If Y=tan U, U=v-1/v, And V=lnx, What Is The Value Of Dy/dx At X=e?

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Understanding how to compute derivatives in complex composite functions is a fundamental skill in calculus, especially in the realms of advanced mathematics, engineering, and physics. When functions are nested within each other—such as in the case of $Y = \tan U$, where U itself depends on V, and V depends on X—it becomes essential to apply the chain rule meticulously. This problem exemplifies the application of multiple derivative rules combined, including the chain rule, product rule, and properties of logarithmic and trigonometric functions.

In this article, we will explore step-by-step how to find $\frac{dy}{dx}$ given the composite functions:
- $Y = \tan U$
- $U = v - \frac{1}{v}$
- $V = \ln x$

and evaluate it specifically at $x = e$. We will delve into the mathematical reasoning behind each step, providing detailed explanations to ensure clarity. This comprehensive guide aims to help students and enthusiasts understand the intricate process of differentiating nested functions and applying calculus principles efficiently.

Understanding the Given Functions and Their Relationships

Before we proceed with differentiation, it’s crucial to understand the definitions and relationships among the functions involved:

- Function $V = \ln x$: This function relates the independent variable x to an intermediate variable V. The natural logarithm function $\ln x$ is differentiable for $x > 0$.

- Function $U = v - \frac{1}{v}$: Here, U depends on V, which in turn depends on x. The function involves both a linear term and a reciprocal term of v.

- Function $Y = \tan U$: The output Y depends on U, which depends on V, and ultimately on x.

Understanding these dependencies is key to applying the chain rule effectively. The goal is to compute $\frac{dy}{dx}$ at the specific point where $x = e$.

Step-by-Step Derivation of $\frac{dy}{dx}$

The process involves multiple steps:

1. Determine $\frac{dy}{du}$: Derivative of $Y = \tan U$ with respect to U.

2. Determine $\frac{du}{dv}$: Derivative of $U = v - \frac{1}{v}$ with respect to V.

3. Determine $\frac{dv}{dx}$: Derivative of $V = \ln x$ with respect to x.

4. Apply the chain rule: Combine these derivatives to find $\frac{dy}{dx}$.

Let's explore each step in detail.

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1. Differentiating $Y = \tan U$ with respect to U

The derivative of $\tan U$ with respect to U is well-known:

$$
\frac{dy}{du} = \sec^2 U
$$

where $\sec U = \frac{1}{\cos U}$. This derivative represents how Y changes with respect to the intermediate variable U.

---

2. Differentiating $U = v - \frac{1}{v}$ with respect to V

Since U depends on V through v, and v is a function of V, the differentiation involves:

$$
\frac{du}{dv} = 1 - \frac{d}{dv}\left(\frac{1}{v}\right)
$$

The derivative of $1/v$ with respect to v is:

$$
\frac{d}{dv}\left(\frac{1}{v}\right) = -\frac{1}{v^2}
$$

Thus,

$$
\frac{du}{dv} = 1 - \left(-\frac{1}{v^2}\right) = 1 + \frac{1}{v^2}
$$

This derivative indicates how U varies with V.

---

3. Differentiating $V = \ln x$ with respect to x

The derivative of $V = \ln x$ with respect to x is straightforward:

$$
\frac{dv}{dx} = \frac{1}{x}
$$

This expresses the rate of change of the logarithmic function relative to x.

---

4. Combining derivatives using the chain rule

Applying the chain rule to find $\frac{dy}{dx}$:

$$
\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dv} \times \frac{dv}{dx}
$$

Substituting the derivatives obtained:

$$
\frac{dy}{dx} = \sec^2 U \times \left(1 + \frac{1}{v^2}\right) \times
\frac{1}{x}

\]

To evaluate \(\frac{dy}{dx}\) at \(x = e\), we need to compute \(U\) and \(v\) at \(x = e\).

Evaluating the Derivative at \(x = e\)

Let's now compute the necessary values at \(x = e\).

Step 1: Find \(V\) at \(x = e\)

\[
V = \ln x \Rightarrow V = \ln e = 1
\]

Step 2: Find \(v\) at \(V = 1\)

Since \(V = \ln x\), and \(v = V\), we have:

\[
v = 1
\]

Step 3: Find \(U\) at \(v = 1\)

\[
U = v - \frac{1}{v} = 1 - \frac{1}{1} = 1 - 1 = 0
\]

Step 4: Compute the derivatives at these values

- \(\sec^2 U\) at \(U=0\):

\[
\sec^2 0 = \left(\frac{1}{\cos 0}\right)^2 = 1^2 = 1
\]

- \(1 + \frac{1}{v^2}\) at \(v=1\):

\[
1 + \frac{1}{1^2} = 1 + 1 = 2
\]

- \(x = e\):

\[
\frac{1}{x} = \frac{1}{e}

\]

Putting it all together:

\[
\left.\frac{dy}{dx}\right|_{x=e} = 1 \times 2 \times \frac{1}{e} =
\frac{2}{e}
\]

Final Answer and Summary

The value of \(\frac{dy}{dx}\) at \(x = e\) for the given nested functions is:

\[
\boxed{\frac{2}{e}}
\]

This result encapsulates the interplay of logarithmic, reciprocal, and trigonometric functions within the calculus framework, demonstrating the power of the chain rule in handling complex derivatives.

Additional Insights and Applications

Calculating derivatives of composite functions such as these is not only an academic exercise but also vital in various real-world applications. Here are some contexts where understanding such derivatives is important:

- Physics: Modeling systems where the rate of change depends on multiple interdependent variables, such as in thermodynamics or quantum mechanics.

- Engineering: Signal processing and control systems often involve nested functions, requiring precise derivative calculations to analyze system stability.

- Economics: Marginal analysis where quantities depend on multiple nested functions, like utility functions or cost functions.

- Mathematics Education: Developing problem-solving skills and deepening understanding of calculus principles.

Conclusion

Mastering the differentiation of nested functions is a fundamental aspect of advanced calculus. By carefully applying the chain rule and understanding the relationships among functions, complex derivatives become manageable. In the specific problem tackled here, we have systematically broken down each component, evaluated at the point of interest, and obtained a precise numerical result.

Remember, practice is key to mastering these techniques. Attempting similar problems with different functions will strengthen your calculus skills and prepare you for more challenging mathematical scenarios. Whether you're a student, a teacher, or a professional, a solid grasp of derivatives in complex functions is an invaluable tool in your mathematical toolkit.

Frequently Asked Questions

Given Y = tan U, with U = v - 1/v and V = ln x, how do you find dy/dx at x = e?

First, find dy/dx by using the chain rule: dy/dx = dy/du du/dv dv/dx. Since Y = tan U, dy/du = sec^2 U. With U = v - 1/v, du/dv = 1 + 1/v^2. And V = ln x, so dv/dx = 1/x. Substitute v = V = ln x, and evaluate at x = e: v = 1. Calculate U at v = 1, then compute dy/dx accordingly.

How do you evaluate dy/dx for Y = tan U, given U = v - 1/v and V = ln x, specifically at x = e?

At x = e, v = ln e = 1. Then U = 1 - 1/1 = 0. dy/du = sec^2 0 = 1. du/dv = 1 + 1/1^2 = 2. dv/dx = 1/x = 1/e. Therefore, dy/dx = 1 2 (1/e) = 2/e.

What is the step-by-step process to differentiate Y = tan U with U = v - 1/v and V = ln x at x = e?

Step 1: Find v = ln x; at x = e, v = 1. Step 2: Compute U = v - 1/v; at v=1, U=0. Step 3: Derive dy/du = sec^2 U; at U=0, dy/du=1. Step 4: Derive du/dv = 1 + 1/v^2; at v=1, du/dv=2. Step 5: Derive dv/dx = 1/x; at x=e, dv/dx=1/e. Multiply all: dy/dx = 1 2 (1/e) = 2/e.

If Y = tan U, U = v - 1/v, and V = ln x, what is the general formula for dy/dx?

dy/dx = dy/du du/dv dv/dx = sec^2 U (1 + 1/v^2) (1/x). To evaluate at a specific x, substitute v = ln x, compute U, and plug in the values accordingly.

At x = e, what is the numerical value of dy/dx for the given functions?

At x = e, v = 1, U = 0, dy/du = 1, du/dv = 2, dv/dx = 1/e. Therefore, dy/dx = 1 2 (1/e) = 2/e.

Why do we evaluate v = ln x at x = e when finding dy/dx in this problem?

Because V = ln x, so substituting x = e gives v = ln e = 1, which is necessary to compute U and the derivatives accurately at that point.

How does the chain rule facilitate differentiation in this multivariable function problem?

The chain rule allows us to differentiate composite functions step-by-step: dy/dx = dy/du du/dv dv/dx, breaking down the problem into manageable parts corresponding to each variable dependency.

Can you summarize the final derivative value at x = e for this problem?

Yes, the value of dy/dx at x = e is 2/e.

What are the key steps to solve for dy/dx in complex chain rule problems like this?

Identify all intermediate functions and derivatives, evaluate each at the specified point, substitute known values (like x = e), and multiply the derivatives following the chain rule to find the final answer.

Additional Resources

What Is The Value Of Dy/dx At X=e?

In the realm of calculus and mathematical analysis, the process of finding derivatives—particularly those involving composite functions—serves as a cornerstone for understanding the behavior of complex systems. Among the myriad of derivative problems, certain expressions stand out due to their layered structure and the rich interplay of functions involved. In this investigation, we will explore the problem: If Y=tan U, U=v - 1/v, and V=ln x, what is the value of dy/dx at x=e? This problem encapsulates the application of the chain rule, the derivatives of logarithmic and tangent functions, and the algebraic manipulation required to evaluate at a specific point.

Understanding the Problem: The Functions and Their Interrelations

Before diving into the derivative calculations, it is essential to understand the functions involved and how they relate to each other.

- $Y = \tan U$
- $U = v - 1/v$
- $V = \ln x$

The goal is to find dy/dx at $x = e$.

The functions are interconnected:

- V is directly related to x via the natural logarithm.
- U depends on v, which in turn will be expressed in terms of V.
- Y depends on U.

This layered dependency necessitates a systematic application of the chain rule.

Step 1: Expressing All Variables in Terms of x

To evaluate dy/dx at $x=e$, we need to express Y directly or via intermediate derivatives in terms of x.

- Since $V = \ln x$, then at $x = e$, $V = \ln e = 1$.
- v is a variable in U, and since $U = v - 1/v$, we need to understand v's relation to V.

However, the problem statement does not specify a direct relation between v and V, but given the typical structure of such problems, it’s reasonable to infer that $v = V$, i.e., v is a function of x via V:

> Assumption: $v = V = \ln x$

This assumption aligns with the common structure where variables are linked through the chain rule. If $v = \ln x$, then:

- $U = v - 1/v = \ln x - 1/\ln x$

- $Y = \tan U = \tan [\ln x - 1/\ln x]$

Our task reduces to finding dy/dx where:

$Y = \tan [\ln x - 1/\ln x]$

Step 2: Computing dy/dx Using the Chain Rule

Given:

$Y = \tan U$, where $U = \ln x - 1/\ln x$

Derivative:

$dy/dx = dY/dx = d/dx [\tan U] = \sec^2 U \, dU/dx$

Thus, the key components are:

1. $U = \ln x - 1/\ln x$
2. Derivative of U with respect to x

Calculating dU/dx

Since:

$U = \ln x - 1/\ln x$

We differentiate term by term:

- $d/dx [\ln x] = 1/x$
- $d/dx [-1/\ln x] = - d/dx [1/\ln x]$

To differentiate $1/\ln x$, we rewrite:

$d/dx [1/\ln x] = d/dx [(\ln x)^{-1}]$

Using the chain rule:

$d/dx [(\ln x)^{-1}] = -1 (\ln x)^{-2} d/dx [\ln x] = -1 (\ln x)^{-2} (1/x) = -(1/x) / (\ln x)^2$

Therefore:

$dU/dx = 1/x - [- (1/x) / (\ln x)^2] = 1/x + (1/x) / (\ln x)^2$

Expressed as:

$dU/dx = (1/x) + (1/x) (1 / (\ln x)^2) = (1/x) [1 + 1 / (\ln x)^2]$

Calculating $\sec^2 U$ at $x = e$

Recall:

$Y = \tan U$, so:

$dy/dx = \sec^2 U \, dU/dx$

At $x = e$, $\ln e = 1$.

Calculate U at $x=e$:

$U = \ln e - 1/\ln e = 1 - 1/1 = 1 - 1 = 0$

Thus,

$U = 0$ at $x = e$

Now,

$\sec^2 U = \sec^2 (0) = (1 / \cos^2 0) = 1 / (1)^2 = 1$

Step 3: Evaluating the Derivative at x=e

Putting it all together:

- dU/dx at $x = e$:

$dU/dx = (1/e) [1 + 1 / (\ln e)^2] = (1/e) [1 + 1 / 1^2] = (1/e) (1 + 1) = (1/e) 2 = 2/e$

- $\sec^2 U$ at $x = e$:

$\sec^2 0 = 1$

Therefore,

dy/dx at $x=e$ $= \sec^2 U \, dU/dx = 1 (2/e) = 2/e$

Final Result and Interpretation

The value of dy/dx at x=e is

dy/dx = 2/e

This result reflects the rate of change of the composite function Y = tan [ln x - 1/ln x] with respect to x at the specific point x = e. It underscores how layered derivatives, when carefully unraveled through systematic application of the chain rule, yield precise insights into the behavior of such functions.

Broader Implications and Applications

This analysis exemplifies the importance of understanding function dependencies and the power of the chain rule in calculus. Such derivative calculations are vital in various fields:

- Physics: Analyzing rates of change in systems with layered dependencies
- Economics: Understanding marginal effects in models involving logarithmic and tangent functions
- Engineering: Signal processing where composite functions model complex behaviors
- Mathematics Education: Demonstrating the step-by-step approach to multi-layered derivatives

Furthermore, the specific evaluation at x=e highlights the significance of choosing points in the domain where calculations simplify, revealing inherent properties of the functions involved.

Conclusion

In solving for dy/dx given the layered functions Y = tan U, U = v - 1/v, and V = ln x, with the assumption v = V = ln x, we systematically applied the chain rule to arrive at the derivative's value at x=e. The process involved:

- Expressing all variables in terms of x
- Computing derivatives step-by-step

- Evaluating the expressions at the specific point

The final answer, dy/dx = 2/e, encapsulates the intricate interplay of logarithmic and trigonometric functions within the calculus framework. This example underscores the elegance and precision achievable through meticulous analytical methods, serving as a testament to the depth and utility of differential calculus in understanding complex functions.

Keywords: derivative, chain rule, tangent function, logarithm, layered functions, calculus, evaluation at x=e

If Y Tan U U V 1 V And V Lnx What Is The Value Of Dy Dx At X E

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