# Precalculus. Please Show All Your Work And Answer All The Questions

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Precalculus is a foundational branch of mathematics that prepares students for the study of calculus. It encompasses a broad spectrum of topics including algebra, functions, trigonometry, and analytic geometry. Mastering precalculus is essential for students pursuing degrees in science, technology, engineering, and mathematics (STEM). In this comprehensive guide, we will explore key concepts, provide step-by-step solutions to typical problems, and answer common questions related to precalculus.

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## Understanding the Scope of Precalculus

Precalculus acts as a bridge between algebra and calculus, equipping students with the necessary skills to analyze functions, understand limits, and model real-world phenomena mathematically.

### Core Topics in Precalculus

- Functions and their graphs
- Polynomial and rational functions
- Exponential and logarithmic functions
- Trigonometry (angles, identities, equations)
- Analytic geometry (conic sections, coordinate systems)
- Sequences and series
- Limits and introductory calculus concepts

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# Functions and Their Graphs

Functions are the backbone of precalculus. Understanding their properties and how to graph them is critical.

## What Is a Function?

A function $f$ assigns exactly one output $f(x)$ for each input $x$. It is often represented as $f: X \to Y$, where $X$ is the domain and $Y$ is the codomain.

## Types of Functions

1. Linear functions: $f(x) = mx + b$
2. Quadratic functions: $f(x) = ax^2 + bx + c$
3. Polynomial functions: degree 3 or higher
4. Rational functions: ratios of polynomials
5. Exponential functions: $f(x) = a^x$
6. Logarithmic functions: $f(x) = \log_a x$
7. Trigonometric functions: sine, cosine, tangent, etc.

## Graphing Functions Step-by-Step

To graph a function:

1. Identify the domain and range
2. Find key points (intercepts, maxima, minima)
3. Determine symmetry (even, odd functions)
4. Plot points and sketch the curve considering asymptotes and end behavior

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# Polynomial and Rational Functions

Polynomial functions are expressions involving variables raised to whole number powers, while rational functions are ratios of such polynomials.

## Analyzing Polynomial Functions

Key steps include:

- Factoring to find roots
- Determining end behavior using leading term
- Finding intercepts and turning points

## Example Problem: Polynomial Function

Suppose $f(x) = 2x^3 - 3x^2 - 12x + 5$

Question: Find the roots of $f(x)$.

Solution:

1. Attempt Rational Root Theorem:

Possible roots are factors of constant term over factors of leading coefficient:

$$
\pm \frac{1, 5}{1, 2} = \pm 1, \pm 5, \pm \frac{1}{2}, \pm \frac{5}{2}
$$

2. Test $x=1$:

$$
f(1) = 2(1)^3 - 3(1)^2 - 12(1) + 5 = 2 - 3 - 12 + 5 = -8 \neq 0

\]

3. Test $x=-1$:

$$
f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 5 = -2 - 3 + 12 + 5 = 12 \neq 0
$$

4. Test $x=5$:

$$
f(5) = 2(125) - 3(25) - 12(5) + 5 = 250 - 75 - 60 + 5 = 120 \neq 0
$$

5. Test $x=-5$:

$$
f(-5) = 2(-125) - 3(25) - 12(-5) + 5 = -250 - 75 + 60 + 5 = -260 \neq 0
$$

6. Test $x=\frac{1}{2}$:

$$
f\left(\frac{1}{2}\right) = 2 \left(\frac{1}{2}\right)^3 - 3 \left(\frac{1}{2}\right)^2 - 12 \left(\frac{1}{2}\right) + 5 = 2 \times \frac{1}{8} - 3 \times \frac{1}{4} - 6 + 5 = \frac{1}{4} - \frac{3}{4} - 6 + 5 = - \frac{1}{2} - 1 = -1.5 \neq 0
$$

7. Test $x=-\frac{1}{2}$:

$$
f\left(-\frac{1}{2}\right) = 2 \left(-\frac{1}{2}\right)^3 - 3 \left(-\frac{1}{2}\right)^2 - 12 \left(-\frac{1}{2}\right) + 5
$$
$$
= 2 \times -\frac{1}{8} - 3 \times \frac{1}{4} + 6 + 5 = -\frac{1}{4} - \frac{3}{4} + 6 + 5 = -1 + 11 = 10 \neq 0
$$

Since none of these are roots, we may need to use synthetic division or numerical methods.

Conclusion: The roots are irrational or complex; further methods like graphing or the quadratic formula (after factoring out known quadratics) can be used.

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# Exponential and Logarithmic Functions

These functions model growth and decay processes and are inverse functions of each other.

## Properties of Exponential Functions

- Domain: \(\mathbb{R}\)
- Range: \((0, \infty)\)
- Growth/Decay depending on base \(a\): \(a>1\) (growth), \(0