:

- Case 1: $x - 3 = \sqrt{5}$
- Case 2: $x - 3 = -\sqrt{5}$

Adding 3 to both sides in each case gives:

- Case 1: $x = 3 + \sqrt{5}$
- Case 2: $x = 3 - \sqrt{5}$

These are the solutions for the quadratic equation in the real number system.

Final Solutions and Their Significance

Having solved the equation, we now identify the specific values of x that satisfy $(x - 3)^2 = 5$.

Explicit Values of x

The solutions are:

- $x = 3 + \sqrt{5}$
- $x = 3 - \sqrt{5}$

Since $\sqrt{5}$ is approximately 2.236, these solutions are approximately:

- $x \approx 3 + 2.236 = 5.236$
- $x \approx 3 - 2.236 = 0.764$

These values are real numbers, meaning the original equation holds true when x takes these values.

Understanding the Nature of Solutions

Because the original equation involves a perfect square, it naturally has two real solutions. This is consistent with the quadratic nature of the equation and demonstrates the importance of considering both positive and negative roots when solving such equations.

Related Concepts in Quadratic Equations

To deepen your understanding, it’s helpful to explore related topics and methods for solving quadratic equations similar to $(x - 3)^2 = 5$.

Standard Form of Quadratic Equations

Most quadratic equations are expressed in the form:

$$ax^2 + bx + c = 0$$

where a, b, and c are constants. The equation $(x - 3)^2 = 5$ can be expanded and rearranged into this standard form.

Expanding:

$$(x - 3)^2 = x^2 - 6x + 9$$

So, the original equation becomes:

$$x^2 - 6x + 9 = 5$$

Subtract 5 from both sides:

$$ x^2 - 6x + 4 = 0 $$

which is a quadratic in standard form.

Using the Quadratic Formula

For equations in the form $ax^2 + bx + c = 0$, the quadratic formula provides a universal solution:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

Applying this to $x^2 - 6x + 4 = 0$:

- $a = 1$
- $b = -6$
- $c = 4$

Calculate the discriminant:

$$ D = b^2 - 4ac = (-6)^2 - 4 \times 1 \times 4 = 36 - 16 = 20 $$

Since $D > 0$, there are two real solutions:

$$ x = \frac{-(-6) \pm \sqrt{20}}{2 \times 1} = \frac{6 \pm \sqrt{20}}{2} $$

Simplify:

$$ x = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5} $$

which matches the solutions obtained earlier.

Graphical Interpretation

Plotting the quadratic function:

$$ y = (x - 3)^2 $$

and the horizontal line $y = 5$, the points where the parabola intersects the line correspond to the solutions. These intersections occur at:

$$ x = 3 \pm \sqrt{5} $$

visualizing the solutions and understanding the quadratic's symmetry about the vertex at $x = 3$.

Practical Applications of Solving Quadratic Equations

Quadratic equations like $(x - 3)^2 = 5$ are not just academic exercises; they have real-world applications in various fields.

Physics and Engineering

- Calculating projectile trajectories where the height depends on quadratic functions.
- Analyzing motion equations involving acceleration and velocity.

Economics and Finance

- Determining break-even points.
- Optimizing profit functions that are quadratic.

Geometry and Design

- Calculating the dimensions of parabolic structures.
- Designing curves with specific properties.

Summary: Key Takeaways

- The equation $(x - 3)^2 = 5$ has two real solutions: $x = 3 + \sqrt{5}$ and $x = 3 - \sqrt{5}$.
- Solving involves taking the square root of both sides and considering both positive and negative roots.
- The solutions approximately are 5.236 and 0.764, respectively.
- Understanding how to manipulate and solve quadratic equations is crucial for tackling a wide range of mathematical and real-world problems.
- The quadratic formula provides a universal method for solving any quadratic equation, including those derived from equations like $(x - 3)^2 = 5$.

Conclusion

Solving the equation $(x - 3)^2 = 5$ demonstrates fundamental algebraic principles, including the use of square roots, absolute value, and quadratic solutions. Recognizing the structure of such equations and applying the appropriate solving strategies can help you navigate more complex algebraic problems with confidence. Remember, whether through factoring, completing the square, or the quadratic formula, understanding these methods equips you with essential tools for mathematical success.

Keywords: quadratic equations, solve $(x - 3)^2 = 5$, quadratic formula, algebra, solving quadratics, real solutions, quadratic solutions, mathematical problem-solving

Frequently Asked Questions

What does the equation $(x + 3)^2 = 5$ represent in terms of solving for x?

It represents a quadratic equation where you need to find the values of x that satisfy the equation by taking the square root of both sides.

How do you solve the equation $(x + 3)^2 = 5$ for x?

First, take the square root of both sides, giving $x + 3 = \pm\sqrt{5}$. Then, subtract 3 from both sides to find the two solutions: $x = -3 + \sqrt{5}$ and $x = -3 - \sqrt{5}$.

What are the solutions to the equation $(x + 3)^2 = 5$?

The solutions are $x = -3 + \sqrt{5}$ and $x = -3 - \sqrt{5}$.

Is the quadratic equation $(x + 3)^2 = 5$ always true for the solutions found?

Yes, substituting either solution back into the original equation confirms that both satisfy the equation, making them valid solutions.

Can the solutions to $(x + 3)^2 = 5$ be simplified further?

No, the solutions are already in their simplest radical form: $x = -3 \pm \sqrt{5}$.

How is this problem related to solving quadratic equations generally?

It demonstrates the process of solving a quadratic by taking square roots, highlighting the importance of considering both positive and negative roots.

Additional Resources

Mathematics

Introduction: Deciphering the Equation and Its Significance

Mathematics, often regarded as the universal language, offers a structured way to understand and interpret the world around us. Among its many branches, algebra stands out for its ability to translate real-world problems into symbolic representations that can be solved systematically. One such problem—"If (x 3)2 = 5, what values of x make the quadratic equation true?"—might seem straightforward at first glance, but delving deeper reveals a rich landscape of concepts, techniques, and logical reasoning.

This article aims to dissect this problem comprehensively, providing clarity on the underlying principles, solving methods, and the importance of understanding such equations. Whether you're a student sharpening your algebra skills or an enthusiast looking to deepen your mathematical insight, this detailed exploration will guide you through every step of the process.

Understanding the Equation: Breaking Down the Components

The Expression "(x 3)2"

Before jumping into solutions, it's crucial to interpret the notation correctly. The expression "(x 3)2" can be ambiguous if not explicitly clarified. Typically, in algebra, such notation might be shorthand or a typographical error. Common interpretations include:

- $(x + 3)^2$: The square of the sum of x and 3.
- $(x - 3)^2$: The square of the difference between x and 3.
- $(x 3)^2$: The square of three times x.
- (x 3) multiplied by 2: If "2" is outside the parentheses, perhaps indicating multiplication.

Given the context of quadratic equations and the typical structure of such problems, the most probable intended expression is:

"$(x + 3)^2$"

and the entire equation is:

$$
(x + 3)^2 = 5
$$

This form makes sense because squaring binomials is a common quadratic expression, and setting it equal to a constant is a standard problem.

Note: If the original problem intended a different operation, such as $(x - 3)^2$ or $2(x + 3) = 5$, the approach would differ accordingly. However, for this review, we'll proceed with the assumption that the original expression is " $(x + 3)^2 = 5$".

Mathematical Foundations: Quadratic Equations and Their Solutions

What is a Quadratic Equation?

A quadratic equation is a second-degree polynomial equation of the form:

$$
ax^2 + bx + c = 0
$$

where $a \neq 0$, and b, c are constants.

In our interpreted case:

$$
(x + 3)^2 = 5
$$

which, when expanded, becomes:

$$
x^2 + 6x + 9 = 5
$$

or, rearranged:

$$
x^2 + 6x + 4 = 0
$$

This is a standard quadratic form, and its solutions can be found using various methods: factoring, completing the square, or applying the quadratic formula.

Why Are Quadratic Equations Important?

Quadratic equations appear in numerous scientific and engineering contexts, from projectile motion to economic modeling. Solving them accurately enables us to understand the behavior of systems modeled by quadratic relationships.

Step-by-Step Solution Process

Step 1: Clarify the Equation

Assuming the most probable scenario:

$$
(x + 3)^2 = 5
$$

Our goal: find all real values of x satisfying this equation.

Step 2: Take the Square Root of Both Sides

To isolate x, we take the square root of both sides:

$$
x + 3 = \pm \sqrt{5}
$$

This step leverages the property that if $A^2 = B$, then $A = \pm \sqrt{B}$.

Step 3: Solve for x

Subtract 3 from both sides:

$$
x = -3 \pm \sqrt{5}
$$

This yields two solutions:

$$
x = -3 + \sqrt{5}
$$
and
$$
x = -3 - \sqrt{5}
$$

Step 4: Approximate the Solutions

Since $\sqrt{5} \approx 2.236$, the approximate solutions are:

- $x \approx -3 + 2.236 = -0.764$
- $x \approx -3 - 2.236 = -5.236$

These are the two real solutions that satisfy the original equation.

Alternative Approach: Expanding and Applying the Quadratic Formula

Suppose the initial expression was intended as the quadratic form:

$$
(x + 3)^2 = 5

\]

which expands to:

$$
x^2 + 6x + 9 = 5
$$

Subtract 5 from both sides:

$$
x^2 + 6x + 4 = 0
$$

Now, the quadratic is in standard form:

$$
ax^2 + bx + c = 0
$$

where:

- $a = 1$
- $b = 6$
- $c = 4$

Applying the quadratic formula:

$$
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Plugging in the values:

$$
x = \frac{-6 \pm \sqrt{6^2 - 4 \times 1 \times 4}}{2}
$$
$$
x = \frac{-6 \pm \sqrt{36 - 16}}{2}
$$
$$
x = \frac{-6 \pm \sqrt{20}}{2}
$$
$$
x = \frac{-6 \pm 2 \sqrt{5}}{2}
$$
$$
x = -3 \pm \sqrt{5}
$$

which confirms our earlier solutions.

Understanding the Solutions: Real vs. Complex Roots

When do solutions exist?

For quadratic equations, the discriminant ($\Delta = b^2 - 4ac$) determines the nature of solutions:

- If $\Delta > 0$, two real solutions.
- If $\Delta = 0$, one real solution (a repeated root).
- If $\Delta < 0$, two complex solutions.

In our case:

$$
\Delta = 36 - 16 = 20 > 0
$$

So, two real solutions exist, which align with our earlier findings.

Additional Considerations: Variations of the Problem

1. Different Interpretations of the Original Expression

If the problem intended:

- "$(x - 3)^2 = 5$"

then solutions would be:

$$
x - 3 = \pm \sqrt{5}
$$
$$
x = 3 \pm \sqrt{5}
$$

- "$2(x + 3) = 5$"

then:

$$
x + 3 = \frac{5}{2}
$$
$$
x = \frac{5}{2} - 3 = -\frac{1}{2}
$$

or

$$
x + 3 = -\frac{5}{2}
$$

$$
x = -\frac{5}{2} - 3 = -\frac{11}{2}
$$

2. Complex Solutions

In cases where the discriminant is negative, solutions involve imaginary numbers:

$$
x = \frac{-b \pm i \sqrt{|b^2 - 4ac|}}{2a}
$$

which broadens the scope of solutions from real to complex numbers.

Real-World Applications and Significance

Understanding how to solve equations like $(x + 3)^2 = 5$ is essential in multiple domains:

- Physics: Calculating projectile trajectories where quadratic equations describe motion.
- Engineering: Signal processing and control systems often involve quadratic equations.
- Economics: Modeling profit and cost functions that are quadratic.
- Computer Graphics: Calculating intersections of curves and surfaces.

Mastery of such algebraic manipulations enhances problem-solving skills and provides foundational knowledge necessary for advanced studies.

Summary and Key Takeaways

- The equation $(x + 3)^2 = 5$ yields two real solutions: $x = -3 \pm \sqrt{5}$.
- The process involves taking square roots, considering both positive and negative roots.
- Expanding the binomial allows us to convert the problem into a standard quadratic form.
- The discriminant determines the nature of solutions.
- Variations in notation or interpretation can lead to different solutions, emphasizing the importance of clarity in mathematical expressions.

Final Thoughts: Navigating the Mathematical Landscape

Solving quadratic equations, especially those involving squares of binomials, is a fundamental skill that opens doors to understanding more complex systems. The problem discussed exemplifies the importance of careful interpretation, methodical solution steps, and awareness of mathematical properties.

By mastering these techniques, learners not only solve specific equations but also develop logical reasoning and analytical skills applicable across scientific disciplines. Whether dealing with theoretical models or real-world problems, the ability to decode and solve quadratic equations remains an invaluable asset in the mathematician's toolkit.

In essence, understanding "If $(x + 3)^2 = 5$, what values

If X 3 2 5 What Values Of X Make The Quadratic Equation True

If X 3 2 5 What Values Of X Make The Quadratic Equation True

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