Please Help: Determine The General Solution Of 3sinx +cosx-5=7sinx

Please Help: Determine The General Solution Of 3sinx +cosx-5=7sinx

---

Introduction

Solving trigonometric equations is a fundamental aspect of mathematics, especially in fields like engineering, physics, and applied sciences. These equations often involve multiple trigonometric functions with varying coefficients, making their solutions both interesting and challenging. One common goal is to find the general solution—the set of all possible solutions—that satisfy a given equation for all values of x.

In this article, we will focus on solving the specific trigonometric equation:

$$
3\sin x + \cos x - 5 = 7 \sin x
$$

Our aim is to determine the general solution to this equation, which involves simplifying, manipulating, and applying trigonometric identities. We will walk through each step in detail, providing insights into the methods used, and ensuring the solution is comprehensive and easy to understand.

Understanding the Equation

The equation provided is:

$$
3\sin x + \cos x - 5 = 7 \sin x
$$

Before attempting to solve, it’s crucial to analyze and simplify it:

- The equation involves both $\sin x$ and $\cos x$.
- The $\sin x$ term appears on both sides.
- The constant term -5 is present on the left side.

The goal is to isolate the trigonometric functions and express the equation in a manageable form.

Simplification of the Equation

Step 1: Bring all terms to one side

Subtract $7 \sin x$ from both sides to group similar terms:

$$
3 \sin x + \cos x - 5 - 7 \sin x = 0
$$

Simplify the $\sin x$ terms:

$$
(3 \sin x - 7 \sin x) + \cos x - 5 = 0
$$
$$
-4 \sin x + \cos x - 5 = 0
$$

Rearranged:

$$
-4 \sin x + \cos x = 5
$$

This is a linear combination of sine and cosine functions, which can be represented as a single sinusoidal function using a suitable method.

Step 2: Express as a single sinusoid

The general form for an expression involving sine and cosine is:

$$
a \sin x + b \cos x = R \sin (x + \alpha)
$$

where:

$$
R = \sqrt{a^2 + b^2}
$$

and

$$
\alpha = \arctan \left(\frac{b}{a} \right)
$$

or, depending on signs, sometimes expressed with cosine.

In our case, the coefficients are:

- $a = -4$
- $b = 1$

Calculate R:

$$
R = \sqrt{(-4)^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17}
$$

Determine α:

$$
\alpha = \arctan \left(\frac{b}{a} \right) = \arctan \left(\frac{1}{-4} \right)
$$

Since $a = -4$ (negative) and $b=1$ (positive), α will be in the second or third quadrant. For simplicity, we will find the principal value:

$$
\alpha = \arctan \left(-\frac{1}{4} \right) \approx -0.245 \text{ radians}
$$

But to express the original equation correctly, we need to consider the phase shift:

$$
a \sin x + b \cos x = R \sin (x + \phi)
$$

where

$$
\phi = \arctan \left(\frac{b}{a} \right)
$$

and the sign conventions are important.

Alternatively, it's often easier to write:

$$
a \sin x + b \cos x = R \sin (x + \theta)
$$

with

$$
\theta = \arctan \left(\frac{b}{a} \right)
$$

but considering the signs:

- Since $a = -4$, $b=1$,
- The angle θ satisfies:

$$
\sin \theta = \frac{b}{R} = \frac{1}{\sqrt{17}}
$$
$$
\cos \theta = \frac{a}{R} = \frac{-4}{\sqrt{17}}
$$

This helps us write:

$$
-4 \sin x + \cos x = R \sin (x + \theta)
$$

where:

$$
\sin \theta = \frac{1}{\sqrt{17}}, \quad \cos \theta = - \frac{4}{\sqrt{17}}
$$

Rewriting the Equation

The previous step allows us to rewrite:

$$
-4 \sin x + \cos x = \sqrt{17} \sin (x + \theta)
$$

Thus, the original simplified equation:

$$
-4 \sin x + \cos x = 5
$$

becomes:

$$
\sqrt{17} \sin (x + \theta) = 5
$$

Solving for x

Step 1: Isolate $\sin (x + \theta)$

Divide both sides by $\sqrt{17}$:

$$
\sin (x + \theta) = \frac{5}{\sqrt{17}}
$$

Check if the right side is within the range of sine function:

$$
\left| \frac{5}{\sqrt{17}} \right| \leq 1
$$

Calculate $\sqrt{17}$:

$$
\sqrt{17} \approx 4.1231
$$

Therefore:

$$
\frac{5}{4.1231} \approx 1.213
$$

Since $1.213 > 1$, the sine value exceeds the maximum possible value of $\sin$ (which is 1). This indicates no real solutions exist because the equation asks for a sine value outside the range $[-1, 1]$.

Conclusion on the Solutions

Because $\sin (x + \theta) = \frac{5}{\sqrt{17}}$ exceeds 1, there are no real solutions to the original equation.

Summary of Results

- The equation simplifies to a form involving a single sinusoid.
- The maximum value of the sine function in this context exceeds 1, which is impossible.
- Therefore, the original trigonometric equation has no real solutions.

Additional Insights and Implications

When do equations have solutions?

Trigonometric equations like:

$$

$$
a \sin x + b \cos x = c
$$

have solutions if and only if:

$$
|c| \leq R = \sqrt{a^2 + b^2}
$$

This is known as the range condition for the linear combination of sine and cosine.

Practical applications

Understanding the conditions for solutions can help in various applications such as:

- Signal processing
- Mechanical vibrations
- Electrical engineering
- Acoustics

Knowing whether an equation admits solutions is crucial in modeling real-world phenomena accurately.

Summary of Step-by-Step Methodology

1. Rewrite the equation to bring all terms to one side.
2. Simplify to a linear combination of sine and cosine.
3. Compute the amplitude R to understand the maximum and minimum possible values.
4. Express the combination as a single sinusoid with a phase shift.
5. Evaluate the feasibility by checking if the right side falls within $[-R, R]$.
6. Conclude whether solutions exist based on the comparison.
7. If solutions exist, find the explicit values of x using inverse sine functions and general solutions involving periodicity.

Final Remarks

In this particular problem, the key insight was recognizing the range condition for the combined sine and cosine function. The solution process demonstrated how to simplify complex trigonometric equations using identities and how to interpret the results in the context of real solutions.

While no real solutions exist for this specific equation, the methodology outlined here applies broadly to similar problems. Whether tackling equations with different coefficients or more complex forms, understanding the range and the combination of sine and cosine functions remains fundamental.

Keywords for SEO Optimization

- Trigonometric equations
- General solution of sine and cosine equations
- Solving $3 \sin x + \cos x - 5 = 7 \sin x$
- Range of sinusoidal functions
- Equations with no real solutions
- Simplifying linear combinations of sine and cosine
- How to solve trigonometric equations
- Trigonometric identities
- Range condition for sine and cosine equations
- Step-by-step trigonometric solution methods
- Applications of trigonometric equations in engineering and physics

---

Conclusion

The process of solving $3 \sin x + \cos x - 5 = 7 \sin x$ reveals important principles about the behavior and limitations of trigonometric functions. Recognizing the maximum possible values helps determine whether solutions exist. In this case, the equation has no real solutions

Frequently Asked Questions

How do I find the general solution for the equation 3sinx + cosx - 5 = 7sinx?

First, rewrite the equation as 3sinx - 7sinx + cosx - 5 = 0, simplifying to -4sinx + cosx = 5. Then, express it in the form Rsin(x + α) = D by calculating R = √((-4)^2 + 1^2) = √17 and α = arctangent(1 / -4). The general solution is x + α = arcsin(5 / R) + 2πn or π - arcsin(5 / R) + 2πn, where n is any integer.

What steps are involved in solving the trigonometric equation 3sinx + cosx - 5 = 7sinx?

The steps include bringing all sine terms to one side to combine like terms, simplifying the equation to a single sinusoidal form, calculating the amplitude R and phase shift α, verifying the value inside the arcsin is within [-1, 1], and then writing the general solutions using the inverse sine function plus multiples of 2π.

Can I use the method of combining sine and cosine into a single sinusoid to solve 3sinx + cosx - 5 = 7sinx?

Yes, rewriting the combined sine and cosine terms as a single sinusoid $R\sin(x + \alpha)$ simplifies the equation. This approach helps identify the solutions more straightforwardly by solving for x after determining R and α.

What is the importance of checking whether the value inside arcsin is within [-1, 1] when solving 3sinx + cosx - 5 = 7sinx?

Because the inverse sine function (arcsin) is only defined for inputs within [-1, 1], verifying that $5 / R$ (or the relevant value) falls within this range ensures real solutions exist. If not, the equation has no real solutions.

How do I express the general solution after rewriting 3sinx + cosx - 5 = 7sinx into a single sinusoid form?

Once in the form $R\sin(x + \alpha) = D$, the general solutions are $x = \arcsin(D / R) - \alpha + 2\pi n$ and $x = \pi - \arcsin(D / R) - \alpha + 2\pi n$, where n is any integer. These encompass all possible solutions to the original equation.

Additional Resources

Understanding and Solving the Equation: 3sinx + cosx - 5 = 7sinx

When approaching trigonometric equations such as 3sinx + cosx - 5 = 7sinx, it’s essential to understand the fundamental principles involved, the methods of solution, and the interpretation of results. This comprehensive review aims to guide you through the process of determining the general solution step-by-step, providing a deep understanding of the underlying concepts and techniques.

Introduction to the Problem

The given equation:

$$3\sin x + \cos x - 5 = 7 \sin x$$

combines sine and cosine functions, which are periodic and fundamental in mathematics, physics, engineering, and many applied sciences. The goal is to find all solutions x that satisfy this equality, considering the periodic nature of the trigonometric functions.

Initial Observations and Simplification

Before delving into solution methods, it’s prudent to analyze the structure of the equation:

$$ 3\sin x + \cos x - 5 = 7 \sin x $$

Key observations:

- The equation contains both $\sin x$ and $\cos x$, but they appear in a linear combination.
- The right side involves only $\sin x$, which suggests consolidating all $\sin x$ terms on one side.
- The constant term -5 indicates the equation is non-homogeneous.

Step 1: Bring all terms to one side:

$$
3 \sin x + \cos x - 5 - 7 \sin x = 0
$$

Step 2: Combine like terms:

$$
(3 \sin x - 7 \sin x) + \cos x - 5 = 0
$$
$$
-4 \sin x + \cos x - 5 = 0
$$

Rearranged:

$$
-4 \sin x + \cos x = 5
$$

This simplified form makes it easier to analyze the equation’s structure.

Rewriting the Equation in a Standard Form

The equation:

$$
-4 \sin x + \cos x = 5
$$

can be viewed as a linear combination of sine and cosine functions:

$$
A \sin x + B \cos x = C
$$

where:

- $A = -4$,
- $B = 1$,
- $C = 5$.

This form is well-understood and has standard solution techniques involving the use of the auxiliary angle method or the amplitude-phase form.

Method 1: Auxiliary Angle Technique

The auxiliary angle method transforms the linear combination $A \sin x + B \cos x$ into a single sinusoid:

$$
R \sin (x + \alpha)
$$

or

$$
R \cos (x - \beta)
$$

where:

$$
R = \sqrt{A^2 + B^2}
$$

and α, β are angles satisfying:

$$
\sin \alpha = \frac{B}{R}, \quad \cos \alpha = \frac{A}{R}
$$

Step 1: Calculate R:

$$
R = \sqrt{(-4)^2 + 1^2} = \sqrt{16 + 1} = \sqrt{17}
$$

Step 2: Find α:

$$
\sin \alpha = \frac{B}{R} = \frac{1}{\sqrt{17}}
$$
$$
\cos \alpha = \frac{A}{R} = \frac{-4}{\sqrt{17}}
$$

Since $\sin \alpha > 0$ and $\cos \alpha < 0$, α lies in the second quadrant:

$$
\alpha = \arcsin \left(\frac{1}{\sqrt{17}} \right)
$$

or equivalently,

$$
\alpha = \pi - \arcsin \left(\frac{1}{\sqrt{17}} \right)
$$

Step 3: Rewrite the original equation:

$$
A \sin x + B \cos x = R \sin (x + \alpha)
$$

which gives:

$$
\sqrt{17} \sin (x + \alpha) = C
$$

or

$$
\sin (x + \alpha) = \frac{C}{\sqrt{17}} = \frac{5}{\sqrt{17}}
$$

Method 2: Solving for x

From the previous step, the problem reduces to solving:

$$
\sin (x + \alpha) = \frac{5}{\sqrt{17}}
$$

Step 1: Check whether the right side’s magnitude is within the sine function’s range:

$$
-1 \leq \frac{5}{\sqrt{17}} \leq 1
$$

Calculate the numerical value:

$$
\sqrt{17} \approx 4.1231
$$
$$
\frac{5}{4.1231} \approx 1.213 \approx 1.213
$$

Since $1.213 > 1$, the sine value exceeds the maximum possible sine value, which indicates no real solutions exist for the original equation.

Implications of the Result

The critical observation from the above is that the sine of any real angle cannot be greater than 1 in absolute value. Because the calculated value:

$$
\frac{5}{\sqrt{17}} \approx 1.213
$$

exceeds 1, the equation:

$$
\sin (x + \alpha) = \frac{5}{\sqrt{17}}
$$

has no real solutions.

Therefore, the original equation has no solutions in the real domain.

Summary of Solution Process

1. Initial Simplification:
- The given equation was simplified to a linear combination of sine and cosine.
2. Transformation to Standard Form:
- Recognized as $A \sin x + B \cos x = C$.
3. Amplitude-Phase Conversion:
- Calculated $R = \sqrt{A^2 + B^2} = \sqrt{17}$.
- Found α satisfying $\sin \alpha = B/R$ and $\cos \alpha = A/R$.
4. Range Check:
- Determined if $\frac{C}{R}$ lies within $[-1, 1]$.
- Found $\frac{5}{\sqrt{17}} > 1$, which is outside the permissible range.

Conclusion:

- Since $\sin (x + \alpha) = \text{value} > 1$, no solutions exist in the real numbers.
- The equation has no real solution.

Understanding the Broader Context

This example underscores an important aspect of solving trigonometric equations: always check the range of the sine or cosine functions after algebraic manipulations. Equations that seem algebraically solvable can sometimes be invalidated due to range restrictions.

In general:

- When solving $A \sin x + B \cos x = C$, the solutions exist only if $|C| \leq R$, where $R = \sqrt{A^2 + B^2}$.
- If $|C| > R$, the equation has no real solutions.

This principle applies broadly across trigonometric equations and is essential for verifying the feasibility of solutions before solving explicitly.

Extensions and Additional Considerations

While this specific problem yields no real solutions, similar equations can have solutions depending on the values of parameters:

- If $|C| \leq R$:
- Solutions are given by:

$$
x + \alpha = \arcsin \left(\frac{C}{R} \right) + 2k\pi
$$
or
$$
x + \alpha = \pi - \arcsin \left(\frac{C}{R} \right) + 2k\pi
$$

- Then, solving for x:

$$
x = -\alpha + \arcsin \left(\frac{C}{R} \right) + 2k\pi
$$
or
$$
x = -\alpha + \pi - \arcsin \left(\frac{C}{R} \right) + 2k\pi
$$

- The solutions are general solutions because of the periodicity of sine.

- In cases where $|C| > R$: no real solutions, but complex solutions may exist involving complex logarithms or exponential forms.

Conclusion and Final Remarks

The process of solving the equation 3sinx + cosx - 5 = 7sinx exemplifies fundamental principles in trigonometry:

- Simplification of equations involving multiple trigonometric functions.
- Conversion to a single sinusoid using the

Please Help Determine The General Solution Of 3sinx Cosx 5 7sinx

Please Help Determine The General Solution Of 3sinx Cosx 5 7sinx

Related Articles

- [What Is Similar About Igneous Rock And Metamorphic Rock Formation?]
- [Given The Triangle As Marked Below, Find The Measure Of Each Angle.]
- [What Props Are Used In Midsummer Night Dream? (Shakespeare Play)]