If Xy + 8ey = 8e, Find The Value Of Y'' At The Point Where X = 0.

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Understanding how to find the second derivative \(y'' \) at a specific point is a fundamental topic in calculus, especially when dealing with implicit functions. The given equation \(X y + 8 e^y = 8 e \) presents a classic implicit differentiation challenge. In this article, we explore the step-by-step process to determine \(y'' \) at the point where \(X = 0 \). Whether you're a student preparing for exams or a calculus enthusiast, this guide will help clarify the concepts involved and provide a detailed solution.

1. Introduction to Implicit Differentiation

Implicit differentiation is used when a function is defined implicitly rather than explicitly. Instead of expressing \(y \) directly as a function of \(x \), the relationship between them is given by an equation involving both variables.

Key Concepts:

- Differentiating both sides of an equation with respect to \(x \).
- Applying the chain rule when \(y \) is a function of \(x \).
- Solving for derivatives \(y' \) and \(y'' \).

Why is it important?

Implicit differentiation allows us to analyze curves defined by complex relationships without solving explicitly for \(y \).

2. The Given Equation and Its Significance

The equation provided is:

\[
X y + 8 e^y = 8 e
\]

Our goal is to find \(y'' \) at the point where \(X = 0 \). To do this, we need to:

- Find the corresponding \(y \) value when \(X = 0 \).
- Determine \(y' \) at that point.
- Compute \(y'' \) at that point.

This process involves multiple steps, starting with substituting \(X = 0 \) into the original equation.

2.1 Finding the Corresponding y at $X = 0$

Substitute $X = 0$:

$$
0 \cdot y + 8 e^y = 8 e
$$
$$
8 e^y = 8 e
$$

Divide both sides by 8:

$$
e^y = e
$$

Recall that:

$$
e^y = e^1 \Rightarrow y = 1
$$

Thus, at $X = 0$, the corresponding y value is:

$$
\boxed{ y = 1 }
$$

3. Differentiating the Equation to Find y'

To find y', differentiate both sides of the original equation with respect to x, considering y as a function of x.

Original Equation:

$$
X y + 8 e^y = 8 e
$$

Differentiation Steps:

- Differentiate $X y$: product rule
- Differentiate $8 e^y$: chain rule
- The right side is constant, derivative is zero

3.1 Differentiating $X y$

$$
\frac{d}{dx}(X y) = y + X y'
$$

- $\frac{dX}{dx} = 1$
- $\frac{dy}{dx} = y'$

3.2 Differentiating $8 e^y$

$$
\frac{d}{dx}(8 e^y) = 8 e^y y'
$$

3.3 Combining the derivatives

Set the derivatives equal to zero:

$$
y + X y' + 8 e^y y' = 0
$$

Rearranged:

$$
y + y' (X + 8 e^y) = 0
$$

Solve for y':

$$
y' = - \frac{y}{X + 8 e^y}
$$

---

4. Evaluating y' at $X=0, y=1$

Recall from earlier:

- $X = 0$
- $y = 1$

Calculate denominator:

$$
X + 8 e^y = 0 + 8 e^1 = 8 e
$$

Calculate numerator:

$$
- y = -1
$$

Thus,

$$
y' = - \frac{1}{8 e}
$$

Result:

$$
\boxed{ y' \big|_{X=0, y=1} = - \frac{1}{8 e} }
$$

5. Differentiating y' to Find y''

Recall:

$$
y' = - \frac{y}{X + 8 e^y}
$$

To find y'', differentiate y' with respect to x:

$$
y'' = \frac{d}{dx} \left(y' \right)
$$

Since y' is a quotient involving y, X, and y itself, we will apply the quotient rule:

$$
y' = \frac{u}{v}
$$

where

$$
u = - y
$$

$$
v = X + 8 e^y
$$

5.1 Computing u' and v'

- $u = - y \Rightarrow u' = - y'$
- $v = X + 8 e^y$

Differentiate v:

$$
v' = \frac{d}{dx} (X + 8 e^y) = 1 + 8 e^y y'
$$

5.2 Applying the quotient rule

$$
y'' = \frac{v u' - u v'}{v^2}
$$

Substitute u, u', v, v':

$$
y'' = \frac{(X + 8 e^y)(- y') - (- y) (1 + 8 e^y y')}{(X + 8 e^y)^2}
$$

Simplify numerator:

$$
= - (X + 8 e^y) y' + y (1 + 8 e^y y')
$$

---

6. Evaluating y'' at $X=0, y=1$

Recall:

- $y' = - \frac{1}{8 e}$
- $X + 8 e^y = 8 e$

Calculate numerator:

$$
- (8 e) \left(- \frac{1}{8 e} \right) + 1 \left(1 + 8 e \times \left(- \frac{1}{8 e} \right) \right)
$$

Simplify each term:

$$
- (8 e) \times \left(- \frac{1}{8 e} \right) = 1
$$

and

$$
8 e \times \left(- \frac{1}{8 e} \right) = -1
$$

Therefore,

$$
\text{Numerator} = 1 + (1) \times (1 - 1) = 1 + 0 = 1
$$

Denominator:

$$
(8 e)^2 = 64 e^2
$$

---

7. Final Calculation of y''

Putting it all together:

$$
y'' = \frac{1}{64 e^2}
$$

Result:

$$
\boxed{ y'' \big|_{X=0, y=1} = \frac{1}{64 e^2} }
$$

---

8. Summary and Key Takeaways

This problem illustrates the process of implicit differentiation leading to the second derivative y''. The key steps involved:

- Substituting $X=0$ to find the corresponding y
- Calculating the first derivative y' via implicit differentiation
- Applying the quotient rule for the second derivative
- Evaluating all derivatives at the specific point

Summary of Results:

- At $X=0$, $y=1$
- $y' = - \frac{1}{8 e}$
- $y'' = \frac{1}{64 e^2}$

Final answer:

$$
\boxed{
\text{The value of } y'' \text{ at } X=0 \text{ is } \frac{1}{64 e^2}
}
$$

9. Additional Tips for Solving Implicit Differentiation Problems

- Always verify the point of interest by substituting values into the original equation.
- When differentiating, carefully apply the product rule, chain rule, and quotient rule as needed.
- Simplify expressions step-by-step to avoid errors.
- Remember to evaluate derivatives at the specific point after differentiation.

10. Conclusion

Implicit differentiation can initially seem challenging, but with systematic steps,

Frequently Asked Questions

What is the given equation and what are the known values when X = 0?

The equation is XY + 8EY = 8E, and when X = 0, the equation becomes 0 Y + 8EY = 8E, simplifying to 8EY = 8E.

How do you find the value of Y when X = 0 in the equation XY + 8EY = 8E?

Substitute X = 0 into the equation to get 8EY = 8E, then divide both sides by 8E to solve for Y, assuming E ≠ 0.

What assumptions are made about the variable E in solving for Y?

It is assumed that E ≠ 0 because dividing by zero is undefined, so the division step to find Y is valid only if E is not zero.

What is the value of Y at X = 0, given the equation?

Y = 1, provided that E ≠ 0, since dividing both sides of 8EY = 8E by 8E yields Y = 1.

Are there any special cases where the value of Y cannot be determined from this equation?

Yes, if E = 0, the original equation reduces to 8E = 0, and the division step becomes invalid, making Y indeterminate in that case.

How does the value of E influence the value of Y at X = 0?

The value of E determines whether Y can be calculated directly; if E ≠ 0, Y = 1; if E = 0, Y cannot be determined from the given equation.

Can the value of Y be any number if E = 0?

No, if E = 0, the equation simplifies to 0 = 0, which is always true, leaving Y undefined and not constrained by the equation.

Is the solution for Y dependent on the value of X?

At X = 0, Y is determined independently of X; for other values of X, the relationship may differ, but at this point, Y is found solely based on E.

What is the main takeaway when solving for Y at X = 0 in this equation?

The key point is that Y equals 1 when E is non-zero, obtained by simplifying the equation at X = 0, highlighting the importance of E in the solution.

Additional Resources

If xy + 8ey = 8e, find the value of y'' at the point where x = 0 is a classic problem in calculus that involves implicit differentiation and the application of second derivatives. This type of problem is common in advanced calculus courses, especially when dealing with implicit functions, and it requires a systematic approach to find the second derivative of y with respect to x at a specific point. In this comprehensive guide, we will walk through the entire process step-by-step, providing clarity on each stage of differentiation, substitution, and calculation, so you can confidently tackle similar problems.

Understanding the Problem

Before diving into the solution, it's essential to understand what the problem asks:

- You're given an implicit relation: xy + 8ey = 8e
- You need to find y'', the second derivative of y with respect to x, at the point where x = 0.

Implicit differentiation is necessary because y is defined implicitly in terms of x. The challenge involves differentiating this relation twice to obtain y'' and then substituting the point's coordinates to compute the value.

Step 1: Recognize the Given Equation

The given equation is:

$$ xy + 8e^y = 8e $$

where:
- x and y are variables,
- e is the base of natural logarithms (approximately 2.71828).

Our goal is to find y'' at the point where x = 0.

Step 2: Find the Corresponding y-value When x = 0

Before differentiating, it's helpful to find the y-coordinate corresponding to x = 0, to facilitate later substitution.

Set x = 0:

$$ (0) \times y + 8e^y = 8e $$
$$ 8e^y = 8e $$
$$ e^y = e $$
$$ y = 1 $$

Thus, the point of interest is (x, y) = (0, 1).

Step 3: Differentiate Implicitly to Find y'

Differentiating the Equation

Differentiate both sides of the original equation with respect to x:

$$ xy + 8e^y = 8e $$

Note that 8e is a constant; its derivative is 0.

Applying the product rule to xy:

$$ \frac{d}{dx}(xy) = y + x \frac{dy}{dx} $$

And the derivative of $8e^y$ with respect to x:

$$ 8e^y \frac{dy}{dx} $$

Putting it all together:

$$ y + x \frac{dy}{dx} + 8e^y \frac{dy}{dx} = 0 $$

Step 4: Solve for y'

Rearranging terms:

$$ y + \left(x + 8e^y \right) \frac{dy}{dx} = 0 $$

So,

$$ \left(x + 8e^y \right) \frac{dy}{dx} = - y $$

Finally,

$$ \frac{dy}{dx} = y' = \frac{ - y }{ x + 8e^y } $$

Evaluate y' at the point (0, 1):

Substitute x = 0 and y = 1:

$$ y' = \frac{ -1 }{ 0 + 8e^{1} } = \frac{ -1 }{ 8e } $$

Step 5: Differentiate y' to Find y''

Next, differentiate y' with respect to x to find y''. Recall:

$$ y' = \frac{ - y }{ x + 8e^y } $$

This is a quotient involving y, which itself depends on x, so we'll need to use the quotient rule or differentiate step-by-step using the chain rule.

Express y' as:

$$ y' = \frac{N}{D} $$

where:

- $ N = - y $
- $ D = x + 8e^y $

Step 6: Compute derivatives of numerator and denominator

- $ N = - y \Rightarrow N' = - y' $
- $ D = x + 8e^y $

Derivative of D:

$$ D' = 1 + 8 e^y y' $$

(because $ d/dx (8e^y) = 8 e^y y' $ via chain rule).

Step 7: Apply the Quotient Rule for y''

Recall the quotient rule:

$$ y'' = \frac{d}{dx} \left(\frac{N}{D} \right) = \frac{N' D - N D'}{ D^2 } $$

Substitute:

$$ N' = - y' $$
$$ D' = 1 + 8 e^y y' $$

Thus,

$$ y'' = \frac{ (- y') (x + 8 e^y) - (- y) (1 + 8 e^y y') }{ (x + 8 e^y)^2 } $$

Simplify numerator:

$$ y'' = \frac{ - y' (x + 8 e^y) + y (1 + 8 e^y y') }{ (x + 8 e^y)^2 } $$

Step 8: Substitute x = 0, y = 1, and y' into the expression

Recall:

- $ y' = - \frac{1}{8 e} $
- $ y = 1 $
- $ x = 0 $

Compute:

- $ D = 0 + 8 e^{1} = 8 e $

Now, plug into the numerator:

$$ - y' (x + 8 e^y) + y (1 + 8 e^y y') $$

Calculate each part:

- $ y' = - \frac{1}{8 e} $
- $ x + 8 e^y = 8 e $

So,

$$ - \left(- \frac{1}{8 e} \right) \times 8 e + 1 \times \left(1 + 8 e \times \left(- \frac{1}{8 e} \right) \right) $$

Simplify step-by-step:

1. First term:

$$ - \left(- \frac{1}{8 e} \right) \times 8 e = - \left(-1 \right) = 1 $$

2. Second term:

$$ 1 + 8 e \times \left(- \frac{1}{8 e} \right) = 1 - 1 = 0 $$

Now, the numerator:

$$ 1 + 0 = 1 $$

And the denominator:

$$ (x + 8 e^y)^2 = (8 e)^2 = 64 e^2 $$

Final Calculation:

$$
\boxed{
y'' = \frac{1}{64 e^2}
}
$$

This is the value of the second derivative of y with respect to x at the point where x = 0.

Summary of the Solution Steps

1. Identify the point: Find y when x = 0, yielding y = 1.
2. Differentiate implicitly: Find y' in terms of x and y.
3. Evaluate y' at the point: y' at (0, 1) is $-\frac{1}{8 e}$.
4. Differentiate y' to find y'': Use the quotient rule, considering y as a function of x.
5. Substitute known values: Plug in x = 0, y = 1, y' into the expression for y''.
6. Simplify to get the final result: $y'' = \frac{1}{64 e^2}$.

Final Remarks

This problem exemplifies the power of implicit differentiation and the importance of careful substitution and algebraic manipulation. The key steps involve differentiating carefully using the chain rule, applying the quotient rule when necessary, and evaluating all expressions at the given point. Mastery of these techniques is essential for tackling more complex calculus problems involving implicit functions and higher derivatives.

Additional Tips

- Always verify the point of interest by substituting x and y into the original equation.
- When differentiating expressions involving y, remember to apply the chain rule.
- Keep track of each step to avoid mistakes, especially when dealing with multiple derivatives and substitutions.
- Practice similar problems to develop intuition for implicit differentiation and second derivatives.

By following this detailed guide, you should now be comfortable with solving for y'' in implicit relations, especially when determining the precise value at a specific point.

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