Select The Correct Answer. Solve The Following Inequality For X.

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Understanding how to solve inequalities for a variable, such as x, is a fundamental skill in algebra that forms the basis for more advanced mathematical concepts. Inequalities express a relationship where one quantity is greater than, less than, or equal to another. Solving these inequalities for x allows us to find the range of values that satisfy the given conditions. This article provides a comprehensive guide to solving inequalities for x, including step-by-step methods, tips, and practice examples to help you master this essential skill.

What Is an Inequality?

An inequality is a mathematical statement that compares two expressions and indicates their relationship using symbols such as:

- Greater than: >
- Less than: <
- Greater than or equal to: ≥
- Less than or equal to: ≤
- Not equal to: ≠

Examples of inequalities:

- 3x + 5 > 11
- -2x ≤ 8
- x - 4 ≥ 0

The goal when solving inequalities is to find all possible values of x that make the inequality true.

Basics of Solving Inequalities

Before diving into complex problems, it's important to understand some fundamental principles:

1. Treat Inequalities Like Equations

Most steps used to solve equations are similar when solving inequalities. However, special rules apply when multiplying or dividing by negative numbers.

2. Remember to Reverse the Inequality When Multiplying or Dividing by a Negative Number

If you multiply or divide both sides of an inequality by a negative number, you must reverse the inequality sign.

Example:

- Starting with: $-2x > 6$
- Divide both sides by -2: $x < -3$ (Note the reversal of the $>$ to $<$)

Step-by-Step Guide to Solving Inequalities for X

Let's explore a systematic approach to solving inequalities.

Step 1: Simplify Both Sides

- Distribute any factors
- Combine like terms
- Simplify expressions as needed

Step 2: Isolate the Variable Term

- Use addition or subtraction to get all x terms on one side
- Keep track of the inequality sign

Step 3: Eliminate Coefficients of X

- Divide or multiply both sides by the coefficient of x to solve for x
- Remember to flip the inequality sign if multiplying/dividing by a negative number

Step 4: Write the Solution Set

- Express the solution as an inequality or interval
- Use interval notation or set builder notation as appropriate

Common Types of Inequalities and How to Solve Them

Understanding different types of inequalities helps in applying the correct methods.

1. Linear Inequalities

These are inequalities where the variable x appears to the first power.

Example:

- $2x + 3 > 7$

Solution:

1. Subtract 3 from both sides: $2x > 4$
2. Divide both sides by 2: $x > 2$

Solution set: $x > 2$

2. Compound Inequalities

These involve two inequalities combined, such as:

- $1 < 2x + 3 \leq 7$

Solution:

1. Subtract 3 from all parts: $-2 < 2x \leq 4$
2. Divide all parts by 2: $-1 < x \leq 2$

Solution set: $x \in (-1, 2]$

3. Absolute Value Inequalities

These inequalities involve absolute value expressions, such as $|x - 3| < 4$.

Solution:

- Rewrite as a compound inequality: $-4 < x - 3 < 4$
- Add 3 to all parts: $-1 < x < 7$

Solution set: $x \in (-1, 7)$

Examples of Solving Inequalities for X

Let's look at some practice problems and their solutions.

Example 1: Solve 3x - 4 ≤ 8

Solution:

1. Add 4 to both sides: 3x ≤ 12
2. Divide both sides by 3: x ≤ 4

Answer: x ≤ 4

Example 2: Solve -5x + 2 > -13

Solution:

1. Subtract 2 from both sides: -5x > -15
2. Divide both sides by -5 (remember to flip the inequality): x < 3

Answer: x < 3

Example 3: Solve |2x - 5| ≥ 7

Solution:

- Rewrite as two inequalities:

a) 2x - 5 ≥ 7 → 2x ≥ 12 → x ≥ 6

b) 2x - 5 ≤ -7 → 2x ≤ -2 → x ≤ -1

Solution set: x ≤ -1 or x ≥ 6

Graphing the Solution Sets

Graphing inequalities visually helps in understanding the solution sets.

Steps to graph:

1. Draw a number line.
2. Mark the critical points from the solution.

3. Use open or closed circles depending on whether the inequality is strict (<, >) or inclusive (≤, ≥).
4. Shade the region that satisfies the inequality.

Example:

- For $x > 2$, draw an open circle at 2 and shade to the right.
- For $x \leq -1$, draw a closed circle at -1 and shade to the left.

Tips for Solving Inequalities

- Always perform operations equally on both sides.
- When multiplying or dividing by negative numbers, flip the inequality sign.
- Keep track of the solution type (strict or inclusive).
- Check your solution by substituting a number from the solution set into the original inequality.
- Practice with different types of inequalities to build confidence.

Practice Problems to Test Your Skills

1. Solve for x: $4x + 1 < 9$
2. Solve: $-3x + 4 \geq 7$
3. Solve: $|x - 2| < 5$
4. Solve: $2(3x - 4) > 8$
5. Solve: $-x/2 \leq 3$

Answers:

1. $x < 2$
2. $x \leq 1$
3. $-3 < x < 7$
4. $6x - 8 > 8 \rightarrow 6x > 16 \rightarrow x > 8/3$
5. $x \geq -6$

Conclusion

Mastering the skill of solving inequalities for x is crucial for progressing in algebra and higher mathematics. By understanding the fundamental principles, carefully following systematic steps, and practicing different types of inequalities, you can confidently tackle any inequality problem. Remember to be attentive to the rules regarding negative numbers, and always verify your solutions. With consistent practice, solving inequalities will become second nature, unlocking a deeper understanding of mathematical relationships and problem-solving skills.

Start practicing today and enhance your algebraic proficiency!

Frequently Asked Questions

What is the first step to solve the inequality 3x - 5 > 7?

Add 5 to both sides to get 3x > 12.

How do you isolate x in the inequality -2x ≤ 8?

Divide both sides by -2, and remember to reverse the inequality sign: x ≥ -4.

Which of the following options correctly solves the inequality 4x + 3 < 15?

Subtract 3 from both sides to get 4x < 12, then divide both sides by 4 to find x < 3.

Why must you reverse the inequality sign when dividing both sides by a negative number?

Because dividing by a negative flips the inequality sign to maintain a true statement.

Solve for x: -5x + 2 ≥ -8.

Subtract 2 from both sides: -5x ≥ -10, then divide both sides by -5 and reverse the sign: x ≤ 2.

What is the solution set for the inequality x/2 < 3?

Multiply both sides by 2 to get x < 6.

Additional Resources

Select The Correct Answer. Solve The Following Inequality For X

Understanding inequalities is a fundamental skill in mathematics that underpins numerous advanced concepts across algebra, calculus, and applied sciences. When faced with an inequality, the ability to select the correct answer and accurately solve for the variable—commonly denoted as x—is crucial for building confidence and proficiency in mathematical reasoning. In this comprehensive guide, we will explore the methodology behind solving inequalities, analyze various types of inequalities, and provide expert insights into selecting the correct solutions effectively.

Understanding Inequalities: The Foundation

Before delving into the process of solving inequalities, it’s essential to understand what inequalities

are and how they differ from equations.

What Are Inequalities?

An inequality is a mathematical statement that compares two expressions using inequality symbols such as:

- Less than (<)
- Greater than (>)
- Less than or equal to (≤)
- Greater than or equal to (≥)

For example:
- x + 3 > 7 indicates that the sum of x and 3 is greater than 7.
- 2x - 5 ≤ 9 indicates that twice x minus 5 is less than or equal to 9.

Unlike equations, inequalities do not have a single solution but rather a range or set of solutions that satisfy the condition.

The Importance of Solving Inequalities

Solving inequalities is vital in various contexts:
- Determining feasible regions in optimization problems.
- Establishing bounds within which variables operate.
- Analyzing real-world situations such as budgeting, engineering tolerances, and probability constraints.

Types of Inequalities and Their Characteristics

Different inequalities require different approaches. Recognizing their type is the first step toward solving them correctly.

Linear Inequalities

These involve variables to the first power only, such as:
- ax + b < c

Example:
Solve 3x - 4 ≥ 2

Solution Approach:
- Isolate x by adding/subtracting constants.
- Divide or multiply both sides by the coefficient of x.
- Remember to reverse the inequality sign if multiplying/dividing by a negative number.

Quadratic Inequalities

Involve x squared (or higher powers), e.g.,:
- $ax^2 + bx + c > 0$

Example:
Solve $x^2 - 5x + 6 < 0$

Solution Approach:
- Factor the quadratic (if possible).
- Find the roots (solutions to the related quadratic equation).
- Use the roots to determine intervals where the inequality holds, based on the parabola's opening direction.

Absolute Value Inequalities

Contain absolute value expressions like:
- $|x - a| < b$ or $|x - a| > b$

Example:
Solve $|2x + 3| \leq 7$

Solution Approach:
- Rewrite as a compound inequality.
- For $|A| \leq B$, solve $-B \leq A \leq B$.
- For $|A| \geq B$, solve $A \leq -B$ or $A \geq B$.

Step-by-Step Guide to Solving Inequalities for X

To accurately solve for x, follow a systematic process:

Step 1: Simplify the Inequality

- Combine like terms.
- Simplify expressions on both sides.
- Remove parentheses by distributing if necessary.

Step 2: Isolate the Variable Term

- Get all x terms on one side.
- Move constants to the other side.

Step 3: Solve for x
- Divide or multiply both sides by the coefficient of x.
- Critical: If multiplying or dividing by a negative number, reverse the inequality sign.

Step 4: Express the Solution Set

- Write the solution as an interval or union of intervals.
- Use set notation for clarity, e.g., $x \in (a, b)$.

Step 5: Verify the Solution

- Test points within the solution set to ensure they satisfy the original inequality.
- Check points outside to confirm they do not satisfy it.

Common Pitfalls and Expert Tips

While solving inequalities may appear straightforward, several common pitfalls can lead to errors. Here are expert insights to avoid them:

1. Forgetting to Reverse the Inequality Sign

- When to reverse: Multiplying or dividing both sides of the inequality by a negative number.

Tip: Always note the sign change rule and double-check after each step.

2. Ignoring the Domain Restrictions

- For inequalities involving square roots or denominators, account for restrictions like $x \neq 0$ (denominator) or the radicand ≥ 0 (square roots).

Tip: Clarify the domain before solving.

3. Overlooking Compound Inequalities

- When solving inequalities like $x > 2$ and $x < 5$, combine the intervals carefully.

Tip: Break down compound inequalities into manageable parts.

4. Misinterpreting Absolute Values

- Remember that $|A| < B$ translates into $-B < A < B$, while $|A| > B$ translates into $A < -B$ or $A > B$.

Tip: Rewrite absolute value inequalities explicitly to avoid misinterpretation.

Choosing the Correct Solution: Practical Strategies

Once the inequality is solved, the ultimate goal is to select the correct answer from multiple choices or determine the solution set accurately. Here are practical strategies:

1. Understand the Solution Set Representation

- Be familiar with interval notation:
- (a, b): open interval, excludes a and b.
- [a, b]: closed interval, includes a and b.
- (a, b] or [a, b): half-open intervals.

Tip: Visualize the solution on a number line to confirm interval correctness.

2. Check Critical Points

- Plug in test points from each interval into the original inequality.
- Confirm which intervals satisfy the inequality.

3. Cross-Verify with the Original Inequality

- After selecting the candidate answer, substitute a representative value to verify correctness.

4. Use Graphical Methods

- Graph the related equation or function.
- Identify regions where the graph satisfies the inequality.

Example:
For x^2 - 4 < 0, graph y = x^2 - 4 and find where y < 0.

Sample Problem and Its Solution

Problem:
Solve -2x + 3 > 7 for x and select the correct answer.

Step-by-Step Solution:

1. Simplify:
\(-2x + 3 > 7\)

2. Isolate x:
\(-2x > 7 - 3\)

\(-2x > 4\)

3. Divide by -2 (remember to reverse the inequality):

\(x < \frac{4}{-2}\)

\(x < -2\)

4. Solution set:
- x belongs to \((-\infty, -2)\)

Check:
- Pick x = -3 (less than -2):

$-2(-3) + 3 = 6 + 3 = 9 > 7$

- Pick x = 0 (greater than -2):

$-2(0) + 3 = 3 > 7?$ No, 3 is not greater than 7, so x cannot be 0 or greater.

Answer:
- The correct solution is x < -2, represented as $(-\infty, -2)$.

Conclusion: Mastering Inequality Solutions for Success

Mastering the skill of solving inequalities for x is an essential component of mathematical literacy. It requires understanding different inequality types, following a structured approach, and applying strategic checks to ensure accuracy. By recognizing common errors, practicing with varied problems, and visualizing solutions graphically, students and professionals alike can develop confidence and precision in their problem-solving abilities.

Whether you are preparing for exams, tackling real-world problems, or advancing into higher mathematics, honing these skills will provide a solid foundation for analytical thinking and logical reasoning. Remember, the key to success lies in careful analysis, methodical execution, and verification at

every step.

Empowered with these insights, you are now equipped to select the correct answer and solve inequalities for x with expert precision.

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