Let Y=[2 6] And U=[6 1], Write Y As The Sum Of A Vector In Span

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Understanding how to express a vector as a sum of vectors within a span is a fundamental concept in linear algebra. This process involves decomposing a given vector into components that lie within certain subspaces or spans of other vectors. In this article, we will explore how to write the vector Y = [2, 6] as the sum of a vector in the span of U = [6, 1] and another vector. We will break down the steps involved, discuss the relevant concepts, and illustrate with detailed examples to clarify the process.

Fundamentals of Vectors and Span

What Is a Vector?

A vector is a mathematical object that has both magnitude and direction. In the context of two-dimensional space, vectors are often represented as ordered pairs, such as [x, y]. They can be added together and scaled by real numbers, following specific rules.

Understanding Span

The span of a set of vectors is the set of all possible linear combinations of those vectors. For example, the span of a single vector U = [6, 1] includes all vectors that can be written as:

$$ c \times U = c \times [6, 1] $$

where c is any real number.

Key Point:
- The span of a set of vectors forms a subspace of the vector space (in this case, $\mathbb{R}^2$).
- Any vector in the span can be expressed as a linear combination of the basis vectors.

Decomposing Vector Y in Terms of U and its Span

Our goal is to express:

$$ Y = [2, 6] $$

as the sum of:

$$ Y = V + W $$

where:

- $V \in \text{Span}(U)$, i.e., $V = c \times U = c \times [6, 1]$, for some scalar c.
- W is a vector orthogonal (or at least not in the span of U), or simply a vector satisfying the decomposition.

In essence, we want to find a scalar c such that:

$$ Y = c \times U + W $$

and W lies outside the span of U.

Step-by-Step Process to Write Y as a Sum of a Vector in Span(U)

Step 1: Find the scalar c such that $c \times U$ is as close as possible to Y

Since $V = c \times U$, our main task is to find c such that:

$$ V = c \times [6, 1] $$

approximates $Y = [2, 6]$.

To find c, we can set up the equation:

$$ Y - V = W $$

which leads to:

$$ [2, 6] - c \times [6, 1] = W $$

If we choose V to be the projection of Y onto U, the residual W will be orthogonal to U. This is a standard approach in projection.

Step 2: Calculate the projection of Y onto U

The projection of Y onto U is given by:

$$
\text{proj}_U (Y) = \left(\frac{Y \cdot U}{U \cdot U} \right) \times U
$$

where:

- $Y \cdot U$ is the dot product of Y and U,
- $U \cdot U$ is the dot product of U with itself.

Calculate the dot products:

$$
Y \cdot U = (2)(6) + (6)(1) = 12 + 6 = 18
$$

$$
U \cdot U = (6)^2 + (1)^2 = 36 + 1 = 37
$$

Now, compute the scalar:

\[

$$
c = \frac{Y \cdot U}{U \cdot U} = \frac{18}{37}
$$

Thus,

$$
V = c \times U = \frac{18}{37} \times [6, 1] = \left[\frac{18}{37} \times 6, \frac{18}{37} \times 1 \right]
$$

which simplifies to:

$$
V = \left[\frac{108}{37}, \frac{18}{37} \right]
$$

Step 3: Find the residual vector W

Now, compute:

$$
W = Y - V = [2, 6] - \left[\frac{108}{37}, \frac{18}{37} \right]
$$

Expressed as:

$$
W = \left[2 - \frac{108}{37}, 6 - \frac{18}{37} \right]
$$

Convert 2 and 6 to fractions with denominator 37:

$$
2 = \frac{74}{37}
$$
$$
6 = \frac{222}{37}
$$

So:

$$
W = \left[\frac{74}{37} - \frac{108}{37}, \frac{222}{37} - \frac{18}{37} \right] = \left[\frac{74 - 108}{37}, \frac{222 - 18}{37} \right]
$$

which simplifies to:

$$
W = \left[-\frac{34}{37}, \frac{204}{37} \right]
$$

Interpreting the Decomposition

The decomposition of Y is:

$$
Y = V + W = \left[\frac{108}{37}, \frac{18}{37} \right] + \left[-\frac{34}{37}, \frac{204}{37} \right]
$$

where:

- $V \in \text{Span}(U)$,
- W is the residual vector orthogonal to U.

This decomposition demonstrates that Y can be expressed as the sum of:

- a scalar multiple of U (the projection component),
- and a residual component W, which is orthogonal to U.

Important:
- If the problem specifically asks to write Y as the sum of a vector in Span(U) and another vector, then this residual W is the explicit complementary vector.
- If the goal is to express Y solely as a sum involving U, then the scalar multiple (projection) suffices, with the residual considered part of the decomposition.

Alternative Approach: Expressing Y as a Linear Combination of U and a Complementary Vector

Suppose we want to write:

$$
Y = a \times U + Z
$$

where:

- a is a scalar,
- Z is orthogonal to U.

This is the projection method, and the previous steps have demonstrated how to compute a and Z.

Summary of steps:

1. Compute the dot product $Y \cdot U$.
2. Compute $U \cdot U$.
3. Find the scalar $a = \frac{Y \cdot U}{U \cdot U}$.
4. Calculate the projection $V = a \times U$.
5. Determine $Z = Y - V$.

This method ensures that:

- $V \in \text{Span}(U)$,
- $Z \perp U$.

Practical Applications of Vector Decomposition

Understanding how to decompose vectors is crucial across multiple disciplines, including:

- Computer Graphics: for transformations and projections.

- Data Science: in principal component analysis and regression.
- Engineering: for signal processing and control systems.
- Physics: in resolving forces and motion components.

By mastering the process of expressing vectors as sums of components in a span, professionals can analyze complex systems, optimize solutions, and interpret data more effectively.

Summary and Key Takeaways

- To express Y = [2, 6] as the sum of a vector in Span(U) and another vector, the projection method is effective.
- The scalar multiple of U that best approximates Y is found via the dot product formula:

$$
a = \frac{Y \cdot U}{U \cdot U}
$$

- The projection vector is:

$$
V = a \times U
$$

- The residual vector:

$$
W = Y - V
$$

completes the decomposition, with $V \in \text{Span}(U)$ and W orthogonal to U.

- This process is fundamental in understanding vector spaces and their subspaces, enabling precise manipulations and

Frequently Asked Questions

How do you express the vector Y = [2, 6] as a sum of a vector in the span of U = [6, 1]?

To express Y as a sum of a vector in the span of U, find a scalar c such that Y = c U. Solving for c: 2 = 6c and 6 = 1c. Since these are different, Y cannot be expressed as a scalar multiple of U, but can be written as the sum of a vector in span(U) and another vector.

What is the process to write Y = [2, 6] as the sum of a vector in span(U) and another vector?

First, find a scalar c such that c U approximates Y. Then, write Y = c U + (Y - c U). The vector c U is in span(U), and (Y - c U) is the residual vector. This residual may or may not be in span(U).

What is the specific vector in span(U) that can be added to another vector

to get Y = [2, 6]?

Since U = [6, 1], find c such that c [6, 1] is close to Y. For example, choose c = 1/3, then c U = [2, 1/3]. The difference Y - c U = [0, 17/3], which is not in span(U). Therefore, Y can be written as the sum of [2, 1/3] in span(U) and the residual [0, 17/3].

Can Y = [2, 6] be written as a linear combination of U = [6, 1]? Why or why not?

No, because Y and U are not scalar multiples of each other, so Y cannot be expressed as a single scalar multiple of U. However, Y can be expressed as a sum involving vectors in span(U) plus a residual vector.

What is the significance of expressing a vector as a sum of vectors in a span?

Expressing a vector as a sum of vectors in a span helps understand its component in the direction of the span and the remaining orthogonal part. It is fundamental in vector decomposition, projections, and solving linear systems.

Additional Resources

Let Y = [2 6] and U = [6 1], Write Y as the Sum of a Vector in Span is a compelling problem that explores fundamental concepts in linear algebra, particularly the ideas of vector spaces, span, and linear combinations. This problem serves as an excellent example for students and enthusiasts aiming to deepen their understanding of how vectors can be decomposed and expressed within specific subspaces. By analyzing how the vector Y can be represented as a sum involving vectors in the span of U, one gains insight into the structure of vector spaces and the practical methods used to manipulate vectors within these spaces.

Understanding the Problem

Before delving into the solution, it's essential to clarify what the problem entails. Given two vectors:

- Y = [2, 6]
- U = [6, 1]

the task is to express Y as a sum of a vector that lies in the span of U and possibly another vector, or more specifically, to write:

$$ Y = v + w $$

where:

- v is a vector in the span of U (meaning v = aU for some scalar a),
- w is a vector in the span of some other vectors or possibly the zero vector, depending on the context.

In many cases, such problems aim to express Y as a linear combination of U and other vectors, often to understand the components of Y relative to a subspace spanned by U.

Key Concepts in Linear Algebra

To approach this problem effectively, a solid grasp of several core concepts is necessary:

Vector Spaces and Subspaces

- A vector space is a collection of vectors where addition and scalar multiplication are defined.
- A subspace is a subset of a vector space that is itself a vector space, closed under addition and scalar multiplication.

Span of a Vector

- The span of a vector U (denoted as span{U}) is the set of all scalar multiples of U:

$$ \text{span}\{U\} = \{ \alpha U : \alpha \in \mathbb{R} \} $$

- Any vector in the span can be written as a linear combination of the basis vectors of that span.

Linear Combinations and Decomposition

- Expressing a vector Y as a sum of vectors in specific spans involves solving for scalars that make such a decomposition possible.
- This process often involves solving linear equations.

Step-by-Step Solution

The goal is to write Y as a sum of a vector in the span of U and possibly another component orthogonal or independent. For simplicity, we'll focus on expressing Y directly as a scalar multiple of U plus a residual vector.

Step 1: Express Y as a scalar multiple of U

Suppose:

$$ Y = \alpha U + W $$

where W is orthogonal (or at least independent) to U.

To find a, we can project Y onto U:

$$
\alpha = \frac{Y \cdot U}{U \cdot U}
$$

- $Y \cdot U$ is the dot product of Y and U.

- $U \cdot U$ is the dot product of U with itself.

Calculating these:

$$
Y \cdot U = (2)(6) + (6)(1) = 12 + 6 = 18
$$
$$
U \cdot U = (6)^2 + (1)^2 = 36 + 1 = 37
$$

Thus:

$$
\alpha = \frac{18}{37}
$$

Step 2: Find the vector in the span of U

The vector v in the span of U is:

$$
v = \alpha U = \frac{18}{37} \times [6, 1] = \left[\frac{18}{37} \times 6, \frac{18}{37} \times 1 \right] = \left[\frac{108}{37}, \frac{18}{37} \right]
$$

Step 3: Compute the residual vector W

Now, W is:

$$
W = Y - v
$$

which gives:

$$
W = [2, 6] - \left[\frac{108}{37}, \frac{18}{37} \right] = \left[2 - \frac{108}{37}, 6 - \frac{18}{37} \right]
$$

Expressed with common denominators:

$$
2 = \frac{74}{37}
$$

$$
6 = \frac{222}{37}
$$

Therefore:

\[
W = \left[\frac{74}{37} - \frac{108}{37}, \frac{222}{37} - \frac{18}{37} \right] = \left[-

\frac{34}{37}, \frac{204}{37} \right]
\]

Final Decomposition

Putting it all together, Y can be expressed as:

$$
Y = v + W = \left[\frac{108}{37}, \frac{18}{37} \right] + \left[-\frac{34}{37}, \frac{204}{37} \right]
$$

with:

- $v \in \text{span}\{U\}$,
- W orthogonal or independent of U (depending on the context, further orthogonal projection can be used).

This decomposition demonstrates how Y relates to U and highlights the usefulness of projections in vector decomposition.

Features, Pros, and Cons of the Method

Features:

- Utilizes dot products to find orthogonal projections.
- Provides a clear geometric interpretation of the vector decomposition.
- Demonstrates the linearity of vector spaces and the concept of span.

Pros:

- Straightforward for low-dimensional vectors.
- Generalizable to higher dimensions with similar methods.
- Offers insight into the structure of vector spaces and linear combinations.

Cons:

- Assumes familiarity with dot products and projections.
- May become computationally intensive with large vectors or multiple basis vectors.
- The orthogonality assumption simplifies the process but may not always be applicable in non-orthogonal contexts.

Extensions and Applications

This problem's principles extend far beyond simple two-vector cases:

- Projection in Machine Learning: Decomposing data vectors into components aligned with feature vectors.

- Signal Processing: Separating signals into components based on basis vectors.
- Computer Graphics: Decomposing movements or transformations into basis vectors.
- Data Compression: Representing data efficiently through basis vectors in vector spaces.

Conclusion

Expressing Y = [2, 6] as a sum of a vector in the span of U = [6, 1] offers a concrete example of fundamental linear algebra concepts. Through projection, we decomposed Y into a component aligned with U and a residual component orthogonal to it. This method exemplifies how vector spaces, spans, and linear combinations intertwine to provide insights into the structure and relationships of vectors in a space.

Such techniques are not only academically interesting but are also practically vital across various scientific and engineering disciplines. Understanding how to decompose vectors into components relative to spans serves as a foundation for more advanced topics like basis transformations, eigenvector analysis, and dimensionality reduction.

In summary, this problem encapsulates the elegance of linear algebra—transforming abstract mathematical notions into tangible tools for analyzing and understanding the geometry of vectors and spaces.

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