Solve Each System Using Elimination. 4y - 2x = 6 , 8y = 4x - 12

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When it comes to solving systems of equations, the elimination method is a powerful technique that simplifies the process by removing one variable through addition or subtraction. In this article, we will explore how to effectively solve the system of equations given by:

- $4y - 2x = 6$
- $8y = 4x - 12$

By applying the elimination method, we can find the values of x and y that satisfy both equations simultaneously. This comprehensive guide will walk you through each step, providing clear explanations, strategies, and tips to master solving systems using elimination.

Understanding the System of Equations

Before diving into the elimination process, it's essential to understand what a system of equations is and how the elimination method works.

What Is a System of Equations?

A system of equations consists of two or more equations with the same set of variables. The goal is to find the values of these variables that satisfy all the equations at once. Systems can be:

- Consistent: Have at least one solution.
- Inconsistent: Have no solution.
- Dependent: Have infinitely many solutions.

The Elimination Method: An Overview

The elimination method involves manipulating the equations to cancel out one variable, making it easier to solve for the remaining variable. This is achieved by:

- Multiplying equations by suitable constants to align coefficients.
- Adding or subtracting equations to eliminate a variable.

- Solving the resulting equation for one variable.
- Substituting back to find the other variable.

Step-by-Step Solution of the System

Let’s now focus on solving the specific system:

- $4y - 2x = 6$ ...(Equation 1)
- $8y = 4x - 12$ ...(Equation 2)

Step 1: Rewrite Equations in Standard Form

For clarity, arrange both equations in the form $Ax + By = C$:

- Equation 1: $4y - 2x = 6$
Rewrite as: $-2x + 4y = 6$

- Equation 2: $8y = 4x - 12$
Rewrite as: $4x - 8y = 12$

This standard form makes it easier to identify coefficients for elimination.

Step 2: Align the Equations for Elimination

Observe the coefficients of x and y:

- First equation: $-2x + 4y = 6$
- Second equation: $4x - 8y = 12$

Notice that the coefficients of x are -2 and 4, and the coefficients of y are 4 and -8, which are multiples of each other. To eliminate one variable, the coefficients should be equal in magnitude.

Step 3: Make the Coefficients of a Variable Opposite

To eliminate x:

- Multiply Equation 1 by 2:

$2 (-2x + 4y) = 2\ 6$
Result: $-4x + 8y = 12$

- Now, compare with Equation 2:

4x - 8y = 12

Step 4: Add or Subtract Equations to Eliminate a Variable

Add the two equations:

(-4x + 8y) + (4x - 8y) = 12 + 12

Simplify:

-4x + 4x + 8y - 8y = 24

0x + 0y = 24

This simplifies to:

0 = 24

Step 5: Interpret the Result

The statement 0 = 24 is a contradiction, which indicates that the system has no solution. In other words, the equations are inconsistent and represent parallel lines that never intersect.

Conclusion: No Solution Scenario

Since the elimination process resulted in a contradiction, the system of equations:

- 4y - 2x = 6
- 8y = 4x - 12

has no solutions. This means there are no values of x and y that satisfy both equations simultaneously. The lines represented by these equations are parallel and do not intersect.

Additional Practice and Tips for Solving Systems Using Elimination

To deepen your understanding of solving systems via elimination, consider the following tips and practice steps.

Tips for Effective Elimination

- Always rewrite equations in standard form: Ensure variables are aligned, and constants are on the right.
- Make coefficients of one variable opposites: Multiply equations appropriately to align coefficients.
- Check for special cases: Systems may be dependent (infinite solutions) or inconsistent (no solution).
- Verify your solution: Substitute your solutions back into the original equations.

Practice Problems

1. Solve the system:

- $3x + 2y = 7$
- $6x - 4y = 2$

2. Solve the system:

- $x + y = 4$
- $2x + 2y = 8$

3. Solve the system:

- $5x - 3y = 10$
- $10x - 6y = 20$

Note: In the last set, observe whether the equations are multiples of each other, indicating dependent equations with infinitely many solutions.

Summary and Key Takeaways

- The elimination method simplifies solving systems by removing one variable through strategic addition or subtraction.

- Properly rewriting equations in standard form and aligning coefficients are critical steps.
- When elimination leads to a contradiction (like 0 = 24), the system has no solution.
- When equations are multiples of each other, they represent dependent systems with infinitely many solutions.
- Always verify your solutions by substituting back into original equations.

Final Thoughts

Mastering the elimination method is essential for solving linear systems efficiently. It’s widely applicable in algebra, calculus, and real-world problem-solving scenarios such as engineering, economics, and physics. Practice solving different types of systems to build confidence and enhance your problem-solving skills.

By understanding the principles demonstrated in solving the system:

- $4y - 2x = 6$
- $8y = 4x - 12$

you are better equipped to approach similar equations with confidence. Remember, the key is to manipulate equations carefully and analyze the results critically. With practice, elimination will become a straightforward and invaluable tool in your algebraic toolkit.

Frequently Asked Questions

What is the first step to solve the system 4y - 2x = 6 and 8y = 4x - 12 using elimination?

Rewrite both equations in standard form and align them to identify how to eliminate one variable, often by making the coefficients of either x or y equal in magnitude.

How can we eliminate one variable in the system 4y - 2x = 6 and 8y = 4x - 12?

Multiply the equations to make the coefficients of either x or y equal and then subtract or add the equations to eliminate that variable.

What is the value of y after applying the elimination method to the system?

After elimination, solving yields $y = 3$.

How do you find the value of x after solving for y in the system?

Substitute $y = 3$ into either original equation and solve for x, which gives $x = 0$.

Can the elimination method be used directly on the given equations without manipulation?

No, you often need to manipulate the equations—such as multiplying one or both equations—to align coefficients for elimination.

What are the steps to solve $4y - 2x = 6$ and $8y = 4x - 12$ using elimination?

First, rewrite both equations in standard form, then multiply to match coefficients, add or subtract to eliminate a variable, solve for the remaining variable, and back-substitute to find the other variable.

Is the system $4y - 2x = 6$ and $8y = 4x - 12$ consistent, and what is the solution?

Yes, the system is consistent, and the solution is $x = 0$, $y = 3$.

What should you check after solving the system using elimination?

Verify the solution by substituting the values back into the original equations to ensure both are satisfied.

What is the importance of aligning coefficients in the elimination method?

Aligning coefficients allows for straightforward addition or subtraction of equations to eliminate one variable efficiently.

Additional Resources

Solve Each System Using Elimination: A Comprehensive Guide

Mathematics, particularly algebra, provides foundational skills that are essential across numerous disciplines. Among these skills, solving systems of equations is a core concept that enables students and professionals alike to analyze real-world problems involving multiple variables. One of the most efficient and widely used methods for solving such systems is the elimination method. This article offers an in-depth exploration of solving systems using elimination, illustrated through the specific example:

- $4y - 2x = 6$
- $8y = 4x - 12$

We will examine the process step-by-step, analyze the rationale behind each move, and discuss key principles that make elimination a powerful technique.

Understanding the System of Equations

Before diving into the elimination process, it’s critical to understand the nature of the system at hand.

The Given Equations

The system provided is:

1. $4y - 2x = 6$
2. $8y = 4x - 12$

At first glance, these equations appear different, but they are interconnected. Recognizing their structure is vital to choosing the right solving method.

Rearranging for Clarity

To facilitate the elimination, it’s often helpful to write both equations in a similar form, ideally equal to zero or to a constant:

- Equation 1: $4y - 2x = 6$
- Equation 2: $8y - 4x = -12$ (by subtracting 4x from both sides of the second equation)

Rearranged as:

- 4y - 2x - 6 = 0
- 8y - 4x + 12 = 0

Alternatively, keeping them in the form with constants on the right:

- 4y - 2x = 6
- 8y = 4x - 12

The choice depends on the method; since elimination typically involves aligning coefficients, rewriting both equations in standard form (ax + by = c) is often the most straightforward.

Applying the Elimination Method Step-by-Step

The elimination method involves manipulating equations to eliminate one variable, thereby solving for the other. It’s particularly effective when the coefficients of a variable are opposites or can be made opposites through multiplication.

Step 1: Standardize the Equations

Express both equations in the standard form:

- Equation 1: 4y - 2x = 6
- Equation 2: 8y = 4x - 12 → 8y - 4x = -12

Now, the system is:

1. -2x + 4y = 6
2. -4x + 8y = -12

This standard form aligns variables systematically, making elimination more straightforward.

Step 2: Determine the Least Common Multiple (LCM) of Coefficients

To eliminate one variable, the coefficients of either x or y should be equal in magnitude and opposite in sign.

- Coefficients of x: -2 and -4
- Coefficients of y: 4 and 8

For the elimination of x:

- Multiply Equation 1 by 2 to match the coefficient of x in Equation 2:

- Equation 1 × 2: $(-2x \times 2) + (4y \times 2) = 6 \times 2$
- Simplifies to: $-4x + 8y = 12$

Now, this new equation and Equation 2 are:

- $(-4x + 8y) = 12$
- $(-4x + 8y) = -12$

Step 3: Subtract Equations to Eliminate x

Subtract the second equation from the first:

$$(-4x + 8y) - (-4x + 8y) = 12 - (-12)$$

Simplify:

$$-4x + 8y + 4x - 8y = 12 + 12$$

Result:

$$0 = 24$$

This is a contradiction—an impossible statement, indicating that the system has no solution.

Interpreting the Result: No Solution or Infinite Solutions?

The outcome of the elimination process is crucial. In this case, the result $0 = 24$ is a contradiction, which signifies inconsistent equations.

Implications of Contradiction

- The system has no solution: The lines represented by the equations are parallel and do not intersect.
- Geometric interpretation: The lines are parallel, with equal slopes but different intercepts.

Hence, the system is inconsistent, and the solution set is empty.

Alternative Approach: Substitution to Verify

While elimination revealed the inconsistency, substitution can serve as a verification tool.

- From Equation 2: $8y = 4x - 12 \rightarrow y = (4x - 12) / 8 = (x - 3) / 2$

Substitute into Equation 1:

$4 [(x - 3)/2] - 2x = 6$

Simplify:

$(4 (x - 3))/2 - 2x = 6$

$(2 (x - 3)) - 2x = 6$

$2x - 6 - 2x = 6$

$-6 = 6$

Again, a contradiction, confirming the absence of solutions.

Key Principles and Insights in Elimination

Understanding how and why elimination works is essential for mastering this technique.

Principle 1: Align Coefficients

To eliminate a variable, coefficients must be equal in magnitude and opposite in sign. Achieving this often involves multiplying equations by suitable constants.

Principle 2: Consistency Check

If elimination results in a false statement (like $0 = 24$), the system has no solutions. If it reduces to a tautology (like 0 = 0), infinitely many solutions exist.

Principle 3: Flexibility in Variable Choice

Choose the variable with the easiest coefficients to eliminate, often the one with the smallest coefficients or the one that simplifies calculations.

Principle 4: Geometric Interpretation

Solutions to systems of linear equations correspond to intersections of lines. Elimination helps find these intersection points efficiently or determine their absence.

Practical Applications of Solving Systems via Elimination

The elimination method isn’t merely an academic exercise; it has real-world applications:

- Engineering: Analyzing forces in structures where multiple equations model different components.
- Economics: Solving supply-demand models involving multiple variables.
- Computer Science: Algorithms for graphics rendering and data analysis.
- Science: Calculating reaction rates involving multiple interacting substances.

Conclusion

The process of solving systems of equations using elimination is a powerful algebraic tool that relies on strategic manipulation of equations to systematically remove variables. The example system:

- $4y - 2x = 6$
- $8y = 4x - 12$

demonstrates how elimination reveals the system’s inconsistency, with no possible solutions due to parallel lines in the geometric interpretation.

Mastering elimination requires understanding the principles of coefficient alignment, recognizing when a system is inconsistent or dependent, and applying logical steps diligently. Whether used in academic exercises or real-world problem-solving, elimination remains an essential technique in the algebra toolkit.

By practicing these methods on diverse systems, students and professionals can build confidence and competence in algebraic reasoning, ultimately enhancing their analytical skills across disciplines.

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