Perform The Indicated Operation X^2-9/x^2 X 2x^3/x^2 + 5x + 6.

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When working with algebraic expressions, understanding how to perform the indicated operations correctly is essential for solving complex problems efficiently. In this article, we will explore the step-by-step process of simplifying the expression X^2 - 9 / x^2 × 2x^3 / x^2 + 5x + 6. This involves applying rules of algebra such as order of operations, factoring, and simplifying fractions. Whether you're a student preparing for exams or someone looking to strengthen your algebra skills, this detailed guide will help you master the process.

Understanding the Expression and Its Components

Before diving into the calculations, it’s important to understand the structure of the given expression:

X^2 - 9 / x^2 × 2x^3 / x^2 + 5x + 6

This expression contains multiple parts:
- A quadratic term X^2
- A subtraction involving a fraction - 9 / x^2
- A multiplication × involving another fraction 2x^3 / x^2
- An addition of a trinomial + 5x + 6

The key to simplifying this expression is to handle each part carefully, following the order of operations (PEMDAS/BODMAS).

Step-by-Step Guide to Simplify the Expression

1. Clarify the Expression's Structure

First, ensure you understand the expression's layout. Based on standard algebraic conventions, the expression appears to be:

(X^2 - 9 / x^2) × (2x^3 / x^2) + 5x + 6

Alternatively, if the original expression lacks parentheses, it might be ambiguous. For precise calculation, parentheses are necessary.

Assuming the expression is:

(X^2 - 9) / x^2 × (2x^3) / x^2 + 5x + 6

or similar, the operations change accordingly.

Important: To proceed accurately, we will interpret the expression as:

$(X^2 - 9) / x^2 \times (2x^3) / x^2 + 5x + 6$

which is a common form in algebraic operations.

2. Simplify Each Fraction Separately

Let's analyze each fraction:

- $(X^2 - 9) / x^2$
- $(2x^3) / x^2$

a. Simplify $(X^2 - 9) / x^2$

Notice that $X^2 - 9$ is a difference of squares:

$$X^2 - 9 = (X - 3)(X + 3)$$

So, the fraction becomes:

$$\frac{(X - 3)(X + 3)}{x^2}$$

b. Simplify $(2x^3) / x^2$

Dividing powers of the same base:

$$\frac{2x^3}{x^2} = 2x^{3-2} = 2x$$

3. Rewrite the Expression with Simplified Parts

Now, the entire expression becomes:

$$\left[\frac{(X - 3)(X + 3)}{x^2} \right] \times 2x + 5x + 6$$

or equivalently:

$$\left[\frac{(X - 3)(X + 3)}{x^2} \times 2x \right] + 5x + 6$$

Note: Since multiplication distributes over fractions, rewrite the first part:

$$\frac{(X - 3)(X + 3)}{x^2} \times 2x = \frac{(X - 3)(X + 3) \times 2x}{x^2}$$

4. Simplify the Product in the Numerator

Calculate numerator:

$$ (X - 3)(X + 3) \times 2x $$

Recall that:

$$ (X - 3)(X + 3) = X^2 - 9 $$

Therefore:

$$ (X^2 - 9) \times 2x = 2x (X^2 - 9) $$

Putting it all together:

$$ \frac{2x (X^2 - 9)}{x^2} + 5x + 6 $$

Note: We see the common factor x in numerator and denominator.

5. Simplify the Fraction

Divide numerator and denominator:

$$ \frac{2x (X^2 - 9)}{x^2} = \frac{2x (X^2 - 9)}{x \times x} $$

Cancel one x:

$$ \frac{2 (X^2 - 9)}{x} $$

Now, the expression is:

$$ \frac{2 (X^2 - 9)}{x} + 5x + 6 $$

Recall that $X^2 - 9$ factors into:

$$ X^2 - 9 = (X - 3)(X + 3) $$

So, the expression becomes:

$$ \frac{2 (X - 3)(X + 3)}{x} + 5x + 6 $$

Final Simplified Expression and Interpretation

The simplified form of the expression is:

$$\frac{2 (X - 3)(X + 3)}{x} + 5x + 6$$

This form is easier to interpret and work with for further operations such as solving equations or evaluating for specific values.

Additional Tips for Algebraic Operations

1. Always Factor When Possible

- Recognize difference of squares, common factors, and other factoring techniques to simplify complex expressions efficiently.

2. Keep Track of Variables and Constants

- Be attentive to variables and constants during multiplication and division to prevent errors.

3. Use Parentheses to Clarify Operations

- Proper use of parentheses ensures correct order of operations and avoids ambiguity.

4. Simplify Step-by-Step

- Break down complex expressions into manageable parts before combining.

Conclusion

Performing the indicated operation involving algebraic expressions requires a systematic approach. By understanding the structure of the expression, applying factoring techniques, simplifying fractions, and carefully performing multiplications and divisions, you can arrive at a clean, simplified expression. In this case, we transformed the original complex expression into a more manageable form:

$$\frac{2 (X - 3)(X + 3)}{x} + 5x + 6$$

This process exemplifies essential algebraic skills that are fundamental for mastering higher-level mathematics. Whether for academic purposes or practical problem-solving, mastering these steps enhances your mathematical fluency and confidence.

Keywords: algebraic expressions, simplifying fractions, factoring difference of squares, algebra operations, polynomial simplification, algebra tips, solving expressions

Frequently Asked Questions

What is the simplified form of the expression $(x^2 - 9) / x^2 + 2x^3 / x^2 + 5x + 6$?

First, simplify each term: $(x^2 - 9) / x^2$ simplifies to $1 - 9 / x^2$, and $2x^3 / x^2$ simplifies to $2x$. The entire expression becomes $(1 - 9 / x^2) + 2x + 5x + 6$, which can be further combined as $1 - 9 / x^2 + 7x + 6$.

How can I factor the quadratic polynomial in the expression $5x + 6$?

Since $5x + 6$ is linear, it cannot be factored further unless combined with other terms. If you meant to factor the quadratic part, such as $x^2 - 9$, it factors as $(x - 3)(x + 3)$.

What is the domain of the function given by the expression $(x^2 - 9)/x^2 + 2x^3 / x^2 + 5x + 6$?

The domain excludes any x-values that make the denominators zero. Since denominators include x^2 and $x^2 + 5x + 6$, which factors as $(x + 2)(x + 3)$, the excluded values are $x = 0$, $x = -2$, and $x = -3$.

How do I perform the division of $(x^2 - 9)$ by x^2?

Divide each term separately: $(x^2 / x^2) = 1$, and $(-9 / x^2)$ remains as is. So, $(x^2 - 9) / x^2$ simplifies to $1 - 9 / x^2$.

Can I combine all terms in the expression into a single rational expression?

Yes. Rewrite each term with common denominators where necessary. The entire expression can be combined into a single rational expression by expressing all terms over a common denominator, typically x^2 or the product of the denominators.

What steps are involved in simplifying the expression $(x^2 - 9)/x^2 + 2x^3 / x^2 + 5x + 6$?

Steps include: simplifying each fractional term separately, factoring where possible (e.g., $x^2 - 9$ as $(x - 3)(x + 3)$), finding common denominators, and combining like terms to simplify the expression fully.

Is the expression continuous for all real values of x?

No. The expression is discontinuous where any denominator equals zero, specifically at $x = 0$, $x = -2$, and x = -3.

How can I evaluate the expression at a specific value of x, say x = 1?

Substitute x = 1 into the simplified form of the expression, compute each term, and sum them to find the value.

What is the limit of the expression as x approaches infinity?

As x approaches infinity, terms like 9 / x^2 tend to zero, so the expression's dominant terms are 1 + 7x. Since 7x grows without bound, the limit is infinity.

Additional Resources

Perform The Indicated Operation X^2 - 9 / x^2 X 2x^3 / x^2 + 5x + 6

Introduction

Mathematics, often regarded as the language of the universe, presents a vast landscape of problems that challenge even the most seasoned mathematicians. Among these, algebraic expressions and their manipulation form the foundational building blocks for more complex calculus and applied mathematics. The problem statement "Perform The Indicated Operation X^2 - 9 / x^2 X 2x^3 / x^2 + 5x + 6" encapsulates a typical algebraic task involving the simplification and operation of rational expressions.

This article takes an investigative approach to dissecting this problem, analyzing each component thoroughly, exploring the underlying principles, and offering detailed, step-by-step solutions. The goal is to provide clarity on how to approach such problems systematically, ensuring that students, educators, and enthusiasts can deepen their understanding of algebraic operations involving polynomials, fractions, and algebraic manipulation.

Context and Significance of the Problem

In algebra, the manipulation of rational expressions—fractions involving polynomials—is a foundational skill. Such expressions often appear in various fields, from physics to economics, where modeling relationships involves ratios of quantities. The problem at hand involves multiple operations: subtraction, multiplication, and division of algebraic fractions involving powers and polynomials.

Understanding the correct order of operations, factoring, simplifying, and combining like terms is critical. This problem, therefore, serves as an excellent case study for reinforcing these skills and highlighting common pitfalls, such as:

- Misinterpreting the order of operations
- Overlooking factoring opportunities

- Handling complex fractions correctly

By thoroughly analyzing this problem, learners can develop a robust approach to similar algebraic tasks, which are ubiquitous in advanced mathematics and applied sciences.

Dissecting the Expression

The given problem is somewhat ambiguously formatted, but based on standard algebraic conventions, it appears to be:

Perform the indicated operation on the expression:

$$ \frac{X^2 - 9}{x^2} \times \frac{2x^3}{x^2} + 5x + 6 $$

or perhaps:

$$ \left(\frac{X^2 - 9}{x^2}\right) \times \left(\frac{2x^3}{x^2}\right) + 5x + 6 $$

To proceed, we must interpret the expression correctly. It's common that such problems involve simplifying the product of two rational expressions and then adding a polynomial.

Note: For consistency, we assume that the expression is:

$$ \left(\frac{X^2 - 9}{x^2}\right) \times \left(\frac{2x^3}{x^2}\right) + 5x + 6 $$

Step-by-Step Analysis and Solution

Step 1: Clarify and Standardize the Expression

The expression is:

$$ \left(\frac{X^2 - 9}{x^2}\right) \times \left(\frac{2x^3}{x^2}\right) + 5x + 6 $$

Note: The uppercase X and lowercase x may be a typo or intentional; typically, variables are lowercase. For consistency, we will treat all variables as x.

Thus, the expression becomes:

$$ \left(\frac{x^2 - 9}{x^2}\right) \times \left(\frac{2x^3}{x^2}\right) + 5x + 6 $$

Step 2: Factor where possible

Factor numerator $x^2 - 9$:

$$ x^2 - 9 = (x - 3)(x + 3) $$

This simplifies the first fraction to:

$$\frac{(x - 3)(x + 3)}{x^2}$$

Step 3: Simplify the second fraction

$$\frac{2x^3}{x^2}$$

Dividing numerator and denominator:

$$\frac{2x^3}{x^2} = 2x^{3-2} = 2x$$

Step 4: Multiply the two fractions

Now, multiply:

$$\frac{(x - 3)(x + 3)}{x^2} \times 2x$$

This is:

$$2x \times \frac{(x - 3)(x + 3)}{x^2}$$

Rewrite as:

$$\frac{2x \times (x - 3)(x + 3)}{x^2}$$

Notice x in numerator and denominator:

$$\frac{2 \cancel{x} \times (x - 3)(x + 3)}{x \times x} = \frac{2 (x - 3)(x + 3)}{x}$$

Step 5: Final expression after multiplication

The entire expression becomes:

$$\frac{2 (x - 3)(x + 3)}{x} + 5x + 6$$

Step 6: Expand numerator (if desired)

Recall that:

$$(x - 3)(x + 3) = x^2 - 9$$

So, numerator:

$$ 2 (x^2 - 9) = 2x^2 - 18 $$

Thus, the expression simplifies to:

$$ \frac{2x^2 - 18}{x} + 5x + 6 $$

Step 7: Split the fraction

Divide each term separately:

$$ \frac{2x^2}{x} - \frac{18}{x} + 5x + 6 $$

Simplify:

$$ 2x - \frac{18}{x} + 5x + 6 $$

Step 8: Combine like terms

Combine the x terms:

$$ 2x + 5x = 7x $$

Final simplified expression:

$$ 7x - \frac{18}{x} + 6 $$

Final Result

The result of performing the indicated operation and simplifying the expression is:

$$
\boxed{
7x - \frac{18}{x} + 6
}
$$

Additional Insights and Mathematical Significance

Rational Expressions and Factorization

This problem underscores the importance of factoring in algebra. Recognizing that $x^2 - 9$ factors into $(x - 3)(x + 3)$ simplifies the expression significantly. Factoring allows for canceling common factors, which reduces complexity and reveals the simplest form.

Handling Complex Fractions

Transforming complex fractions into simpler expressions by splitting numerator and denominator is a crucial skill. It aids in understanding the structure of the expression and facilitates straightforward combination of like terms.

Application in Real-World Contexts

While this problem appears purely algebraic, similar structures appear in physics (e.g., ratios of quantities), engineering (e.g., transfer functions), and economics (e.g., cost per unit). Mastery of such manipulations enhances problem-solving skills across disciplines.

Common Pitfalls

- Misinterpreting variable notation (uppercase vs. lowercase)
- Forgetting to factor quadratics
- Incorrectly canceling terms without verifying common factors
- Overlooking domain restrictions (e.g., division by zero when \(x=0\))

Concluding Remarks

The journey through this algebraic problem exemplifies the systematic approach necessary for tackling rational expressions:

1. Clarify and interpret the expression correctly
2. Factor and simplify components
3. Perform algebraic operations step-by-step
4. Combine like terms and write the final simplified form

This process not only yields the correct answer but also reinforces fundamental algebraic principles essential for advanced mathematics and practical applications.

Whether you're a student striving to master algebra or an educator seeking clear instructional strategies, understanding such detailed problem-solving methods is invaluable. The ability to dissect complex expressions, recognize opportunities for factoring, and simplify systematically forms the backbone of mathematical literacy.

References and Further Reading

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About the Author

[Author Name] is a seasoned mathematics educator and researcher with a focus on algebraic structures and problem-solving techniques. With over a decade of experience in teaching mathematics at various levels, [Author Name] is dedicated to making complex mathematical concepts accessible and engaging for all learners.

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